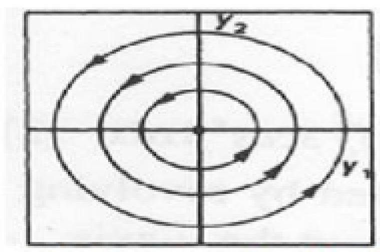
Example 3.16 Sketch the phase portrait of the system

$$x_1' = 2x_1 + 2x_2, \;\; x_2' = 4x_1 - 2x_2 \qquad (2.14)$$

Sol. The eigenvalue are $\lambda_1 = 2i, \lambda_2 = -2i, \;\; \alpha = 0, \beta = 2$

Then $J = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$, and $M = \begin{bmatrix} 2 & -2 \\ 4 & 0 \end{bmatrix}$ the phase portrait of Jordan form is



$\lambda_1 = 2i, \lambda_2 = -2i$: Centers

And the phase portrait of system x_1, x_2 is

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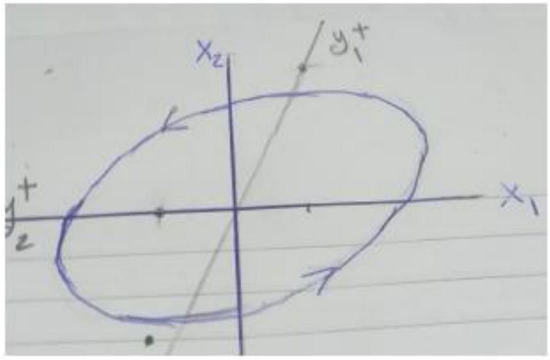


Homework: Sketch the phase portrait of the systems $1. x_1' = 2x_1,\ \ x_2' = x_1 + x_2$.

$2.\ x_1' = -x_1 + 2x_2,\ \ x_2' = -2x_1 - x_2 \qquad 3.\ x'' - 2x' + x = 0$

3.4 Linear systems of three dimensional

In this case we have three eigenvalues which are (a) 3 distinct real; (b) 2 complex and one real; (c) 2 equal and one distinct (d) 3 equal. then the Jordan form are

$$a.\ \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix},\ b.\ \begin{bmatrix} \alpha & -\beta & 0 \\ \beta & \alpha & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix},\ c.\ \begin{bmatrix} \lambda_0 & 1 & 0 \\ 0 & \lambda_0 & 0 \\ 0 & 0 & \lambda_1 \end{bmatrix},\ d.\ \begin{bmatrix} \lambda_0 & 1 & 0 \\ 0 & \lambda_0 & 1 \\ 0 & 0 & \lambda_0 \end{bmatrix}$$

we can partitioned in the diagonal blocks of dimensions one or two and use theorem 2.2, $\lambda_1, \lambda_2, \lambda_3, \alpha, \beta$. For example if we discuss the Jordan in (b.) can be partitioned into $J = \begin{bmatrix} \alpha & -\beta & 0 \\ \beta & \alpha & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$

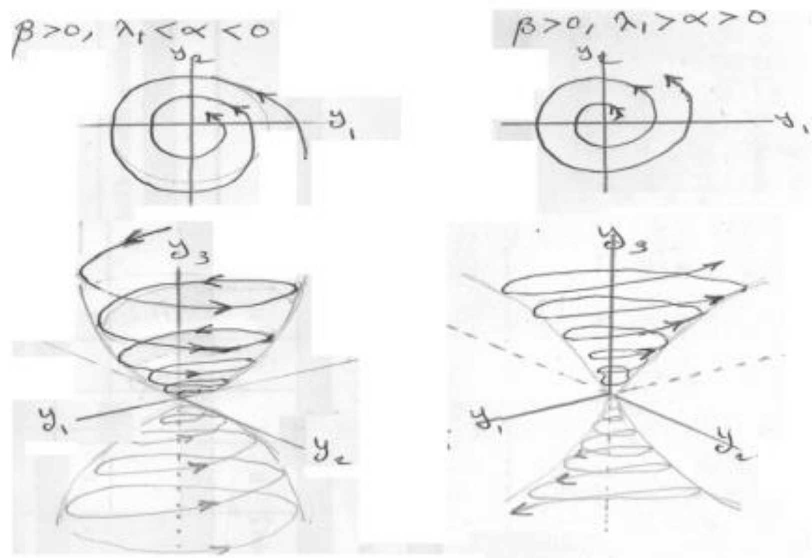
	α	$-\beta$	0	$\rightarrow \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \; \dot{y}_3 = \lambda_3 y_3 \;\; (2.15)$
$J =$	β	α	0	
	0	0	λ_3	

From (2.15) we get

$$\dot{y}_1 = \alpha y_1 - \beta y_2, \;\; \dot{y}_2 = \beta y_1 + \alpha y_2 \rightarrow r\,\dot{r} = \alpha r^2 \rightarrow r = ce^{\alpha t} \rightarrow \; e^t = kr^{\frac{1}{\alpha}}$$

And $\dot{y}_3 = \lambda_3 y_3 \rightarrow \; y_3 = k_1 e^{\lambda_3 t} = k_1 (e^t)^{\lambda_3} = k_1 (kr^{\frac{1}{\alpha}})^{\lambda_3} = k_1 k^{\lambda_3} r^{\frac{\lambda_3}{\alpha}}$

$$y_3 = K\left(\sqrt{y_1^2 + y_2^2}\right)^{\frac{\lambda_3}{\alpha}}$$



Example 3.17. Sketch the phase portrait of the system

$\dot{X}=\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}X$, the eigenvalues are $\pm i,-1$ then $J=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$

$$J=\left[\begin{array}{cc|c}0 & -1 & 0\\ 1 & 0 & 0\\ \hline 0 & 0 & -1\end{array}\right] \rightarrow \begin{bmatrix}\dot{y}_1\\ \dot{y}_2\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix},\ \ \dot{y}_3=-y_3$$

And the eigenvector are $V_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $V_2=\begin{bmatrix}0\\1\\0\end{bmatrix}$, $V_3=\begin{bmatrix}0\\0\\1\end{bmatrix}$ then $M=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$