

Chapter -4-

Application for differential equations

Differential equations as mathematical models

Introduction

we introduce the notion of a differential equation as a mathematical model and discuss some specific models .

(1) Newton's law of cooling/warming

According to Newton's law empirical law of cooling /warming :

The rate at which the temperature of a body changes is proportional to the difference between the temperature of a body and the temperature of the surrounding medium the so-called ambient temperature.

Solution

$T(t)$:the temperature of a body at time t

: the temperature of the surrounding medium T_m

: The rate at which the temperature of the body changes $\frac{dT}{dt}$

The Newton's law of cooling /warming translates into the mathematical statement

$$\frac{dT}{dt} \propto T - T_m$$

$$\frac{dT}{dt} = k(T - T_m)$$

k is a constant of proportionality

$$\frac{dT}{k(T - T_m)} = dt$$

$$\frac{1}{k} \ln(T - T_m) = t + c$$

Is either case cooling or warming if T_m is a constant, it stands to reason that $k < 0$

(2) Population growth

The idea that the rate at which the population of a country growth at a certain time is proportional to the total population of the country at that time.

The more people there are at time t the more there are going to be in the future.

Solution

Let $p(t)$: the total population at time t

Then this assumption can be expressed as:

$$\frac{dp}{dt} \propto p$$

$$\frac{dp}{dt} = kp$$

K is a constant of proportionality

$$\frac{dp}{p} = k dt$$

$$\ln p = kt + a$$

$$p = ce^{kt}$$

(3) Melting calf

If the average of melting calf is proportional to an equal calf. to find the follow over plus calf after t time where q_o is the calf in head start.

Solution

Let q :melting calf after time t

$\frac{dq}{dt}$: the average of melting calf

$K > 0$ is a constant of proportionality

$$\frac{dq}{dt} \propto q$$

$$\frac{dq}{dt} = -kq$$

$$\frac{dq}{q} = -kdt$$

$$\ln q = -kt + c$$

$$\text{Let } q = q_o, t = 0$$

$$\ln q_o = c$$

$$\ln q = -kt + \ln q_o$$

$$\ln \frac{q}{q_o} = -kt$$

$$\frac{q}{q_o} = e^{-kt}$$

$$\therefore q = q_o e^{-kt}$$