



Introduction to Mechanics of Soilds

Detailed Syllobus

Simple Stress

1. Normal stress
2. Bearing Stress
3. Shearing Stress
4. Allowable Stresses and Factor of Safety
5. Applications:
 - (a) Design of Axially Loaded Members
 - (b) Riveted And Bolted Connections
 - (c) Thin-Walled Pressure Tubes and Vessels

Simple Strain

1. Strain Definiution
2. Stress-Strain Diagram
3. Hook's Law
4. Defelection of Axially Loaded Members
5. Generalized Hook's Law
6. Statically Indeterminate Problems of Axially Loaded Members
7. Thermal Stresses

Torsion

1. Application of Method of Sections
2. Basic Assumptions
3. Torsion Formula
4. Design of Circular Members in Torsion
5. Angle of Twist of Circular Members
6. Statically Indeterminate Torsional Members
7. Torsion of Solid Non-Circular Members
8. Torsion of Thin-Walled Hollow Members

Axial Force, Shear, and Bending Moment

1. Sign Conension
2. Axial Force, Shear, and Bending Moment Diagrams: Direct Approach
3. Shear and Bending Moment Diagrams: Summation Approach

Bending Stresses in Beams

1. Limitations of Theory
2. Basic Kinematic Assumptions
3. Flexural Formula
4. Economic Sections
5. Beams of Two Materials

Shearing Stresses in Beams

1. Some Preliminaries
2. Shear Flow
3. The Shearing Stress Formula for Beams



4. Limitations of Shearing Stress Formula

Deflection of Beams

1. Strain-Curvature and Moment-Curvature Relations
2. Direct Integration Method
3. Moment-Area Method

Compound Stresses

1. Superposition and its Limitations
2. Skew Bending
3. Eccentrically Loaded Members
4. Superposition of shearing Stresses

Transformation of Stresses

1. The Basic Problem
2. Equations for the Transformation of Plane Stresses
3. Principal Stresses
4. Maximum Shearing Stresses
5. Importance of Transformation of Stresses
6. Mohr's Circle of Stresses

Columns

1. Buckling and Stability
2. Columns with Pinned Ends
3. Columns with Eccentric Axial Loads
4. The Secant Formula for Columns



Text Book:

Introduction to Mechanics of Solids, 1979, by E. P. Popov

References:

1. Mechanics of Materials "2nd Edition", 1979, by E. P. Popov.
2. Mechanics of Materials "7th Edition", 2009, by J. M. Gere.
3. Strength of Materials "3rd Edition", 1980, by F. L. Singer.
4. Elements of Strength of Materials "5th Edition", 1971, by S. Timoshenko and Yong.

The following table contains the major subjects that will be given and their location in the text book. The minimum number of lectures required to cover each subject is also mentioned. The time of each lecture is 2 hours.

No.	Subject	Location	No. of lectures
1	Stress and Axial Loads	Chapter3	6
2	Strain, Constitutive Laws and Axial Deformation	Chapter4	6
3	Torsion	Chapter5	6
4	Axial Force, Shear and Bending Moments	Chapter2	6
5	Bending Stress in Beams	Chapter6	8
6	Shearing Stress in Beams	Chapter7	8
7	Deflection of Beams	Chapter11	6
8	Compound Stresses	Chapter8	4
9	Transformation of Stress and Strain	Chapter9	4
10	Buckling of Columns	Chapter14	4



Professor Egor Popov, 1913-2001



Engineering Pioneer Egor Popov UC Berkeley Professor Emeritus of Civil and Environmental Engineering Egor Popov passed away after a brief illness, on Thursday, April 19, 2001, at Alta Bates Hospital in Berkeley, California.

Born in Kiev, Russia, in 1913, Popov and his family escaped to Manchuria during the Bolshevik Revolution and eventually made their way to the U.S. His father, Dr. Paul Popov, became a prominent physician in San Francisco.

Professor Popov began his engineering studies at UC Berkeley, and continued with graduate work at Caltech, MIT, and Stanford where he received his Ph.D. He was active in teaching and research for more than 50 years, having first joined UC Berkeley's Department of Civil Engineering in 1946. Within this department he was the first chair of the division of structural engineering and structural mechanics. Many of his former graduate students went on to become distinguished engineers, including those who went on to become faculty in PEER: M. S. Agababian, H. Krawinkler, and F. Filippou.

Popov's seminal engineering textbook, *Mechanics of Materials* went to a second edition in 1976, and in 1990 he authored *Engineering Mechanics of Solids*. His engineering expertise was used by NASA to solve buckling problems and played a key role in the structural analysis of the Alaskan pipeline and the San Francisco-Oakland Bay Bridge.

Popov pioneered the study of inelastic behavior and the seismic response of reinforced concrete and steel buildings. His research interests covered a wide spectrum of topics in earthquake engineering, including cyclic testing and modeling; the development of the eccentrically braced frame concept; the seismic resistance of steel connections; and the development of friction devices to retrofit existing structures. Popov was a long-time faculty participant of the Earthquake Engineering Research Center and a faculty participant of the Pacific Earthquake Engineering Research Center. Elected to the National Academy of Engineering in 1976, he was honored in 1999 with the Earthquake Engineering Research Institute's highest honor, the George W. Housner Medal.

Popov's enthusiasm for his work was equal to his contributions. His most recent research report, coauthored by Shakhzod M. Takhirov, was published just days before his death (*Experimental Study of Large Seismic Steel Beam-to-Column Connections*, PEER report 2001/01).

<http://peer.berkeley.edu/news/2001spring/popov.html>

11/14/2006



DEDICATION



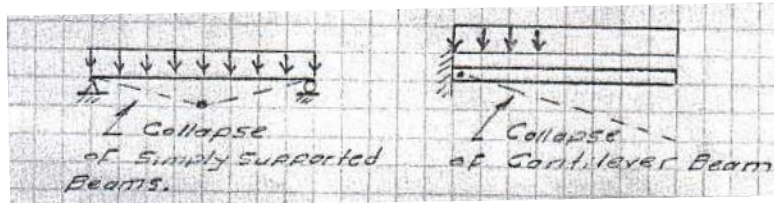
Professor Egor Popov

This edition of the AISC Seismic Provisions is dedicated to the memory of Professor Egor Popov. Professor Popov was a Professor for over 50 years at the University of California at Berkeley, and a long time member of the AISC Committee on Specifications. Professor Popov focused a major portion of his career improving the understanding and seismic performance of steel structures. He was instrumental in the development of seismic design provisions for steel structures for over thirty years, and initiated the activity of AISC in this regard in the late 1980's. As Chair of TC113 (the predecessor of TC9), he led the publication of the first two editions of the AISC Seismic Provisions. Until the time of his death at the age of 88 early in 2001, Professor Popov remained a very active member of TC9 in the role of Vice Chair. His contributions to the development of these provisions and understanding of the seismic performance of steel buildings is unequalled, and will long be remembered and appreciated by AISC, the steel industry and the structural engineering profession. It is entirely fitting that these provisions be dedicated to the memory of Professor Egor Popov.

1.1 Purpose and Scope

The adequacy of structural member can be checked based on the following three criteria:

1. Strength: see Fig.1



2. Deformation: see Fig.2

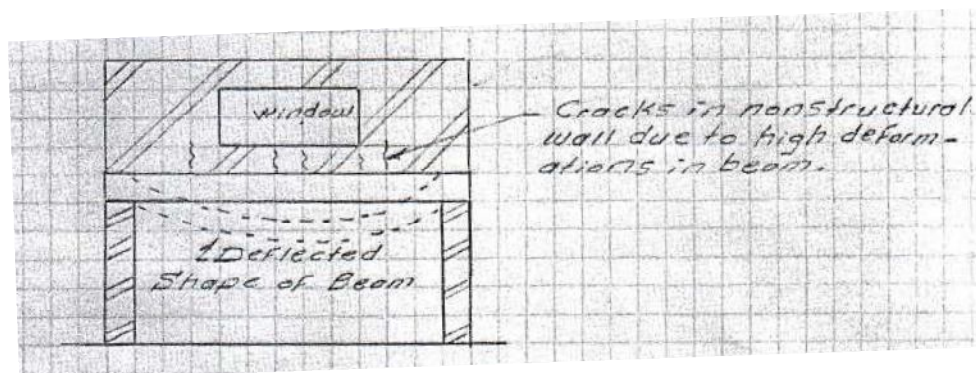


Fig.2: Beam with Large Deformation (Inadequate Stiffness)

3. Stability: see Fig.3

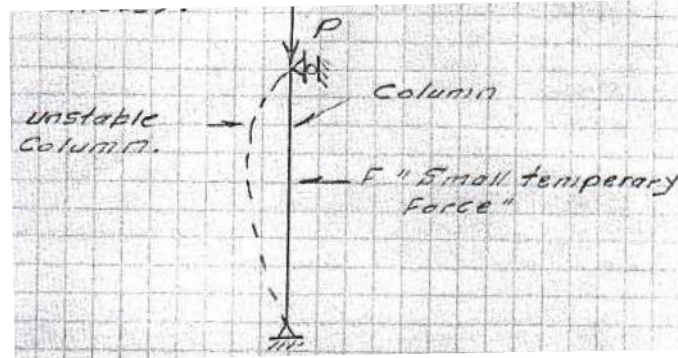


Fig.3: Unstable Column

Based on above three criteria the more rigorous title of the subject is: "Mechanics of Soilds". Any structure is generally built up from the following members:

1. Members under axial forces such as truss members (see Fig.4). Strength of these members will be studied in Chapter 4. Stability of these members will be studied in Chapter 14.

