Chapter 1: Functions

1.1 Functions , Domain and Range

Def: A function f from a set D to a set Y is a rule that assigns a unique (single) element $f(x) \in Y$ to each element $x \in D$.

The set D of all possible input values is called the **domain** of the function. The set of all values of f(x) as x varies throughout *D* is called the **range** of the function.

EXAMPLE 1

Function	**Domain (x)**	**Range (y)**
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

Solution:

1- $y = x^2$ gives a real y-value for any real number x, so the domain is $(-\infty,\infty)$.

The range of $y = x^2$ is $[0, \infty)$ because the square of any real number is nonnegative and $x = \sqrt{y}$, x to be real $y \geq 0$.

2- $y = 1/x$ gives a real y-value for every x except x = 0. For the rules of arithmetic, we cannot divide any number by zero. The domain is $\mathbb{R}\backslash\{0\}$. The range of y = 1/x, can be found by x = 1/y is the input assigned to the output value y. Then range is $\mathbb{R}\backslash\{0\}$.

3- $y = \sqrt{x}$ gives a real y-value only if $x \geq 0$ so the domain is $[0,\infty)$.

The range of $y = \sqrt{x}$ can be found by $y \geq 0$ and $x = y^2$ so range $=[0, \infty)$

4- $y = \sqrt{4 - x}$: $4 - x \geq 0 \rightarrow 4 \geq x$. The formula gives real y-values for all $x \geq 4$.

The range : first $y \geq 0$, second $x = 4 - y^2 \rightarrow$ range $=[0, \infty)$.

The range of $y = \sqrt{x}$ can be found by y≥0 and $x = y^2$ so range =[0, ∞)

4- $y = \sqrt{4-x}$: 4 - x ≥0 → 4≥x. The formula gives real y-values for all x≥4.

The range : first y≥0 , second $x = 4 - y^2$→ range =[0, ∞).

5- $y = \sqrt{1-x^2}$ gives a real y-value if $1 - x^2 \geq 0$ → (1-x)(1+x) ≥0

Domain= [-1,1].

Range: . First y≥0 , second $x^2 = 1 - y^2$ → $x = \pm\sqrt{1-y^2}$ which means that we get the same solution above i.e. y=[-1,1] this implies that the range should be [0.1].

1.2 Graphs of Functions

If f is a function with domain D, its graph consists of the points in the Cartesian plane whose coordinates are the input-output pairs for *f.* In set notation, the graph is {(x,f(x)) / *x∈D}.*

EXAMPLE 1*:* The graph of the function f(x) = x + 2

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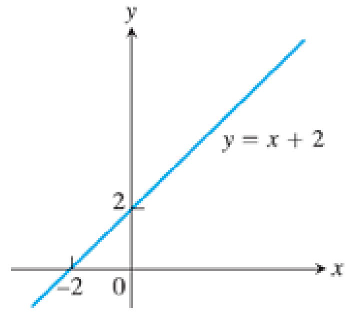
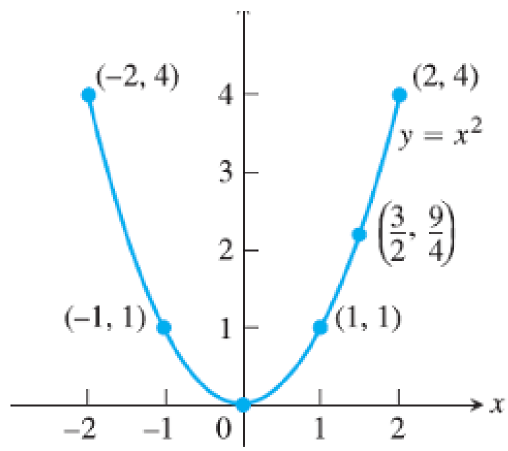


EXAMPLE 2: Graph the function $y = x^2$ over the interval $[-2, 2]$.



1.3 Piecewise-Defined Functions