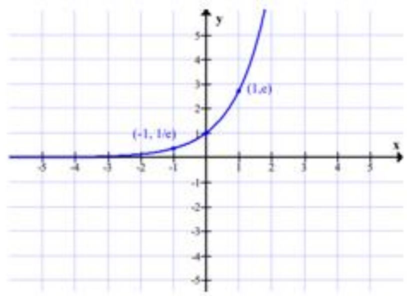
**The Natural Exponential Function $e^x$**

The most important exponential function is the natural exponential function, whose base is the special number e. The number e is irrational, and its value is 2.718281828. The number e is defined as the number to which the expression $\left(1+\frac{1}{n}\right)^n$ Since e > 1, the graph of the natural exponential function is as below:




**Properties of exponential functions**

1. The domain is the set of all real numbers: D = R

2. The range is the set of positive numbers: R = (0, +∞). (This means that a x is always positive, that is a x > 0 for all x. The equation ax = negative number has no solution)

3. There are no x-intercepts

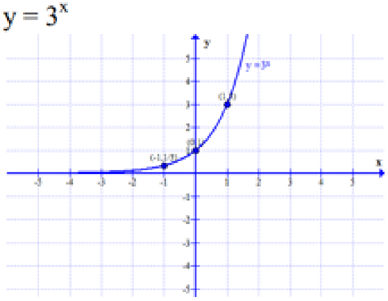
4. The y-intercept is (0, 1)

**Example:**

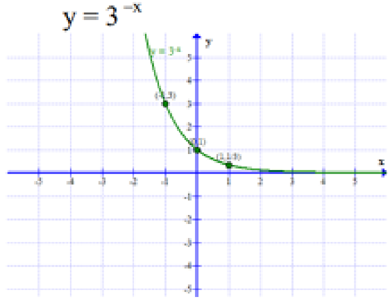
Use shifting to graph $f(x) = 3^{-x} - 2$. Start with a basic function and use one shift at a time. This function is obtained from the graph of $y = 3^x$ by first reflecting it about y-axis (obtaining $y = 3^{-x}$ ) and then shifting the graph down by 2 units. Make sure to plot the three points on the graph of the basic function!

Remark: Function $y = 3^x$ has a horizontal asymptote, so remember to shift it too when performing shift up/down.

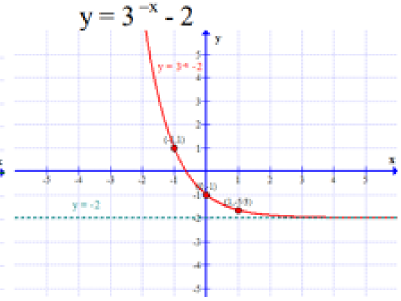
$y = 3^x$



$y = 3^{-x}$



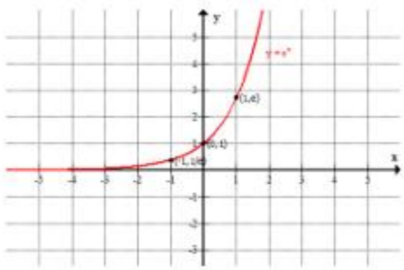
$y = 3^{-x} - 2$



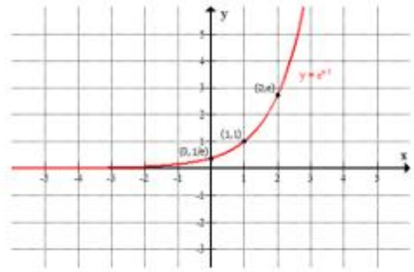
**Example:**

Graph $f(x) = 3e^{2x-1}$ .