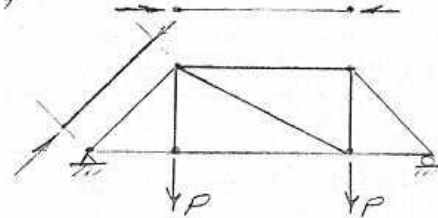


Fig.4:Members under axial forces

2. Members under torque(see Fig.5). Strength and deformation of these members will be studied in Chapter 5.

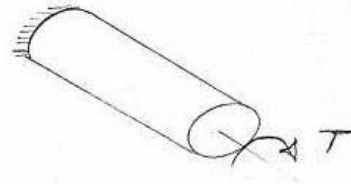


Fig.5:Members under Torque

3. Members under bending moment (see Fig.6). Strength of these members will be studied in Chapters 6&7. Deformation will be studied in Chapter 11.



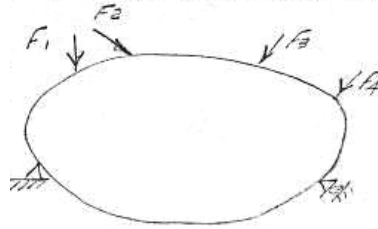
Fig.6:Members under Bending Moment

4. Members under compound stresses. Strength of these members will be studied in Chapter 8.

Review of Reactions:



Consider a body of any shape acted upon by the forces in the figure below:



In engineering mechanics, it would be started by determining reactions based on the equations of equilibrium. It was showed that the equations of equilibrium of coplaner force system can take any one of the following three equivalent forms:

1st Set : $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$.

2nd Set: $\Sigma F_x = 0$, $\Sigma M_O = 0$, $\Sigma M_A = 0$, where the line OA must not be along x-axis.

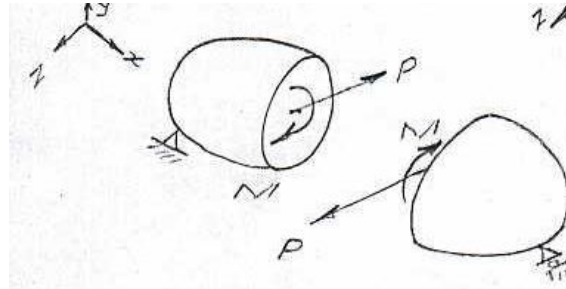
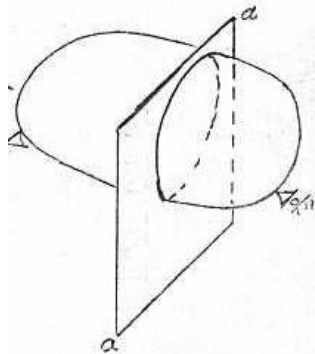
3rd Set: $\Sigma M_O = 0$, $\Sigma M_A = 0$, $\Sigma F_B = 0$, where the points O, A and B must not be on the same line.

H.W. Problems: 2.1 and 2.2 (Popov).

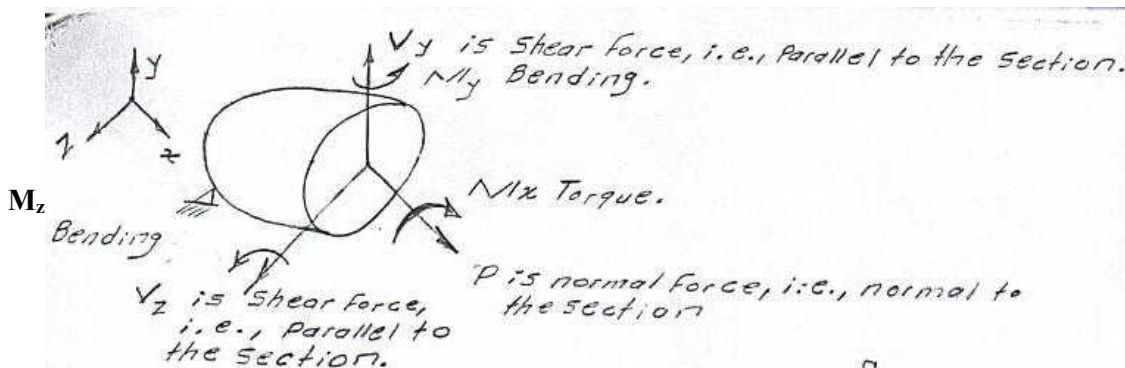
Internal Forces:

In mechanics of materials an additional investigation is necessary for the internal distribution of forces.

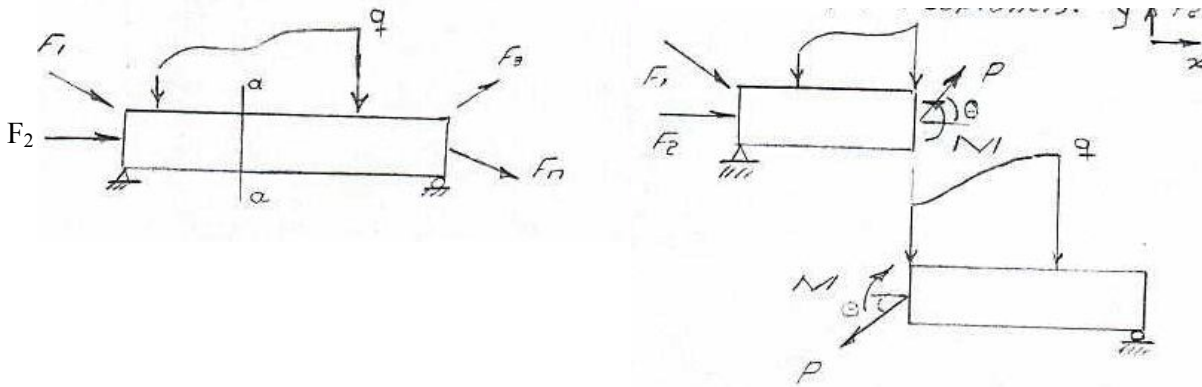
This is done by passing an imaginary section through the body. One will find generally a force and a couple as shown below:



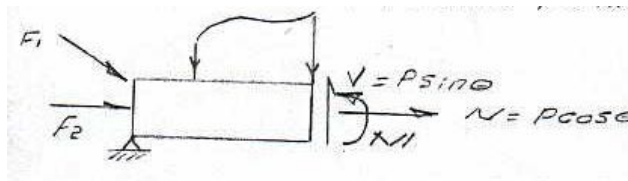
Force P and couple M are necessary to maintain equilibrium of free body diagram of either segment. For convenience, the internal force and couple can be analyzed into their rectangular components



For plane problems, any imaginary section will contain a force and a couple (Coplaner):



The force P on any side of section can be resolved into two rectangular components, one is normal to the section (Normal force) and the other is parallel to the section (Shear force)





Chapter Three Simple Stress and Axial Force

Normal Stress (Simple Stress)

For section a-a of the body shown, only normal force "P" will occur to maintain the equilibrium of the upper or lower parts. If the force P can be assumed to uniformly distributed on the whole area of section, then the normal stress can be defined as:

$$\sigma = \frac{P}{A} \quad \text{--- Eq. a}$$

where σ is normal stress "Greek letter symbol read as Sigma"

Since stress represent the intensity of force on an area, stress is measured in units of force divided by units of area.

In English System

$$\text{Pound/inch}^2 = \frac{\text{lb}}{\text{in}^2} = \text{psi}$$

$$\text{or } \frac{\text{kilo-Pound}}{\text{inch}^2} = \frac{\text{kips}}{\text{in}^2} = \text{ksi}$$

In SI System

$$\frac{\text{Newton}}{\text{meter}^2} = \frac{\text{N}}{\text{m}^2} = \text{Pascals} = \text{Pa}$$

$$\text{or } \frac{\text{Newton}}{10^6 \text{ mm}^2} = \frac{10^6 \text{ N}}{\text{mm}^2} = 10^6 \text{ Pa} = \text{MPa}$$

In M.K.S System

$$\frac{\text{Kilogram}}{\text{meter}^2} = \frac{\text{kg}}{\text{m}^2} \text{ or } \frac{\text{kg}}{\text{cm}^2}$$

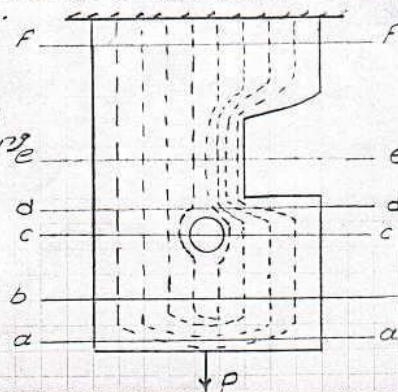
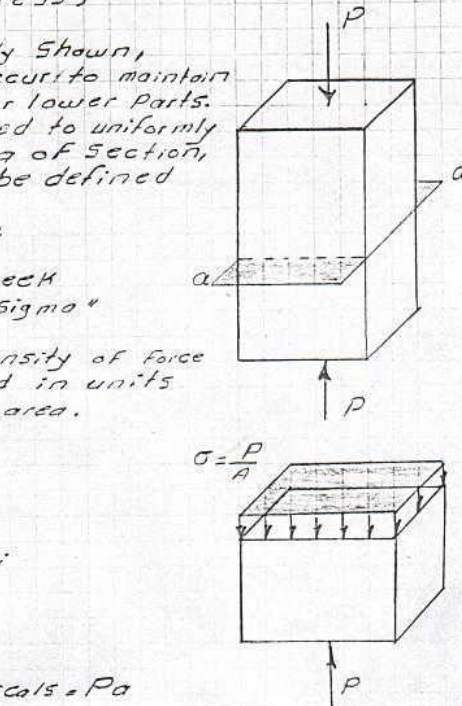
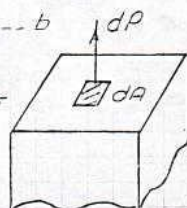
The force can be assumed uniformly distributed if the following conditions have been satisfied:-

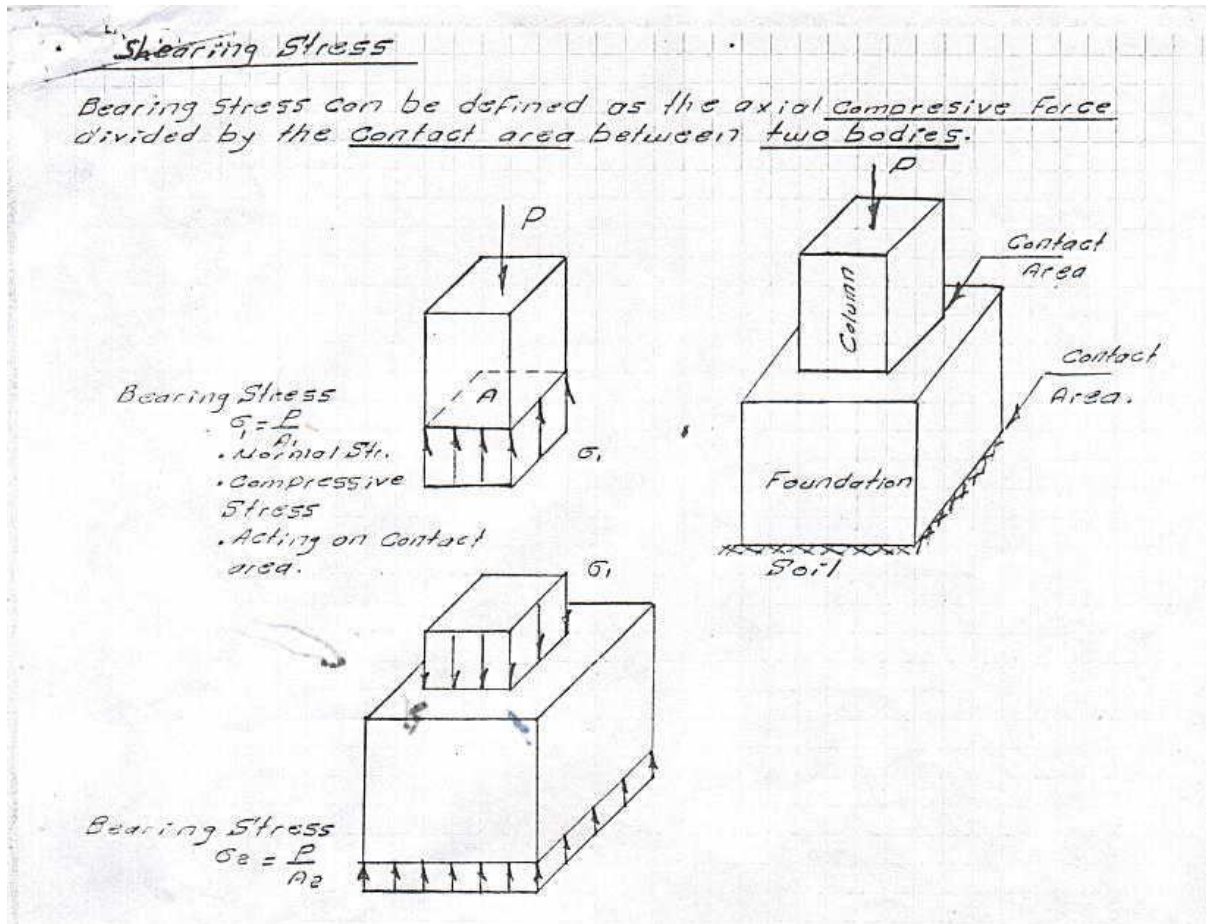
1. Must pass through centroid of section (Sec. e-e).
2. There is no abrupt changes in section (Sec. a-a and d-d).
3. Section is not too close to point where the load is applied (Section a-a).

General definition of normal stress (uniformly or non-uniformly distributed) can be written in the following form:-

$$\sigma = \frac{dP}{dA} \quad \text{--- b}$$

From eq. b, it's clear that stress concept is hold true only at a point.







Simple Stress

Problem 3.3

Solution

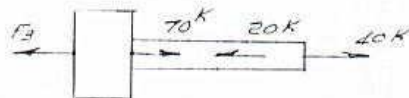
Sec. 1-1

$$\begin{aligned} \oplus \sum F_x &= 0 \\ 40 - F_1 &= 0 \Rightarrow F_1 = 40 \text{ K} \leftarrow \\ \sigma_1 &= \frac{F_1}{A_1} = \frac{40}{2} \Rightarrow \sigma_1 = 20 \text{ Ksi} \oplus \end{aligned}$$

Sec. 2-2

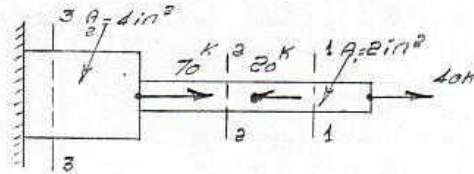
$$\begin{aligned} \oplus \sum F_x &= 0 \\ -F_2 - 20 + 40 &= 0 \\ F_2 &= 20 \text{ K} \leftarrow \\ \sigma_2 &= \frac{F_2}{A_2} = \frac{20}{2} \Rightarrow \sigma_2 = 10 \text{ Ksi} \oplus \end{aligned}$$

Sec. 3-3



$$\begin{aligned} \oplus \sum F_x &= 0 \\ -F_3 + 70 - 20 + 40 &= 0 \Rightarrow F_3 = 90 \text{ K} \leftarrow \\ \sigma_3 &= \frac{F_3}{A_3} = \frac{90}{4} = 22.5 \Rightarrow \sigma_3 = 22.5 \text{ Ksi} \oplus \end{aligned}$$

$$\sigma_{max} = \text{Maximum} [\sigma_1, \sigma_2, \sigma_3] \Rightarrow \sigma_{max} = 22.5 \text{ Ksi} \oplus$$

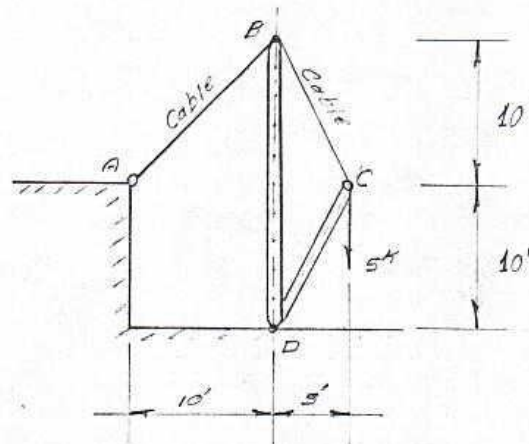
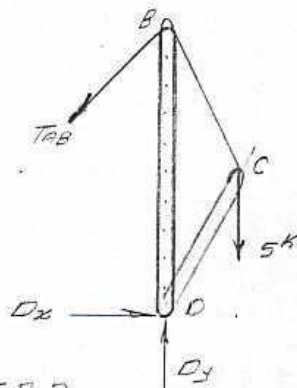


Problem 3.12

Solution

Whole frame as F.B.D.

$$\begin{aligned} \oplus \sum M_D &= 0 \\ 5 \times 5 - T_{BC} \times \frac{1}{\sqrt{2}} \times 20 &= 0 \\ T_{BC} &= 1.77 \text{ K} \end{aligned}$$



Joint B as F.B.D.

