



تفاضل وتكامل ٢ م...



Find $\lim_{x \rightarrow 0^+} \ln f(x)$. Since

$$\ln f(x) = \ln(1+x)^{1/x} = \frac{1}{x} \ln(1+x),$$

L'Hôpital's Rule now applies to give

$$\begin{aligned}\lim_{x \rightarrow 0^+} \ln f(x) &= \lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x} \quad \frac{0}{0} \\ &= \lim_{x \rightarrow 0^+} \frac{\frac{1}{1+x}}{1} \\ &= \frac{1}{1} = 1.\end{aligned}$$

If $\lim_{x \rightarrow a} \ln f(x) = L$, then

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} e^{\ln f(x)} = e^L.$$

Here a may be either finite or infinite.

Therefore, $\lim_{x \rightarrow 0^+} (1+x)^{1/x} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} e^{\ln f(x)} = e^1 = e$.

EXAMPLE 8 Find $\lim_{x \rightarrow \infty} x^{1/x}$.

Solution The limit leads to the indeterminate form ∞^0 . We let $f(x) = x^{1/x}$ and find $\lim_{x \rightarrow \infty} \ln f(x)$. Since

$$\ln f(x) = \ln x^{1/x} = \frac{\ln x}{x},$$

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L'Hôpital's Rule gives

$$\begin{aligned}\lim_{x \rightarrow \infty} \ln f(x) &= \lim_{x \rightarrow \infty} \frac{\ln x}{x} \quad \frac{\infty}{\infty} \\ &= \lim_{x \rightarrow \infty} \frac{1/x}{1} \\ &= \frac{0}{1} = 0.\end{aligned}$$

Therefore $\lim_{x \rightarrow \infty} x^{1/x} = \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} e^{\ln f(x)} = e^0 = 1$. ■

2 INTEGRATION:

1) The Definite Integral

$$S_n = \sum_{k=1}^n f(c_k) \Delta x_k = \sum_{k=1}^n f(c_k) \left(\frac{b-a}{n} \right),$$

$$\Delta x_k = \Delta x = (b-a)/n \text{ for all } k$$

$$J = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(c_k) \left(\frac{b-a}{n} \right) = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(c_k) \Delta x$$

$$\Delta x = (b-a)/n$$



Upper limit of integration
Integral sign
Lower limit of integration
The function is the integrand.
 $\int_a^b f(x) dx$
 x is the variable of integration.
Integral of f from a to b
When you find the value of the integral, you have evaluated the integral.

Rules satisfied by definite integrals

- Order of Integration: $\int_b^a f(x) dx = -\int_a^b f(x) dx$
- Zero Width Interval: $\int_a^a f(x) dx = 0$

A Definition

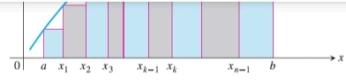
A Definition

when $f(a)$ exists



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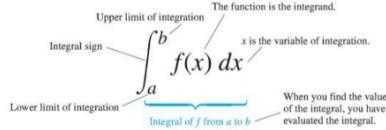
$$S_n = \sum_{k=1}^n f(c_k) \Delta x_k = \sum_{k=1}^n f(c_k) \left(\frac{b-a}{n} \right),$$



$\Delta x_k = \Delta x = (b - a)/n$ for all k

$$J = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(c_k) \left(\frac{b-a}{n} \right) = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(c_k) \Delta x$$

$\Delta x = (b - a)/n$



Rules satisfied by definite integrals

1. Order of Integration: $\int_b^a f(x) dx = - \int_a^b f(x) dx$ A Definition
2. Zero Width Interval: $\int_a^a f(x) dx = 0$ A Definition when $f(a)$ exists
3. Constant Multiple: $\int_a^b kf(x) dx = k \int_a^b f(x) dx$ Any constant k
4. Sum and Difference: $\int_a^b (f(x) \pm g(x)) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$
5. Additivity: $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$
6. $f(x) \geq g(x)$ on $[a, b] \Rightarrow \int_a^b f(x) dx \geq \int_a^b g(x) dx$
 $f(x) \geq 0$ on $[a, b] \Rightarrow \int_a^b f(x) dx \geq 0$ (Special Case)

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EXAMPLE:

Let $\int_{-1}^1 f(x) dx = 5$, $\int_1^4 f(x) dx = -2$, and $\int_{-1}^1 h(x) dx = 7$.
Then:

1. $\int_4^1 f(x) dx = - \int_1^4 f(x) dx = -(-2) = 2$
2. $\int_{-1}^1 [2f(x) + 3h(x)] dx = 2 \int_{-1}^1 f(x) dx + 3 \int_{-1}^1 h(x) dx = 2(5) + 3(7) = 31$
3. $\int_{-1}^4 f(x) dx = \int_{-1}^1 f(x) dx + \int_1^4 f(x) dx = 5 + (-2) = 3$

2.1 Integration by Substitution

THEOREM Substitution in Definite Integrals: If g' is continuous on the interval $[a, b]$ and f is continuous on the range of $g(x) = u$, then

$$\int_a^b f(g(x)) \cdot g'(x) dx = \int_{g(a)}^{g(b)} f(u) du.$$

EXAMPLE: Evaluate $\int_{-1}^1 3x^2 \sqrt{x^3 + 1} dx$.

