<u>Properties of the conjunction operators:</u> **($\wedge$) خواص أداة الوصل**

Let p, q and r are three propositions. Using the truth table, show that:

1. $p \wedge q = q \wedge p$ (commutative خاصية الإبدال)
2. $(p \wedge q) \wedge r = p \wedge (q \wedge r)$ (associative خاصية التجميع) **(H.W.)**
3. $p \wedge p = p$ (Idempotent lawقانون تساوي القوى)
4. $p \wedge T = p$ (Identity law)
5. $p \wedge F = F$ (Domination Law)
6. $p \wedge \sim p = F$

<u>Solution</u>:

1. $p \wedge q = q \wedge p$

p	q	$p \wedge q$	$q \wedge p$
T	F	**F**	**F**
T	T	**T**	**T**
F	T	**F**	**F**
F	F	**F**	**F**

3. $p \wedge p = p$

p	p	$p \wedge p$
T	T	**T**
F	F	**F**

4. $p \wedge F = F$

p	F	$p \wedge F$
T	F	**F**
F	F	**F**

<u>2. Disjunction operator</u> (أو) أداة الفصل--English word (**or**), symbol ($\vee$).

Let p and q be two propositions. The disjunction of p and q is denoted by ''$p \vee q$'' and read ''p or q''.

We say that ''$p \vee q$'' is true when p is true **or** q is true **or both** are true. If both p and q are false, then $p \vee q$ is false.

إذا كانت كل من p و q عبارتين بسيطة. تكون العبارة المركبة ''$p \vee q$'' المرتبطة بأداة الفصل ($\vee$) كاذبة إذا كانت كل من العبارة p و q عبارة كاذبة. وتكون العبارة

‘‘$p \vee q$’’ صادقة فيما عدا ذلك (أي إذا كانت إحدى العبارتين البسيطتين على الأقل صادقة).

Disjunction		
p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

The truth table for the disjunction of two propositions

<u>Example1.14</u>: (H. W) Let p, q and r are three propositions such that

p: dogs can fly

$q: x - x = 0,\ x \in \mathbb{R}$

$r: -3 \in N$

Find the truth value of the following statements:

a) $(p \vee q) \vee r$

b) $\sim q \vee r$

c) $\sim(\sim p \vee q)$

d) $(p \wedge q) \vee (q \vee r)$

<u>Solution of (c)</u>:

$\sim(\sim p \vee q) = \sim(T \vee T) = \sim T = F$

<u>Example1.15</u>: Let p and q are two primitive propositions such that

p: Today is Friday (T)

q: It is raining today (T)

What is the disjunction of the propositions p and q? Discuss the truth value of ‘‘$p \vee q$’’.

Solution: The disjunction ‘‘$p \vee q$’’ is

‘‘Today is Friday or it is raining today’’

‘‘$p \vee q$’’ means that today is **either** Friday **or** raining **or** both.

The compound proposition ($p \vee q$) is **false** if:

‘‘Today is not Friday or it is not raining today’’

The compound proposition ($p \vee q$) is **true** if:

‘‘Today is Friday or it is raining today’’

‘‘Today is not Friday or it is raining today’’

‘‘Today is Friday or it is not raining today’’

Properties of the disjunction operator: **(∨) خواص أداة الفصل**

Let p, q and r are three propositions. Using the **truth table**, show that:

1. $p \vee q = q \vee p$ (خاصية الإبدال) **(H. W)**
2. $(p \vee q) \vee r = p \vee (q \vee r)$ (خاصية التجميع)
3. $p \vee p = p$ (قانون تساوي القوى) **(H. W)**
4. $p \vee T = T$ (Domination Law) **(H. W)**
5. $p \vee F = p$ (Identity Law) **(H. W)**
6. $p \vee \sim p = T$ **(H. W)**

<u>Solution2</u>: $(p \lor q) \lor r = p \lor (q \lor \mathrm{r})$

p	q	r	$p \lor q$	$q \lor \mathrm{r}$	$(p \lor q) \lor r$	$p \lor (q \lor \mathrm{r})$
T	T	T	T	T	**T**	**T**
F	F	F	F	F	**F**	**F**
F	T	T	T	T	**T**	**T**
F	T	F	T	T	**T**	**T**
F	F	T	F	T	**T**	**T**
T	F	F	T	F	**T**	**T**
T	T	F	T	T	**T**	**T**
T	F	T	T	T	**T**	**T**

3. <u>Conditional operator</u> أداة الشرط: English word (**if…then**), Arabic word (**إذا كان فان**), symbol (→).

Let p and q be two propositions. The conditional statement ‘‘$p \rightarrow q$’’ is the proposition ‘‘if p then q’’. The conditional statement ‘‘$p \rightarrow q$’’ is **false** when p is true and q is false, otherwise ‘‘$p \rightarrow q$’’ is **true**.

إذا كانت كل من p و q عبارة بسيطة فان العبارة المركبة (if…then)

يرمز لها بالرمز (→).

تكون العبارة $(\text{if } p \text{ then } q)$ **كاذبة** في حالة واحدة فقط عندما تكون p عبارة صادقة و q عبارة كاذبة. تكون العبارة $(\text{if } p \text{ then } q)$ **صادقة** فيما عدا ذلك.