$$\begin{aligned} \oplus \sum M_C &= 0 \\ F_{BD} \times 5 - 1.77 \times \frac{1}{\sqrt{2}} \times 15 &= 0 \end{aligned}$$

$$\therefore F_{BD} = 3.75 \text{ K}$$

From table of appendix,

$$A = 8.399 \text{ in}^2$$

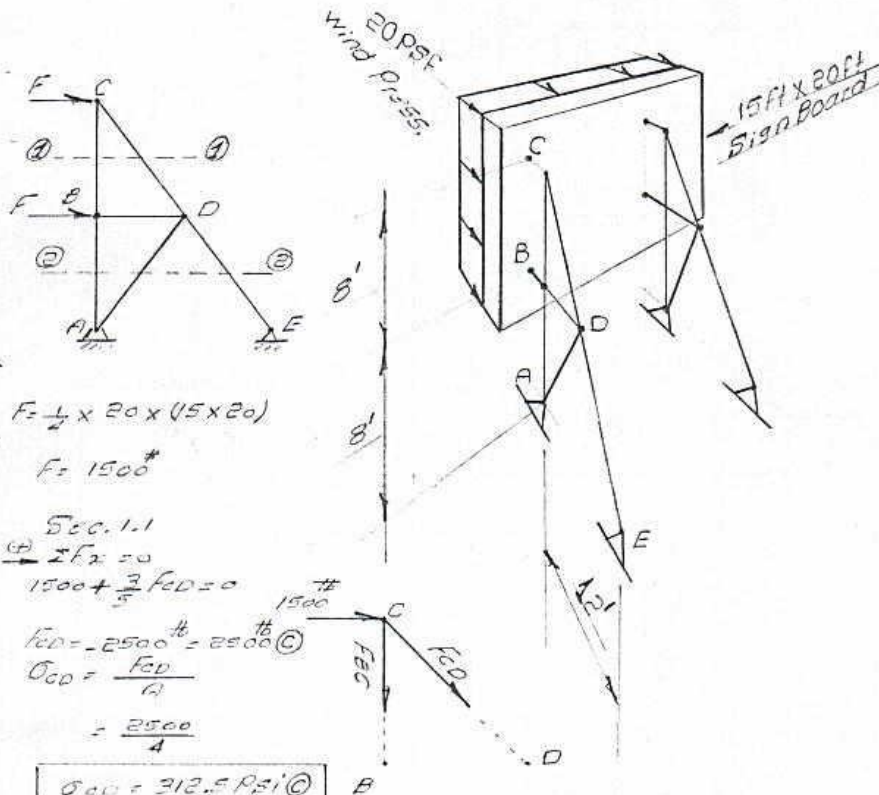
$$\therefore \sigma_{BD} = \frac{F_{BD}}{A} = \frac{3.75}{8.399} = .446 \text{ Ksi} \Rightarrow \sigma_{BD} = 446 \text{ PSI} \oplus$$



Problem 3.13

Solution

- All members are actually 2 in by 4 in.
- Assume one quarter of the total wind force acts at B and at C.

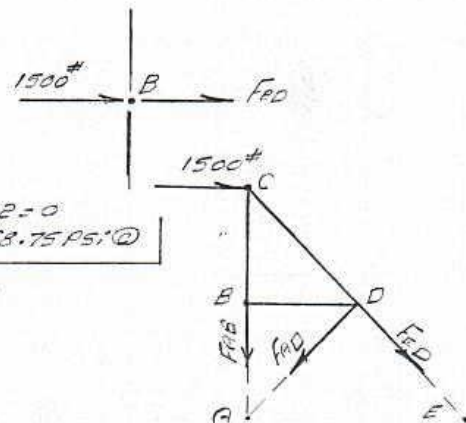


$\sum F_y = 0 \Rightarrow -F_{BC} - (-2500 \times \frac{4}{5}) = 0 \Rightarrow F_{BC} = 2000^{\#} \text{ (T)}$
 $\sigma_{BC} = 250 \text{ PSI (T)}$

Joint B
 $\sum F_x = 0 \Rightarrow F_{BD} = -1500^{\#} = 1500^{\#} \text{ (C)}$
 $\sigma_{BD} = 187.5 \text{ PSI (C)}$

Sec. 2.2
 $\sum M_A = 0 \Rightarrow 1500 \times 8 + 1500 \times 16 + F_{ED} \times \frac{4}{5} \times 12 = 0$
 $\Rightarrow F_{ED} = -3750^{\#} = 3750^{\#} \text{ (C)} \Rightarrow \sigma_{ED} = 468.75 \text{ PSI (C)}$
 $\sum M_C = 0$
 $F_{AD} \times \frac{3}{5} \times 16 - 1500 \times 8 = 0 \Rightarrow F_{AD} = 1250^{\#} \text{ (T)}$
 $\sigma_{AD} = 156.25 \text{ PSI (T)}$

$\sum M_D = 0$
 $F_{AB} \times 6 - 1500 \times 8 = 0 \Rightarrow F_{AB} = 2000^{\#} \text{ (T)}$
 $\sigma_{AB} = 250 \text{ PSI (T)}$





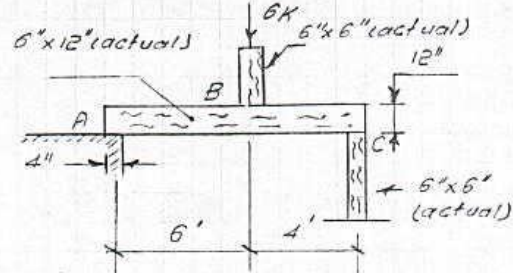
Bearing Stress

Example 1. Determine the bearing stress caused by the applied forces at A, B, and C for the structure shown in the figure.

Solution

$$\sigma_B = \text{bearing stress @ B.} \\ = \frac{6K}{6 \times 6} = .167 \text{ Ksi}$$

$$\sigma_B = .167 \text{ Ksi} \\ \text{bearing}$$



$$\uparrow \sum \mathcal{M}_C = 0 \\ R_A \times 10 - 6 \times 4 = 0 \Rightarrow R_A = 2.4 \text{ K}$$

$$\sigma_A = \frac{2.4K}{4 \times 6} = .1 \text{ Ksi} \Rightarrow \sigma_A = .1 \text{ Ksi} \\ \text{bearing}$$

$$\uparrow \sum F_y = 0 \\ 2.4 - 6 + R_C = 0 \Rightarrow R_C = 3.6 \text{ Kips}$$

$$\therefore \sigma_C = \frac{3.6}{6 \times 6} = .1 \text{ Ksi} \Rightarrow \sigma_C = .1 \text{ Ksi} \\ \text{bearing}$$



Shearing Stress.

Example 1. A hole is to be punched out of the plate shown below. Determine the Shear Stress.

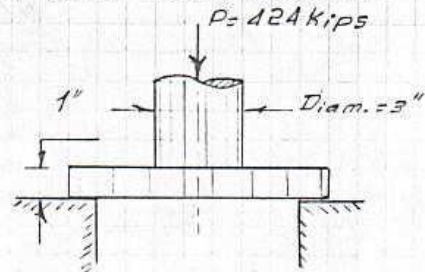
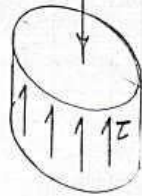
Solution

$$\tau = \frac{P}{A}$$

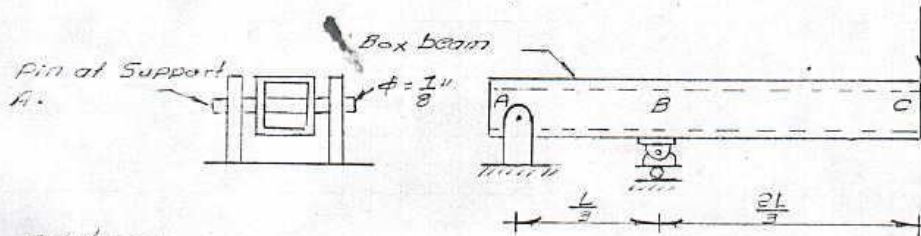
$$= \frac{424 \text{ kips}}{(3 \times \pi) \times 1''}$$

$$\tau = 44.9 \text{ ksi}$$

424 kips



Example 2 For the box beam shown below. Determine the average Shear Stress in the pin at A.



Solution

$$\sum F_x = 0$$

$$R_{Ax} = 0$$

$$\sum M_B = 0$$

$$R_{Ay} \times \frac{L}{3} - 3000 \times \frac{2L}{3} = 0 \Rightarrow R_{Ay} = 6000$$

$$\tau_{avg} = \frac{6000}{2 \times (\frac{\pi}{4} \times \frac{1}{2}^2)} = 4991.6 \text{ psi}$$

double shear

$$\tau = 4991.6 \text{ psi}$$

Notes

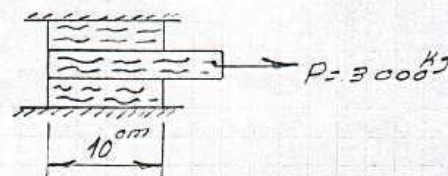
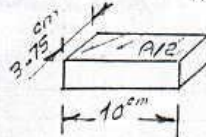
As will be shown in chapter 7, the Shear Stress through section is not distributed uniformly. Then the object

Example 3. Three pieces of wood having $3.75 \text{ cm} \times 3.75 \text{ cm}$ square cross sections are glued together and to the foundation as shown below. If the horizontal force $P = 3000 \text{ Kg}$, what is the average Shearing Stress in each of the glued joints.

Solution

$$\tau_{avg} = \frac{P}{A} = \frac{3000}{10 \times 3.75 \times 2}$$

$$\tau_{avg} = 40 \text{ Kg/cm}^2$$





Example 4. In Figure shown below a lever is attached to a 10.41 cm spindle by means of a square key 6 mm x 6 mm x 2.5 long. Calculate the average shear stress in the key.

Solution

Force Supported by the Key = F

$\sum M_o = 0$

$75 \times 35 - F \times 2.5 = 0$

$\therefore F = 1050 \text{ Kg.}$

$\tau_{avg} = \frac{F}{A}$

$= \frac{1050 \text{ Kg}}{.6 \times 2.5}$

$\tau_{avg} = 700 \text{ Kg/cm}^2$

Alloable Stresses; Factor of Safety:

In laboratory, specimens of known material are carefully prepared to desired dimensions. Then these specimens are subjected to successively increasing known forces. The force necessary to cause rupture is called the ultimate load. By dividing this ultimate load by the original cross-sectional area of the specimen, the ultimate strength (stress) of the material is obtained.