SOL:
$$\int_{-1}^1 3x^2 \sqrt{x^3 + 1} dx$$
 Let $u = x^3 + 1$, $du = 3x^2 dx$.
When $x = -1$, $u = (-1)^3 + 1 = 0$.
When $x = 1$, $u = (1)^3 + 1 = 2$.

$$\begin{aligned} &= \int_0^2 \sqrt{u} du \\ &= \frac{2}{3} u^{3/2} \Big|_0^2 \\ &= \frac{2}{3} [2^{3/2} - 0^{3/2}] = \frac{2}{3} [2\sqrt{2}] = \frac{4\sqrt{2}}{3} \end{aligned}$$

(a) $\int_{\pi/4}^{\pi/2} \cot \theta \csc^2 \theta d\theta = \int_1^0 u \cdot (-du)$

Let $u = \cot \theta$, $du = -\csc^2 \theta d\theta$.
 $-du = \csc^2 \theta d\theta$.
When $\theta = \pi/4$, $u = \cot(\pi/4) = 1$.
When $\theta = \pi/2$, $u = \cot(\pi/2) = 0$.

$$\begin{aligned} &= - \int_1^0 u du \\ &= - \left[\frac{u^2}{2} \right]_1^0 \end{aligned}$$





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EXAMPLE:
 Let $\int_{-1}^1 f(x) dx = 5$, $\int_1^4 f(x) dx = -2$, and $\int_{-1}^1 h(x) dx = 7$.
 Then: $\int_4^1 f(x) dx = -\int_1^4 f(x) dx = -(-2) = 2$

1. $\int_4^1 f(x) dx = -\int_1^4 f(x) dx = -(-2) = 2$
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2.1 Integration by Substitution

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$$\int_a^b f(g(x)) \cdot g'(x) dx = \int_{g(a)}^{g(b)} f(u) du.$$

EXAMPLE: Evaluate $\int_{-1}^1 3x^2 \sqrt{x^3 + 1} dx$.

SOL:

$$\begin{aligned} & \int_{-1}^1 3x^2 \sqrt{x^3 + 1} dx && \text{Let } u = x^3 + 1, du = 3x^2 dx. \\ & && \text{When } x = -1, u = (-1)^3 + 1 = 0. \\ & && \text{When } x = 1, u = (1)^3 + 1 = 2. \\ & & & \\ & = \int_0^2 \sqrt{u} du && \text{Evaluate the new definite integral.} \\ & & & \\ & = \frac{2}{3} u^{3/2} \Big|_0^2 && \\ & & & \\ & \text{EXAMPLE} &= \frac{2}{3} \left[2^{3/2} - 0^{3/2} \right] &= \frac{2}{3} [2\sqrt{2}] = \frac{4\sqrt{2}}{3} \end{aligned}$$

(a) $\int_{\pi/4}^{\pi/2} \cot \theta \csc^2 \theta d\theta = \int_1^0 u \cdot (-du)$

$$\begin{aligned} & \text{Let } u = \cot \theta, du = -\csc^2 \theta d\theta. \\ & \text{When } \theta = \pi/4, u = \cot(\pi/4) = 1. \\ & \text{When } \theta = \pi/2, u = \cot(\pi/2) = 0. \\ & = -\int_1^0 u du \\ & = -\left[\frac{u^2}{2} \right]_1^0 \\ & = -\left[\frac{(0)^2}{2} - \frac{(1)^2}{2} \right] = \frac{1}{2} \end{aligned}$$

(b) $\int_{-\pi/4}^{\pi/4} \tan x dx = \int_{-\pi/4}^{\pi/4} \frac{\sin x}{\cos x} dx$

$$\begin{aligned} & \text{Let } u = \cos x, du = -\sin x dx. \\ & \text{When } x = -\pi/4, u = \sqrt{2}/2. \\ & \text{When } x = \pi/4, u = \sqrt{2}/2. \\ & = -\int_{\sqrt{2}/2}^{\sqrt{2}/2} \frac{du}{u} \\ & = -\ln |u| \Big|_{\sqrt{2}/2}^{\sqrt{2}/2} = 0 & \text{Integrate, zero width interval} \end{aligned}$$

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THEOREM:
 Let f be continuous on the symmetric interval $[-a, a]$.

(a) If f is even, then $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$.

 If f is odd, then $\int_{-a}^a f(x) dx = 0$.

Evaluate $\int_{-2}^2 (x^4 - 4x^2 + 6) dx$.

Since $f(x) = x^4 - 4x^2 + 6$ satisfies $f(-x) = f(x)$, it is even on the symmetric interval $[-2, 2]$, so

$$\begin{aligned} \int_{-2}^2 (x^4 - 4x^2 + 6) dx &= 2 \int_0^2 (x^4 - 4x^2 + 6) dx \\ &= 2 \left[\frac{x^5}{5} - \frac{4}{3}x^3 + 6x \right]_0^2 \end{aligned}$$