جامعة بغداد

كلية التربية للعلوم الصرفة / ابن الهيثم

*** قسم الفيزياء ***

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محاضرات مادة الميكانيك التحليلي

المرحلة الثالثة

** النظري **

التدريسيين

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Chapter One
Basic principles
"Vectors"

* Notation :-

If you have two Vectores $\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$
$\vec{B} = \hat{i}B_x + \hat{j}B_y + \hat{k}B_z$

then

1- Equality of Vectores

$\vec{A} = \vec{B}$

$A_x = B_x$

$A_y = B_y$

$A_z = B_z$

2- Vector Addition

$\vec{A} + \vec{B} = [A_x + B_x , A_y + B_y , A_z + B_z]$

3- Multiplication by a scalar عدد

if c is a scalar and A a vector,

$\therefore c\vec{A} = c[A_x, A_y, A_z] = [cA_x, cA_y, cA_z] = Ac$

4- Vector subtraction :-

subtraction is defined as follows :-

$\vec{A} - \vec{B} = [A_x - B_x , A_y - B_y , A_z - B_z]$

5- The Commutative Law of Addition التبادل

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$

6- The Associative Law قانون التجميع

$A + (B + C) = (A + B) + C$

7 - The Distributive Law

$$c(\vec{A}+\vec{B}) = c\vec{A} + c\vec{B}$$

* Magnitude of a Vector :-

The Magnitude of Vector A, denoted by $|\vec{A}|$

$$|\vec{A}| = (A_x^2 + A_y^2 + A_z^2)^{1/2}$$

* Unit Coordinate Vectors :-

A unit vector is one whose Magnitude is Unity. The three unit Vectors

$\hat{i} = [1, 0, 0]$
$\hat{j} = [0, 1, 0]$
$k = [0, 0, 1]$

* The Scalar product :-

Given two Vectors $\vec{A}$ and $\vec{B}$, the scalar product or (dot) product, is the scalar defined by the equation :

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$
$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$\therefore \cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$
$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} , \hat{k} \cdot \hat{i} = \text{zero}$$

1. $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
2. $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
3. $\vec{A} \cdot \vec{A} = |\vec{A}|^2$

* If the dot product $\vec{A} \cdot \vec{B}$ is equal to zero, then $\vec{A}$ is perpendicular to $\vec{B}$, provided neither $\vec{A}$ nor B is null.

* if θ is the angle between $\vec{A}$ and $\vec{B}$ then the projection of $\vec{B}$ on $\vec{A}$ is just

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$\boxed{B \cos\theta = \frac{\vec{A} \cdot \vec{B}}{A}}$$

and the projection of $\vec{A}$ on $\vec{B}$

$$\vec{A} \cdot \vec{B} = AB \cos\theta \Rightarrow \boxed{A \cos\theta = \frac{\vec{A} \cdot \vec{B}}{B}}$$

* The Vector product :.

Given two Vectors , the Vector product or (cross product) is given by :.

$$\vec{A} \times \vec{B} = AB \sin\theta \; \hat{n}$$
$$\vec{A} \times \vec{B} = |\vec{A} \times \vec{B}| \; \hat{n}$$

where n :. is aunit Vector normal to the plane of the two Vectors $\vec{A}$ and $\vec{B}$.

$$\therefore \hat{n} = \frac{\vec{A} \times \vec{B}}{AB \sin\theta}$$

and

$$\vec{A} \times \vec{B} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$$

1. $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
2. $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
3. $m(\vec{A} \times \vec{B}) = (mA) \times B = A \times (mB)$

* $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \text{zero}$

  $\hat{j} \times \hat{k} = \hat{i}$

  $\hat{i} \times \hat{j} = \hat{k}$

  $k \times \hat{i} = \hat{j}$

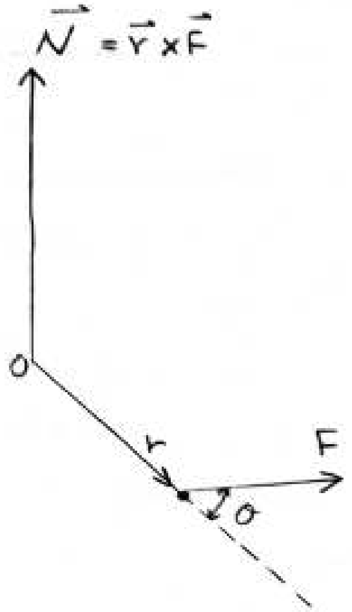
* $\hat{k} \times \hat{j} = -\hat{i}$

  $\hat{j} \times \hat{i} = -\hat{k}$

  $\hat{i} \times k = -\hat{j}$

عزم القوة

Torque of a Force ($\vec{N}$) :.