For design of members, the stress level called allowable stress is set considerably lower than the ultimate strength. The necessity is for the following reasons:

1. the exact magnitudes of the forces that may act upon the structure are seldom accurately known.
2. Materials are not uniform.



Allowable Stress

Problem 3.6

$$a. \sigma_{\text{Bearing}} = \frac{P}{A}$$

$$= \frac{12}{6 \times 6} = 0.333 \text{ Ksi}$$

$$\sigma_{\text{Bearing}} = 0.333 \text{ Ksi}$$

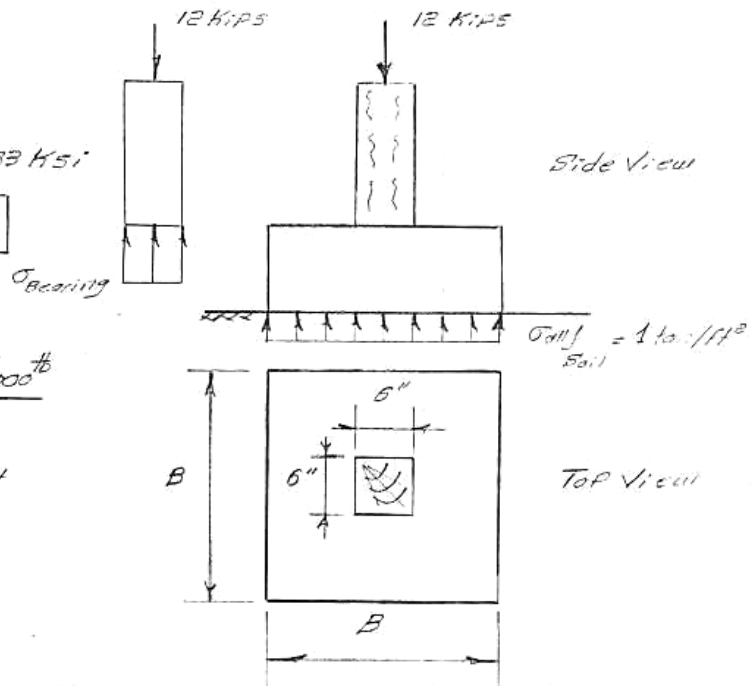
$$b. \sigma_{\text{Bearing}} = \frac{P}{B \times B}$$

$$1 \text{ ton/ft}^2 = 2000 \text{ lb/ft}^2 = \frac{12000 \text{ lb}}{6 \times 6}$$

$$\therefore B \times B = 6 \text{ ft}^2$$

$$\therefore B = \sqrt{6} = 2.449 \text{ ft}$$

$$\text{or } B = 29.39 \text{ in}$$



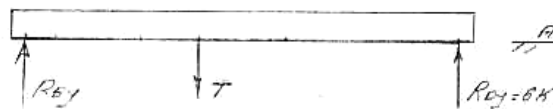
Problem 3.8

F.B.D. of whole structure

$$\sum M_A = 0$$

$$R_{Dy} \times 18'' - 9 \times 12'' = 0 \Rightarrow R_{Dy} = 6 \text{ Kips}$$

Part BCD as F.B.D.



$$\sum M_B = 0$$

$$T \times 6 - 6 \times 15 = 0 \Rightarrow T = 15 \text{ Kips} \oplus$$

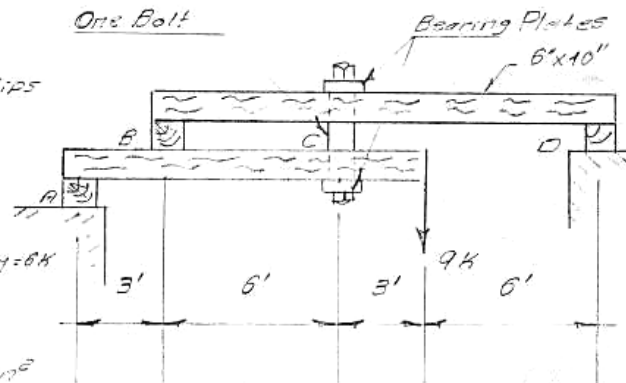
$$\sigma_{\text{tensile}} = \frac{T}{A} \Rightarrow A = \frac{15}{0.333} = 45 \text{ in}^2$$

$$\Rightarrow A = \frac{(D_{\text{rod}})^2 \cdot \pi}{4} = 45 \Rightarrow D_{\text{rod}} = 1.0''$$

$$\sigma_{\text{allow}} = \frac{T}{A} \Rightarrow A = \frac{15 \text{ Kips}}{0.333} = 45 \text{ in}^2$$

$$A = \frac{15 \text{ Kips}}{0.333} = 45 \text{ in}^2$$

$$\Rightarrow A = 45 \text{ in}^2$$

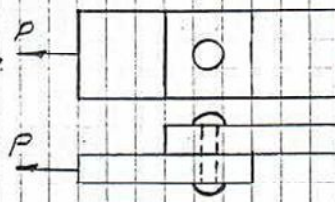




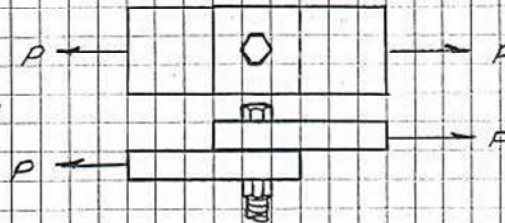
Connections 1. Introduction

most Structures are members that must be connected to each other. Three types of connection are usually used in Structures assembly as follows:-

1. Rivets and Ordinary Bolts Connections

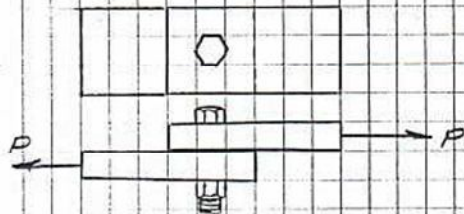


Rivet Connection



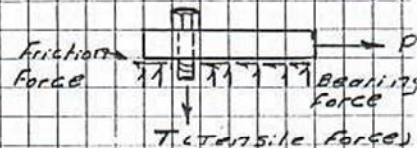
Ordinary Bolt Connection

2. High Strength Bolts Connections



High Strength Bolts Connection

When bolts or rivets are tightened during the construction, tensile force will be formed in the body of bolts or rivets, the reaction due to the action of this tensile force will be a bearing force that acts on the contact surfaces of the connected parts. The friction forces will be created on the contact surfaces of the connected parts.



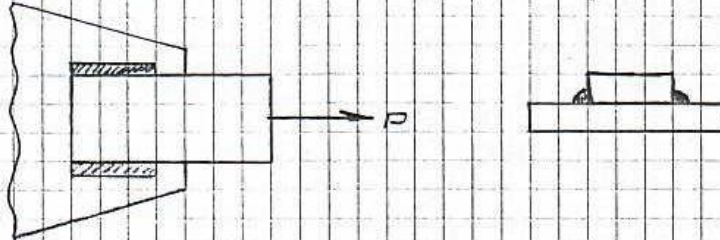
In high strength bolts, the tensile force in body of bolt is very large that, the connection strength is based on friction force that form on contact surfaces of the connected parts.

In rivets and ordinary bolts, the tensile force in body of rivet



will be small and can't be considered as a design basis

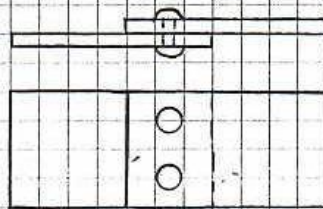
3. Welding Connection



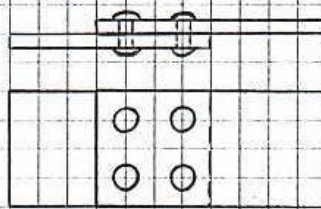
High Strength bolt connection and welding connection are out the scope of our study. Behaviors of rivets and ordinary bolts connection are nearly the same, then they will be studied together.

2. Types of Connection

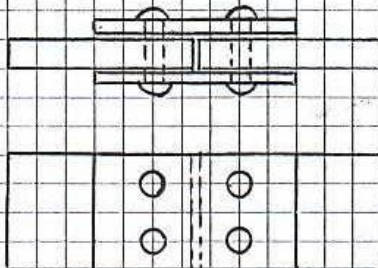
Several typical connections, with self-explanatory titles are shown in Fig. below.