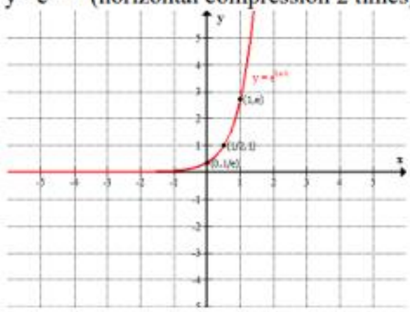
Basic function: $y = e^{x}$



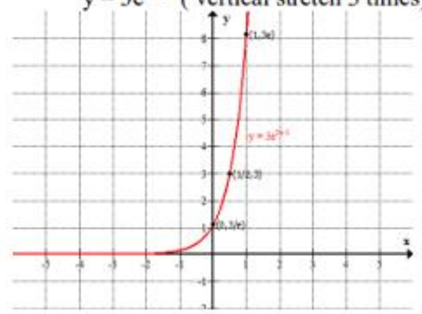
$y = e^{x-1}$ (shift to the right by1)



$y= e^{2x-1}$ (horizontal compression 2 times)



$y = 3e^{2x-1}$ ( vertical stretch 3 times)



***Example:*** Solve $4^{x^2} = 2^x$

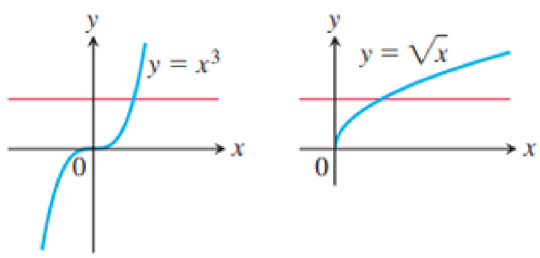
## 1.11 Inverse Functions and Logarithms

### One-to-One Functions

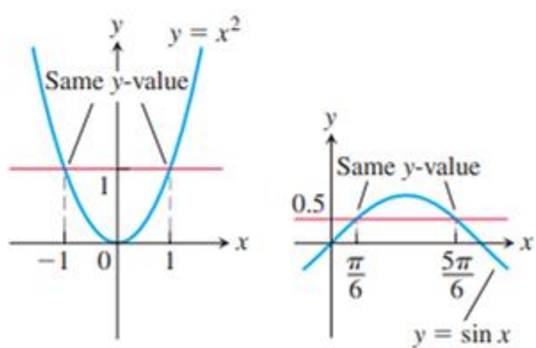
A function $f(x)$ is one-to-one on a domain D if $f(x1) \neq f(x2)$ whenever $x1 \neq x2$ in D.

Example:

(a) $f(x) = \sqrt{x}$ is one-to-one on any domain of nonnegative numbers because $\sqrt{x_1} \neq \sqrt{x_2}$ whenever $x_1 \neq x_2$.




(b) g(x) = sin x is not one-to-one on the interval $[0, \pi]$ because $sin\ (\pi/6) = sin\ (\frac{5\pi}{6})$.




(b) Not one-to-one: Graph meets one or more horizontal lines more than once.

**The Horizontal Line Test for One-to-One Functions**

A function $y = f(x)$ is one-to-one if and only if its graph intersects each hori-zontal line at most once.

**Inverse Functions**

Since each output of a one-to-one function comes from just one input, the effect of the function can be inverted to send an output back to the input from which it came.

**DEFINITION**

Suppose that $f$ is a one-to-one function on a domain D with range R. The inverse function $f^{-1}$ is defined by

$$f^{-1}(b) = a \quad \text{if} \quad f(a) = b.$$

The domain of $f^{-1}$ is $R$ and the range of $f^{-1}$ is $D$.

The symbol $f^{-1}$ for the inverse of $f$ is read "$f$ inverse." The "$-1$" in $f^{-1}$ is *not* an exponent; $f^{-1}(x)$ does not mean $1/f(x)$. Notice that the domains and ranges of $f$ and $f^{-1}$ are interchanged.

**Example:**

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 3 | 4.5 | 7 | 10.5 | 15 | 20.5 | 27 | 34.5 |

| $y$ | 3 | 4.5 | 7 | 10.5 | 15 | 20.5 | 27 | 34.5 |
|---|---|---|---|---|---|---|---|---|
| $f^{-1}(y)$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |