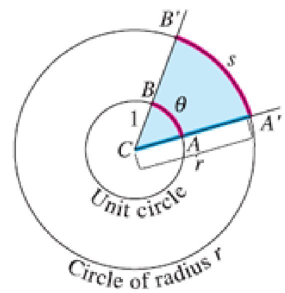
1.9 Trigonometric Functions

Angles

Angles are measured in degrees or radians. One radian is the angle subtended at the centre of a circle by an arc that is equal in length to the radius of the circle, that is, $\theta = s / r$, where θ is the subtended angle in radians, s is arc length, and r is radius.



Let the circle is a unit circle having radius $r = 1$, one complete revolution of the unit circle is 360 degree has arc length $2r*\pi = 2\pi$ radians, so we have

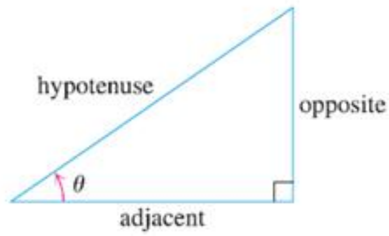
$$\pi \text{ radians} = 180°$$

$$1 \text{ radian} = \frac{180}{\pi} (\approx 57.3) \text{ degrees} \qquad \text{or} \qquad 1 \text{ degree} = \frac{\pi}{180} (\approx 0.017) \text{ radians.}$$

Degrees	**−180**	**−135**	**−90**	**−45**	**0**	**30**	**45**	**60**	**90**	**120**	**135**	**150**	**180**	**270**	**360**
θ (radians)	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

$\tan(x + \pi) = \tan x$
$\cot(x + \pi) = \cot x$

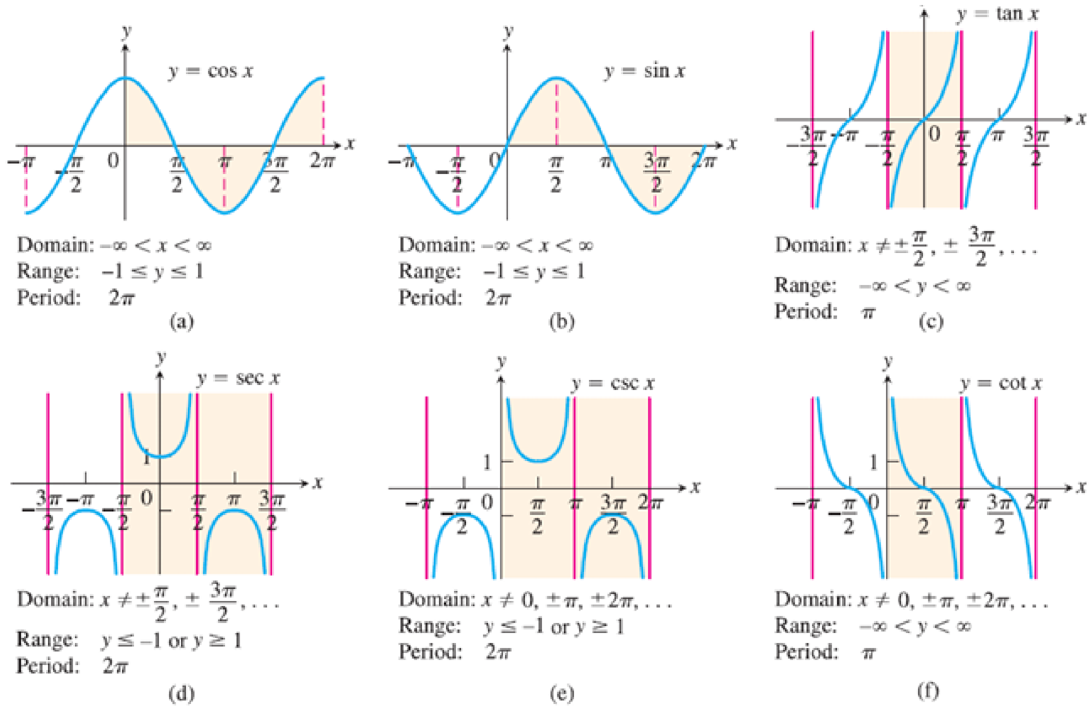
$\sin(x + 2\pi) = \sin x$
$\cos(x + 2\pi) = \cos x$
$\sec(x + 2\pi) = \sec x$
$\csc(x + 2\pi) = \csc x$

Even

$\cos(-x) = \cos x$
$\sec(-x) = \sec x$

Odd

$\sin(-x) = -\sin x$
$\tan(-x) = -\tan x$
$\csc(-x) = -\csc x$
$\cot(-x) = -\cot x$



$$\cos^2\theta + \sin^2\theta = 1.$$

$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

1.10 Exponential Functions

The function of the form is the exponential function.

A function of the form $f(x) = a^x, a > 0$, $a \neq 1$ is called an exponential function with base a. Its domain is the set of all real numbers. For an exponential function f we have $\frac{f(x+1)}{f(x)} = a$.

For integer and rational exponents, the value of an exponential function $f(x) = a^x$ is obtained arithmetically as follows. If x = n is a positive integer, the number a^n is given by multiplying a by itself n times:

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}.$$

If $x = 0$, then $a^0 = 1$, and if $x = -n$ for some positive integer n, then

$$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n.$$

If $x = 1/n$ for some positive integer n, then

$$a^{1/n} = \sqrt[n]{a},$$

which is the positive number that when multiplied by itself n times gives a. If $x = p/q$ is any rational number, then

$$a^{p/q} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p.$$

Rules for Exponents:

If $a > 0$ and $b > 0$, the following rules hold true for all real numbers x and y.

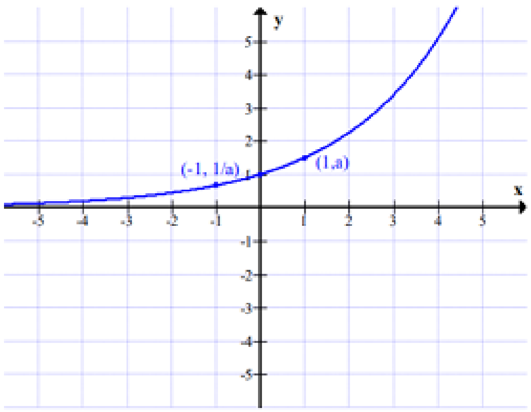
1. $a^x \cdot a^y = a^{x+y}$ **2.** $\frac{a^x}{a^y} = a^{x-y}$

3. $(a^x)^y = (a^y)^x = a^{xy}$ **4.** $a^x \cdot b^x = (ab)^x$

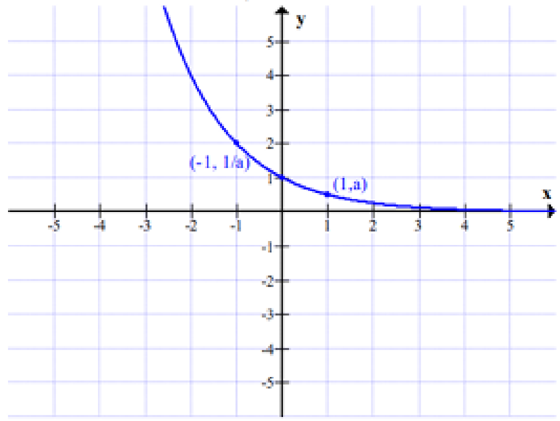
5. $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

The graph of an exponential function depends on the value of a.

a> 1



0 < a< 1



Points on the graph: (-1, 1/a), (0,1), (1, a)