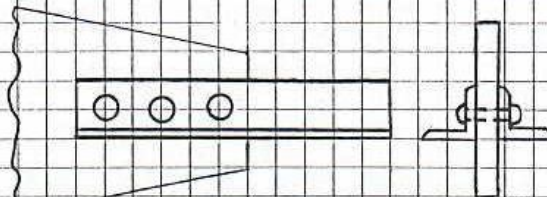
Single Riveted Lap Joints



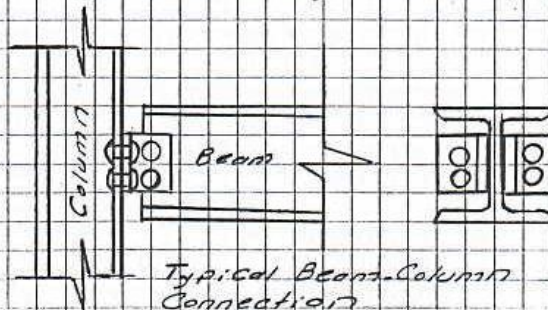
Double Riveted Lap Joint



Butt Riveted Joint



Detail of Angle Attachment



Typical Beam-Column Connection

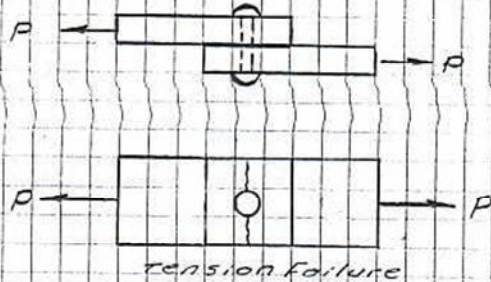


3. Methods of Failure of a Riveted Joint

Rivets and Ordinary bolts joints may fail in a number of different ways:-

a. Failure in Tension

A joint transmitting a tensile force may fail in a plate weaker by the rivet holes. For example, in a Single - riveted lap joint, the net area in either plate across the section A-A in Fig. below is the least area, and a tear would occur there.



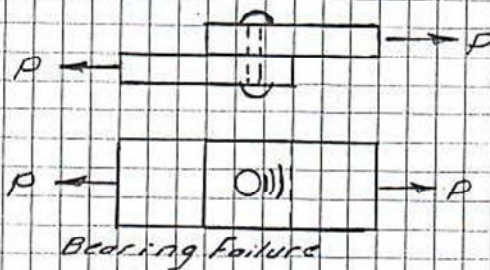
b. Failure in Shear

In riveted or bolted joints, the rivets or bolts themselves may fail in shear.



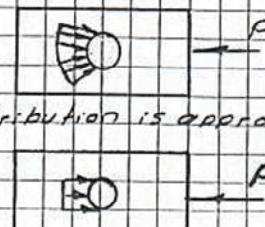
c. Failure in Bearing

A riveted joint may fail if a rivet crushes the material of the plate against which it bears



The stress distribution is very complicated in this type of failure and is somewhat like that shown below:

In practice, distribution is approximately as shown

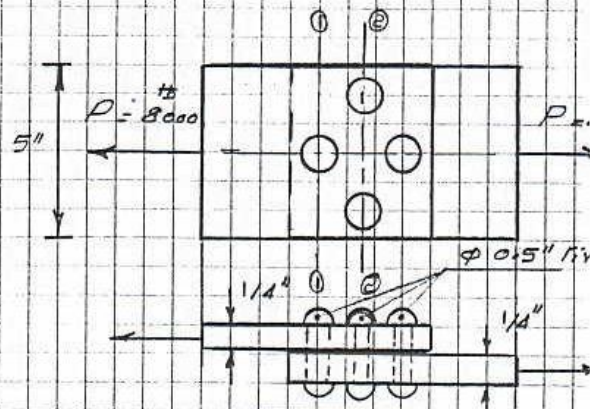




Example 1

For the lap joint shown,
determine the following:

1. Tensile Stress in the gross area.
2. Tensile Stress in the first and second rows of rivets.
3. Shearing Stress in the rivets.
4. Bearing stress on the side of the rivets.



Solution

$$1. \sigma_{gross} = \frac{P}{A} = \frac{8000}{(5 \times 2.5)} = 6400 \text{ PSI}$$

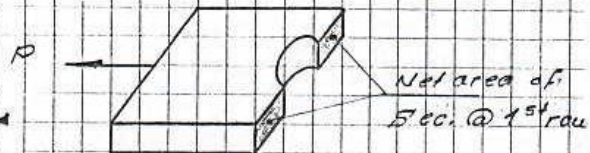


$$2. \sigma_{net1} = \frac{P}{A}$$

$$1^{st} \text{ row}$$

$$A = (5 - 0.5) \times 2.5 = 1.125 \text{ in}^2$$

$$\therefore \sigma_{net1} = \frac{8000}{1.125} = 7111.1 \text{ PSI}$$



$$\sigma_{net2} = \frac{P_2}{A'}$$

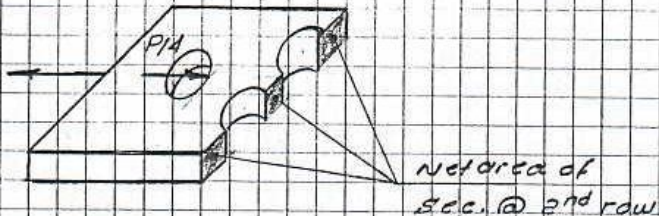
$$2^{nd} \text{ row}$$

$$A' = (5 - 2 \times 0.5) \times 1/4$$

$$A' = 1 \text{ in}^2$$

$$P_2 = P - P/4 = \frac{3}{4} P = 6000$$

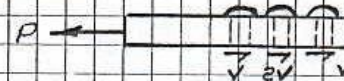
$$\therefore \sigma_{net2} = \frac{6000}{1} = 6000 \text{ PSI}$$



$$3. V = \text{Shear Force Per rivet}$$

$$= \frac{P}{\text{No. of rivets}} = \frac{8000}{4}$$

$$V = 2000 \text{ #}$$

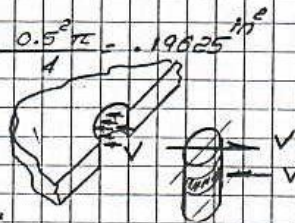


$$\tau_{rivets} = \frac{V}{A_{of rivet}}, A = \frac{0.5^2 \pi}{4} = .19625 \text{ in}^2$$

$$\therefore \tau_{rivets} = 10191 \text{ PSI}$$

$$4. A_{bearing} = 0.5 \times 2.5 = .125$$

$$\sigma_{bearing} = \frac{V}{A_{bearing}} = \frac{2000}{.125}$$





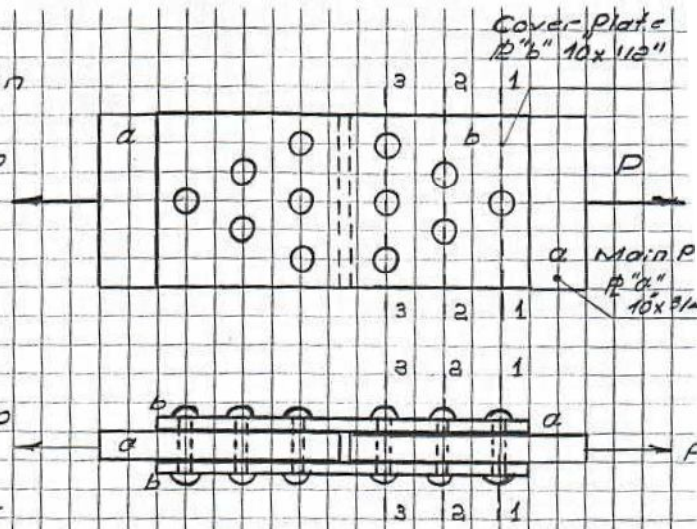
Example 2

For the butt joints shown
if: $P = 80$ kips

ϕ For all rivets = 1"

Determine the following P

1. Tensile Stress in main plate at first and second rows of rivets.
2. Tensile Stress in cover plate at third row of rivets.
3. Shearing Stress in the rivets.
4. Bearing Stress on the side of rivets in the main and cover plates.



Solution

1. Tensile Stress in Main Plate

$$\sigma_{main} = \frac{80}{10 - 1 \times 3/4} = 11.85 \text{ Ksi}$$

$$\sigma_{main} = \frac{80 - 80}{10 - 2 \times 1} = 11.1 \text{ Ksi}$$

2. Tensile Stress in Cover Plate

$$\sigma_{cover} = \frac{80}{10 - 3 \times 1} = 11.4 \text{ Ksi}$$

3. Shearing Stress in Rivets

$$80 = (V + 2V + 3V) \times 2$$

$$V = \frac{80}{12} = 6.67 \text{ K}$$

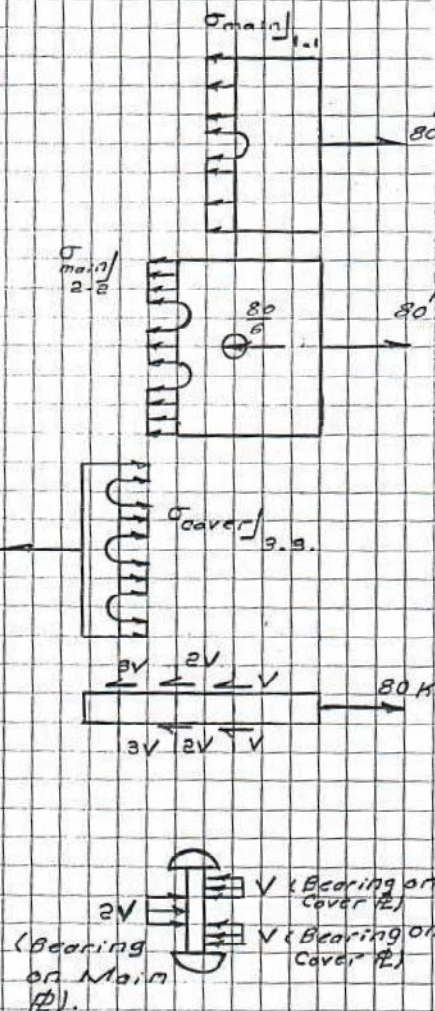
$$A_{\text{rivet}} = \frac{\pi \phi^2}{4} = 0.785 \text{ in}^2$$

$$\tau = \frac{V}{A_{\text{rivet}}} = \frac{6.67}{0.785} = 8.497 \text{ Ksi}$$

4. Bearing Stress

$$\sigma_{\text{bearing}} = \frac{2V}{1 \times 3/4} = \frac{2 \times 6.67}{1 \times 3/4} = 17.79 \text{ Ksi}$$

$$\sigma_{\text{bearing}} = \frac{V}{1 \times 1/2} = \frac{6.67}{1 \times 1/2} = 13.34 \text{ Ksi}$$





Biaxial Tension and Compression (Thin-Walled Pressure Vessels)

The study of thin-walled pressure vessels is important due to the following two reasons:-

1. It is important to reach some conclusions regarding the effect of biaxial stress on the resistance to failure of various materials.
2. Cylindrical and spherical pressure vessels are exceedingly important in industry.



