Diagonally Dominant Matrix

A square matrix is said to be diagonally dominant if for every row of the matrix, the magnitude of the diagonal entry in a row is larger than or equal to the sum of magnitudes of all the other (non−diagonal) entries in that row (i.e. $|a_{\text{ii}}| \geq \sum_{i \neq j} |a_{ij}|$). However a matrix is called strictly diagonally dominant if $|a_{\text{ii}}| > \sum_{i \neq j} |a_{ij}|$ and strictly diagonally dominant is non- singular.

For example,

$\boldsymbol{A} = \begin{bmatrix} 3 & -2 & 1 \\ 1 & -3 & 2 \\ -1 & 2 & 4 \end{bmatrix}$ is diagonally dominant since $|3| \geq |-2| + |1|$, $|-3| \geq |1| + |2|, |4| \geq |-1| + |2|$. But $\boldsymbol{B} = \begin{bmatrix} -2 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & -2 & 0 \end{bmatrix}$ is not diagonally dominant since $|-2| < |2| + |1|$, $|0| < |1| + |-2|$.

$\boldsymbol{C} = \begin{bmatrix} -4 & 2 & 1 \\ 1 & 6 & 2 \\ 1 & -2 & 5 \end{bmatrix}$ is strictly diagonally dominant since $|-4| > |2| + |1|$, $|6| > |1| + |2|, |5| > |1| + |-2|$.

Exercises

Classify the following matrices as diagonally dominant, strictly diagonally dominant or unknown:

$$\boldsymbol{A} = \begin{bmatrix} 2 & -1 & 1 \\ 1 & -4 & 2 \\ -1 & 2 & 4 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} -6 & 2 & 1 \\ 1 & 4 & 2 \\ 1 & -2 & 7 \end{bmatrix}.$$