Solution of differential equation

A solution of a differential equation is an expression for the dependent variable in terms of the independent one(s) which satisfies the relation. The general solution includes all possible solutions and typically includes arbitrary constants (in the case of an ODE) or arbitrary functions (in the case of a PDE.)

Solution of differential equation

any function φ defined on an interval I and possessing at least n derivatives that are continuous on I ,which when substitute into an n-th order O.D.E. reduces the equation to an identity, is said to be a **solution** of the equation on the interval I .

is a solution of $F(x,y,y',y'',\ldots\ldots\ldots,y(n))=0$ if $\varphi(x)$

Then $F($x $,\varphi(x),\varphi'(x),\varphi''(x),\ldots\ldots..\varphi(n)(x))=0$ for all x in I.

Example 1:

prove that $y=e^3x$ is a solution of $y'-3y=0$

Solution:-

$y'=3e3x$ Substituted in given equation obtain: $3e3x-3e3x=0$

Example 2:

Prove that $y=x^2$ is a solution of $x^2y''-3xy'+4y=0$?

Solution:-

$$y'=2x\ ,y''=2$$

Substituted in given equation obtain: $2x2-6x2+4x2=0$

$\therefore y=x2\ is\ a\ solution\ of\ given\ equation$

Example 3:

Prove that $y=xlnx-x$ ***is a solution of*** $xy'=x+y\ ;x>0$

Solution

$y' = 1 + lnx - 1 = lnx$

$\therefore xlnx = x + xlnx - x$

$\therefore xlnx = xlnx \;\; \rightarrow \therefore$

$y = xlnx - x \; is \; a \; solution \; of \; given \; equation$

Example 4

is $x = ce^{-\mathrm{kt}}$ **a solution of** $\frac{\boldsymbol{dx}}{\boldsymbol{dt}} = -\boldsymbol{kx}$?

-:Solution

$x' = -cke^{-\mathrm{kt}} \quad \therefore -cke^{-\mathrm{kt}} = -kce^{-\mathrm{kt}}$

$\therefore x = ce^{-\mathrm{kt}} \;\; is \; a \; Solution \; of \; \frac{\boldsymbol{dx}}{\boldsymbol{dt}} = -kx$

Exercises

1- prove that :

a- $y = \sqrt{1 - x2} \; is \; a \; solution \; of \; yy' + x = 0 \; on(-1,1).$

b- $\boldsymbol{y} = \frac{1}{16}x^4 \; \boldsymbol{is \; a \; solution \; of} \; y' = xy^{1/2}$

2- Is $y = e^{5x}$ $\boldsymbol{a \; solution \; of}$ $y'' - y' + y = 0$?

3- Is $y = xe^{x}$ $\boldsymbol{a \; solution \; of \; y'' - 2y' + y = 0}$?

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Types of the solution for O.D.E

1-The general solution

Is a solution of O.D.E. has arbitrary constants.

.Ex

1- $y=sinx+c$ $is\ a\ general\ solution\ of$ $y'=cosx$

2- $y = x^4 + xc_1 + c_2$ $is\ a\ general\ solution\ of$ $y'' = 12x^2$

3- $y = ce^x$ $is\ a\ general\ solution\ of$ $y' = y$

2-The particular solution

Is a solution of O.D.E. we get it from the general solution.

Ex.1

$y=sinx$ $(c=0)$

$y=sinx+12$ $(c=12)$

$y=sinx-1$ $(c=-1)$

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Are particular solutions of $y'=cosx$

Ex.2

$y = x^4$ $(c_1 = c_2 = 0)$

$y = x^4 + x - 1$ $(c_1 = 1, c_2 = -1)$

$y = x^4 + \sqrt{2x} - 3$ $(c_1 = \sqrt{2}, c_2 = -3$

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Are particular of $y'' = 12x^2$

Ex.3

$y = e^x \qquad (c = 1)$

$y = -e^x \qquad (c = -1)$

$y = \sqrt{2}e^x \qquad (c = \sqrt{2})$

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Are particular solutions 0f y'=y

3-The singular solution

Any solution of O.D.E. it is not get it from the general solution.

Ex.1

y=0 is a singular solution of $y' = y$

Ex.2

y=0 is a singular solution of $y' = x\sqrt{y}$ on (−∞,∞)

$y = \left(\frac{1}{4}x^2 + c\right)^2$ is a general solution of $y' = x\sqrt{y}$

If c=0 then $y = \frac{1}{16}x^4$ is particular solution of $y' = x\sqrt{y}$

4-The complete solution

The general solution of O.D.E. is called complete if all solution for O.D.E. is a particular solution from the general solution.

Ex.1

The general solution $y = sinx + c\ of\ y' = cosx$ is complete

Ex.2

The general solution $y = cx - c^2$ is not complete since $y = \frac{x^2}{4}$ is a singular solution of O.D.E.

$$(y')^2 - xy' + y = 0$$

5-**Implicit solution of an O.D.E .**

A relation $G(x, y) = 0$ is said to be an implicit solution of an O.D.E. $F(x, y, y', y'', \dots \dots, y(n)) = 0$

On an interval I, provided that there exists at least one function φ that satisfies the relation as well as the O.D.E. on I.

Note:

Implicit solution means a solution in which dependent variable is not separated and explicit means dependent variable is separated.

EX .

Let us consider a differential equation

$$x + yy' = 0$$

The relation $x^2 + y^2 = 25$

Is an implicit solution of the $x + yy' = 0\ , for\ all\ x \in\ (-5,5)$

$\therefore y = \varphi_1(x) = \sqrt{25 - x2}\ , y = \varphi_2(x) = -\sqrt{25 - x2}$