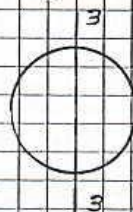
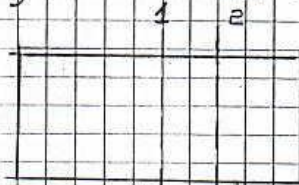
Cylindrical Pressure Vessels



Spherical Pressure Vessel

Analysis of Cylindrical Pressure Vessel

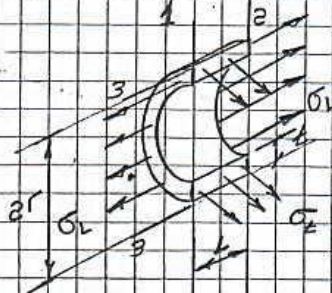
Analysis of Cylindrical Pressure Vessel will be considering a segment that isolated from this vessel by passing two planes perpendicular to the axis of cylinder and one additional longitudinal plane through the same axis.



where
 σ_L = Longitudinal Stress

σ_t = Tangential Stress
(Hoop Stress)

σ_L can be computed from left or right side of section



where P the internal pressure in excess of the external pressure (gauge pressure)



$$\begin{aligned} \sum F_x = 0 \\ \sigma_L \times (2\pi r) \times t - p \times (\pi r^2) = 0 \\ \therefore \sigma_L = \frac{pr}{2t} \end{aligned}$$

where

r is the radius of cylindrical vessel.
 t is the wall thickness of vessel.
 p is the gage pressure.

The σ_t (tangential stress) can be computed from Section 3.3

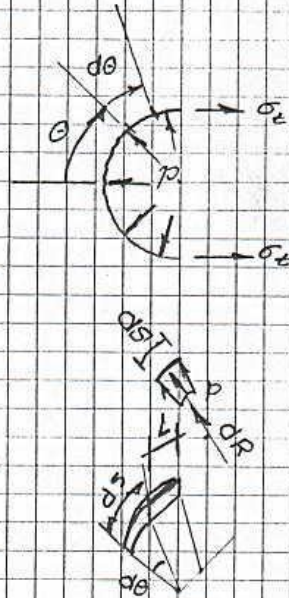
$$dR = (p d\theta L) = R(r d\theta)L$$

$$\begin{aligned} \sum F_x = 0 \\ -2 \int_0^{\pi/2} dR \cos\theta + \sigma_t \times (L \times t) = 0 \\ \therefore 2\sigma_t (L \times t) = 2 \int_0^{\pi/2} dR \cos\theta \end{aligned}$$

$$\begin{aligned} &= 2 \int_0^{\pi/2} (R r L) \cos\theta d\theta \\ &= 2 p r L \int_0^{\pi/2} \cos\theta d\theta \\ &= 2 p r L \times [\sin\theta]_0^{\pi/2} \end{aligned}$$

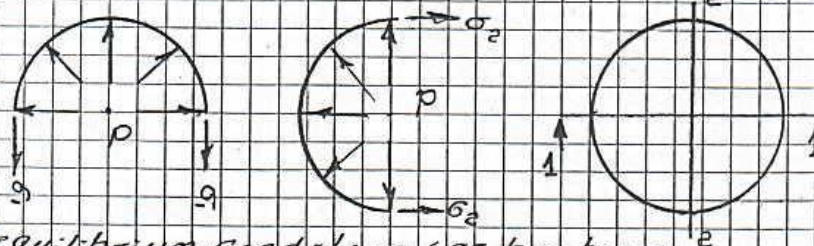
$$2\sigma_t (L \times t) = 2 p r L$$

$$\sigma_t = \frac{pr}{t}$$



Analysis of Spherical Pressure Vessel

For sphere any section which passes through the center of the center of sphere yield the following results



Based on equilibrium condition (as has been done above for σ_L) one can show that

$$\sigma_1 = \sigma_2 = \sigma = \frac{pr}{2t}$$



Example 1

A water main pipe of 2 meter diameter and 2cm thickness is subjected to an internal pressure of 15 kg/cm². Calculate the circumferential (tangential or hoop) and longitudinal stress in the pipe.

Solution $P = P_i$ - Atmospheric Pressure $\rightarrow P = P_i - (20) = P_i$

$$\sigma_L = \frac{Pr}{2t} = \frac{15 \times (100)^2}{2 \times 2} = 375 \text{ kg/cm}^2 \quad \Delta$$

$$\sigma_t = \frac{Pr}{t} = \frac{15 \times 100}{2} = 750 \text{ kg/cm}^2 \quad \Delta$$

Example 2 A gas cylinder of internal diameter 1.5m is 3cm thick. Find the allowable pressure of the gas inside the cylinder, if the tensile stress in the material not to exceed 1000 kg/cm².

Solution

$\therefore \sigma_t = 2\sigma_L$ $\therefore$ the pressure will be computed based on σ_t .

$$\sigma_t = \frac{Pr}{t} \rightarrow 1000 = \frac{P \times 150/2}{3} \rightarrow P = 40 \text{ kg/cm}^2 \quad \Delta$$

Example 3

A water main 80cm diameter contains water at a pressure of 10 kg/cm². Find the thickness of metal required for the water main. Given the permissible stress is 200 kg/cm².

Solution :

As the $\sigma_t = 2\sigma_L$, the thickness of pipe will be computed based on σ_t

$$\sigma_t = \frac{Pr}{t} \rightarrow 200 \frac{\text{kg}}{\text{cm}^2} = \frac{10 \frac{\text{kg/cm}^2}{\text{cm}^2} \times 40 \text{ cm}}{t} \rightarrow t = 2 \text{ cm} \quad \Delta$$

Example 4.

A spherical vessel of 3m diameter is subjected to an internal pressure of 15 kg/cm². Find the thickness of the plate required if maximum stress is not to exceed to 675 kg/cm².

Solution

$$\sigma = \frac{Pr}{2t} \rightarrow 675 = \frac{15 \times \frac{300}{2}}{2t} \rightarrow t = 1.67 \text{ cm}$$

$$\text{Say } t = 1.75 \text{ cm} \quad \Delta$$



Chapter Four Simple Strain

Strain Definition:

The elongation per unit of length is termed linear strain.

— Physically:

$$\epsilon = \int_{l_0}^l \frac{dl}{l_0} = \frac{1}{l_0} [l]_{l_0}^l = \frac{1}{l_0} (l - l_0) = \frac{\Delta l}{l_0}$$

In general, a body also may be deformed as in Fig. (4.2)

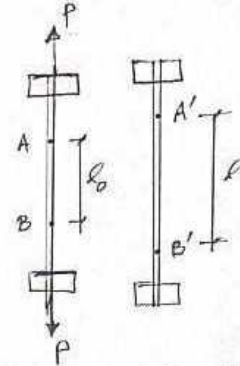


Fig. (4.1): Axial Deformation

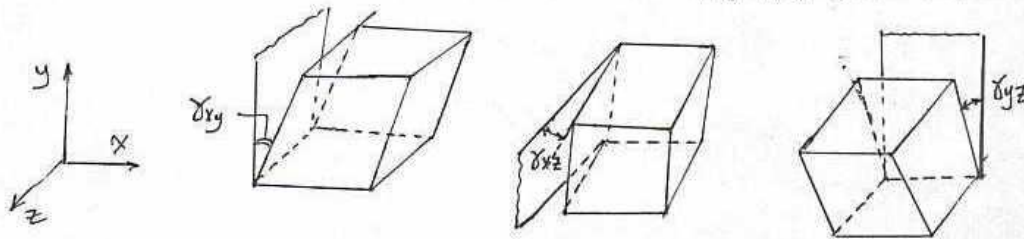


Fig. (4.2): Shearing Deformation

Such deformations cause a change in the initial right angles of the body. This angle change defines the shearing strain.

— Mathematically:—

Point A moves to A' by displacement (u).

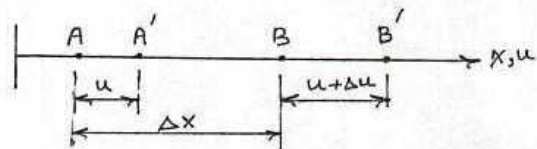


Fig. (4-3)

u: is rigid body motion (R.B.M.)