<u>ملاحظة</u>:إذا كانت p عبارة خاطئة فان العبارة ‘if p then q’ تكون غير قابلة للاختبار وبالتالي فأنها لايمكن أن تكون خاطئة.

The following is the truth table:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

<u>Remark</u>1.16: In the conditional statement ‘‘$p \to q$’’, p is called the **hypothesis** فرضية and q is called the **conclusion نتيجة** .

<u>Remark</u>1.17: The conditional statement can be expressed in the following equivalent ways:

a) ‘‘p implies q’’

b) ‘‘q if p’’ ,

c) ‘‘q only if p’’,

d) ‘‘p is sufficient condition for q’’,

e) ‘‘q is a necessary condition for p’’.

<u>Example1.18</u>: Find the truth value of the following statements:

1. If fish fly, then $3 + 2 = 5$

   $F \to T = T$

2. If fish walk, then $3 + 2 = 6$

   $F \to F = T$

Example1.19: Let p, q, and r are three propositions such that

p: 3 is an odd number

$q: x+y=y+x,\ x,y \in \mathbb{R}$

r: Winter is hot

Find the truth value of the following statements:

1) $(p \rightarrow q) \vee (r \rightarrow q)$ **(H. W)**
2) if $(p \wedge q)$ then $(q \vee \sim r)$
3) $(p \wedge r) \vee (q \rightarrow p)$ **(H. W)**

Solution2:

if $(p \wedge q)$ then $(q \vee \sim r)$

if $(T \wedge T)$ then $(T \vee T)$

if T then $T = T$

Example1.20: Find the truth value of the following statements:

1. The statement: ''**If** x is negative **then** $-5x$ is positive''

$$\text{T} \rightarrow T = T$$

2. The statement: "**If** 9 > 5 **then** dogs don't fly"

$$\text{T} \rightarrow T = T$$

3. The statement: ''**If** ($x > 0$ and $x^2 < 0$) **then** $x \geq 10$''

If (T and F**) then** $F(or\ T)$

If F **then** $F(or\ T) = T$

4. The statement: ''**If** $x > 0$ **then** ($x^2 < 0$ or $2x < 0$)''

$$\text{T} \rightarrow (F\ or\ F) = T \rightarrow F = F$$

Definition 1.21: Let p and q are two propositions, then

1. The proposition ‘‘$q \rightarrow p$’’ is called the **converse** of ‘‘$p \rightarrow q$’’.

   The proposition ‘‘$\sim p \rightarrow \sim q$’’ is called the **inverse** of ‘‘$p \rightarrow q$’’.

Example1.22: What is the converse and the inverse of the conditional statement:

‘‘**if** $x > 5, x \in N$ **then** $x > 3$’’?

What is the truth value of the statement and its inverse and converse?

Solution: The statement ‘‘**if** $x > 5$ **then** $x > 3$’’

The truth value: T → T= T

The **converse** is ‘‘**if** $x > 3$ **then** $x > 5$’’

The truth value: for $x = \{4,5\}$; T → F= F

For $x = \{6,7, \dots\}$, }; T → T= T

The **inverse** is **if** $x \leq 5$ **then** $x \leq 3$’’

The truth value: for $x = \{1,2,3\}$; T → T= T

For $x = \{4,5\}$, }; T → F= F

Properties of the conditional operator: **(→) خواص أداة الشرط**

Let p, q and r are three propositions. Using the **truth table** show that (**H. W.**)

1. $p \rightarrow q \neq q \rightarrow p$
2. $(p \rightarrow q) \rightarrow r \neq p \rightarrow (q \rightarrow r)$
3. **Find** the truth value of: $p \rightarrow T,\ p \rightarrow F,\ p \rightarrow \sim p,\ p \rightarrow p$

4. Bi-conditional operator أداة الشرط المزدوج: English word (if and only if)**,** Arabic word (إذا وفقط إذا), **symbol** (↔)

Let p and q be propositions. The *bi-conditional* statement ‘‘$p \leftrightarrow q$’’ is the proposition ‘‘p if and only if q’’. The bi-conditional statement is **true** when p and q have the same true value and is **false** otherwise.

لتكن كل من **p** و **q** عبارة بسيطة. تكون العبارة المركبة ‘‘**p** إذا وفقط إذا **q**’’ والتي يرمز لها بالرمز (↔) صادقة في حالة تشابه قيم صدق العبارتين وكاذبة فيما عدا ذلك.

T	T	T
T	F	F
F	T	F
F	F	T

The truth table for the bi-conditional of two propositions

Remark1.23: There are some other ways to express ‘‘$p \leftrightarrow q$’’:

‘‘p iff q’’

‘‘if p then q , and if q then p’’

‘‘p is necessary and sufficient for q’’.

Example1.24: Find the truth value of ‘‘$x > 0 \leftrightarrow 2x > 0$’’

Solution: The statement is true because

If $x > 0$ then $2x > 0$ and if $2x > 0$ then $x > 0$

Example1.25: Find the truth value of ‘‘$x > 0 \leftrightarrow x^2 > 0$’’ (**H. W.**)