Let a Force $\vec{F}$ act at a point $P(x, y, z)$, as shown in Figure and :-

$$\vec{OP} = \vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$

the torque ($\vec{N}$) is defined as the cross product

$$\vec{N} = \vec{r} \times \vec{F}$$

ex) Find a unit vector normal to the plane containing the two vectors $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2k$?

$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n} = |\vec{A} \times \vec{B}| \, \hat{n}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(2-1) + \hat{j}(-1-4) + \hat{k}(-2-1)$$

$$= \hat{i} - 5\hat{j} - 3k$$

$$|\vec{A} \times \vec{B}| = \sqrt{1+9+9} = \sqrt{35}$$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{\hat{i} - 3\hat{j} - 3\hat{k}}{\sqrt{19}}$$

ex) Given the two vectors $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$, find

1. $\vec{A} \cdot \vec{B}$, $\vec{A} \times \vec{B}$
2. Find the angle between $\vec{A}$ and B.

① $$\vec{A} \cdot \vec{B} = (2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 2 - 1 - 2 = -1$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(2-1) + \hat{j}(-1-4) + \hat{k}(-2-1)$$

$$= \hat{i} - 5\hat{j} - 3\hat{k}$$

② $$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$A = |\vec{A}| = \sqrt{4+1+1} = \sqrt{6}$$

$$B = |\vec{B}| = \sqrt{1+1+4} = \sqrt{6}$$

$$\cos\theta = \frac{-1}{6} \Rightarrow \theta = \cos^{-1}\left(\frac{-1}{6}\right) = 99.6^\circ$$

Problems p.(19)

① Given two Vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}$
$\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$ Find

ⓐ the Value of $|\vec{A} - \vec{B}|$

ⓑ the Value of $\vec{A} \cdot \vec{B}$

ⓒ the angle between $\vec{A}$ and $\vec{B}$

ⓐ $\vec{A} - \vec{B} = (\hat{i} + \hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} - 3\hat{k})$
$= -\hat{j} + 4\hat{k}$

$|\vec{A} - \vec{B}| = \sqrt{(-1)^2 + (4)^2} = \sqrt{17}$

ⓑ $\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})$
$= 1 + 2 - 3 = zero$ ∴ المتجهين متعامدين

ⓒ $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{AB}$

$A = |\vec{A}| = \sqrt{1+1+1} = \sqrt{3}$
$B = |\vec{B}| = \sqrt{1+4+9} = \sqrt{15}$

$\because \vec{A} \cdot \vec{B} = zero$

$\therefore \cos\theta = zero \rightarrow$ ∴ This Vectors are perpendicular

(2) For the same vectors as in problem (1) express in $\hat{i}\,\hat{j}\,\hat{k}$ Form

(a) $2\vec{A}+3\vec{B} = [2(\hat{i}+\hat{j}+\hat{k}) + 3(\hat{i}+2\hat{j}-3\hat{k})]$

$= [(2\hat{i}+2\hat{j}+2\hat{k}) + (3\hat{i}+6\hat{j}-9\hat{k})]$

$= 5\hat{i}+8\hat{j}-7\hat{k}$

(b) $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & -3 \end{vmatrix}$

$= \hat{i}(-3-2)+\hat{j}(1+3)+\hat{k}(2-1)$

$= -5\hat{i}+4\hat{j}+\hat{k}$

(c) $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$:

$\vec{A}+\vec{B} = (\hat{i}+\hat{j}+\hat{k})+(\hat{i}+2\hat{j}-3\hat{k})$

$= (2\hat{i}+3\hat{j}-2\hat{k})$

$\vec{A}-\vec{B} = -\hat{j}+4\hat{k}$

$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -2 \\ 0 & -1 & 4 \end{vmatrix}$

$= \hat{i}(12-2)+\hat{j}(0-8)+\hat{k}(-2-0)$

$= 10\hat{i}-8\hat{j}-2\hat{k}$

(3) The Vector $\vec{A} = \hat{i} + \hat{j} + 2\hat{k}$ is perpendicular to the Vector $\vec{B} = 2\hat{i} - 3\hat{j} + q\hat{k}$.
what is the value of q.

$\because \vec{A}$ is perpendicular to $\vec{B}$

$\therefore \vec{A} \cdot \vec{B} = 0$

$$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} - 3\hat{j} + q\hat{k})$$

$$2 - 3 + 2q = 0$$

$$-1 + 2q = 0$$

$$2q = 1 \Rightarrow q = \frac{1}{2}$$

(4) Find the length of projection of the Vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on Vector $\vec{B} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$A\cos\theta = \frac{\vec{A} \cdot \vec{B}}{B}$$

$$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \cdot (2\hat{i} - 3\hat{j} + 4\hat{k})$$

$$= 2 - 3 + 4 = 3$$

$$B = |\vec{B}| = \sqrt{(2)^2 + (-3)^2 + (4)^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$$

$$\therefore A\cos\theta = \frac{3}{\sqrt{29}}$$