Point B moves to B' by (u + \Delta u)

\Delta u: is the stretch (deformation) in the element \Delta x

$$\epsilon_x = \lim_{\Delta x \rightarrow 0} \frac{\Delta u}{\Delta x} = \frac{du}{dx}$$



If the body strained in orthogonal directions (2-directions) as shown in Fig. (4-4):

$$\epsilon_y = \frac{dv}{dy}$$

And, similarly:

$$\epsilon_z = \frac{dw}{dz}$$

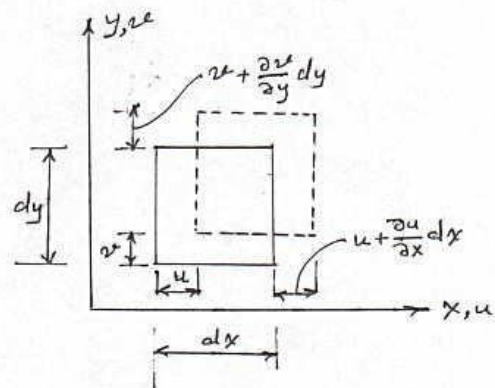


Fig. (4-4)

In the case of shearing strain,

Consider Fig. (4-5) below:

$\frac{\partial v}{\partial x}$: is the slope of initially horizontal side of the element.

$\frac{\partial u}{\partial y}$: is the slope of initially vertical side of the element.

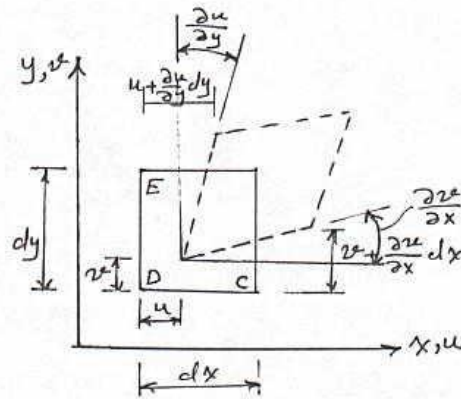


Fig. (4-5)

The right angle CDE is reduced by $\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$. This called shearing strain in xy coordinate γ_{xy} .

$$\therefore \gamma_{xy} = \gamma_{yx} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Similarly:

$$\gamma_{xz} = \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$\text{And, } \gamma_{yz} = \gamma_{zy} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$

γ_{xy} , γ_{xz} and γ_{yz} are called Engineering shearing strains. The mathematical shearing strains are defined by:

$$\epsilon_{xy} = \epsilon_{yx} = \frac{\gamma_{xy}}{2} = \frac{\gamma_{yx}}{2}$$

$$\epsilon_{xz} = \epsilon_{zx} = \frac{\gamma_{xz}}{2} = \frac{\gamma_{zx}}{2}$$

$$\epsilon_{yz} = \epsilon_{zy} = \frac{\gamma_{yz}}{2} = \frac{\gamma_{zy}}{2}$$



Stress - Strain Diagram:

In tensile strength test on a steel specimen, the applied tensile stress can be plotted with observed tensile strain as shown for mild steel.

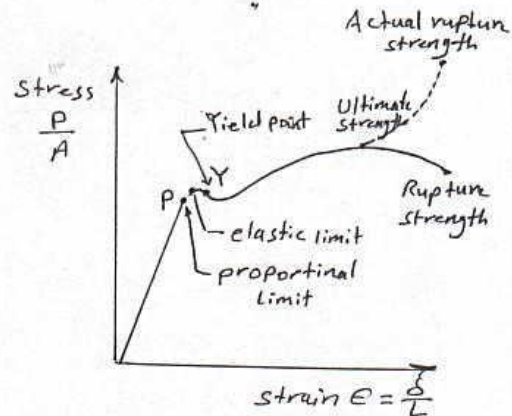


Fig. (4-6): Stress - strain Diagram for mild steel

Proportional Limit: Maximum stress in which the stress is a linear function of the strain.

Elastic Limit: The maximum stress at which there is no permanent deformation when the load is entirely removed.

Yield Point: The point at which there is an increase in strain with no increase in stress.

In brittle materials:

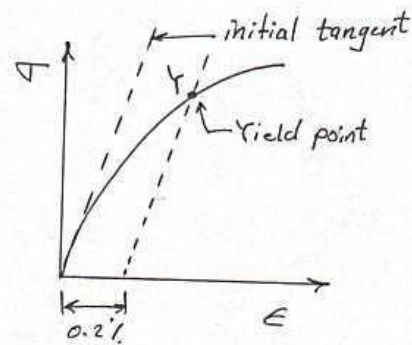
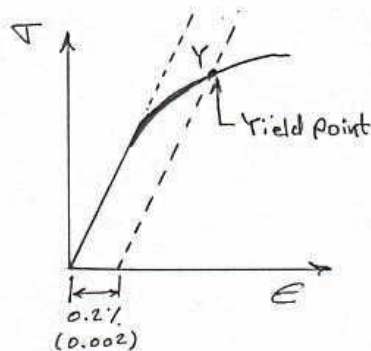


Fig. (4-7)

Modulus of Resilience: Is the area under stress-strain diagram and strain axis up proportional limit.

Or: It is the ability of the material to absorb energy. $U_0 = \frac{1}{2} \sigma_p \epsilon_p$



Modulus of Toughness: Is the area under the stress-strain diagram.

Elasticity: Is the ability of the material to recover completely its original dimensions upon removal of the load.

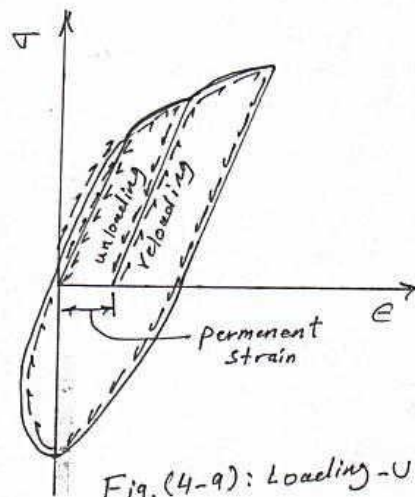
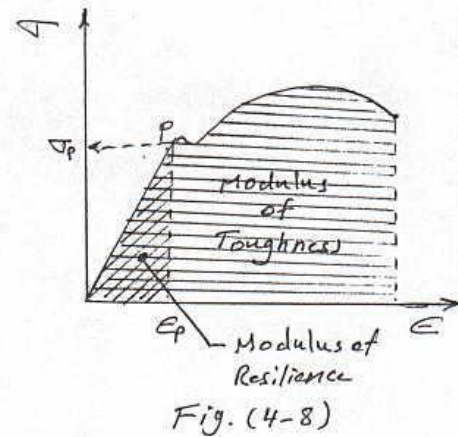


Fig. (4-9): Loading-Unloading-Reloading

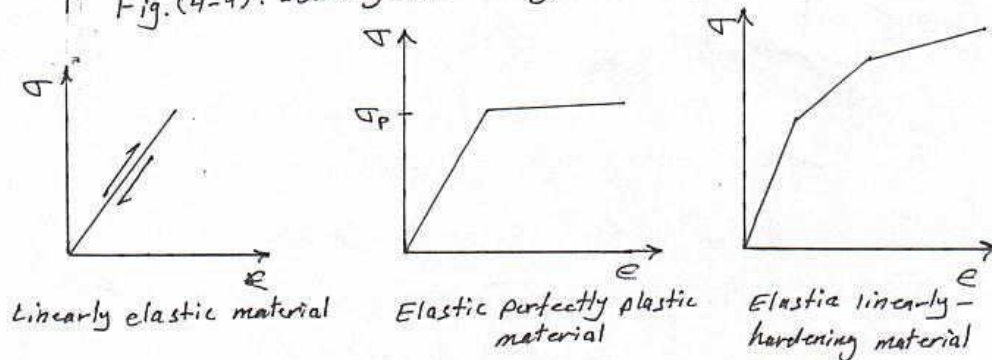


Fig. (4-10): Idealized stress-strain diagram

Ramberg and Osgood, developed the following equation to represent a wide range of stress-strain curves:

$$\frac{\sigma}{E_0} = \frac{\sigma}{\sigma_0} + \frac{3}{7} \left(\frac{\sigma}{\sigma_0} \right)^n$$

where, E_0 & σ_0 correspond the yield point. The exponent n determines the shape of the curve. Then $E = \frac{d\sigma}{d\epsilon}$



Hook's Law:

When the stress be in the proportional limit (the straight line in the stress-strain curve), the slope of that line is the ratio of stress to strain. It is called the modulus of elasticity E .

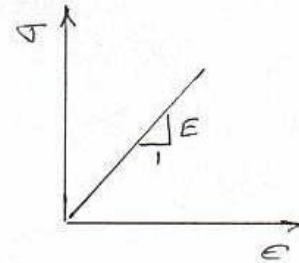


Fig. (4-11)

$$E = \frac{\sigma}{\epsilon}$$

or $\sigma = E\epsilon$ (Hook's Law).

Originally, Hook's Law was in the form:

$$\sigma \propto \epsilon$$

Thomas Young, 1807 introduced the proportionality constant known as Young's modulus (E).

Deflection of Axially Loaded Members:

Consider Fig. (4-12) shown below:

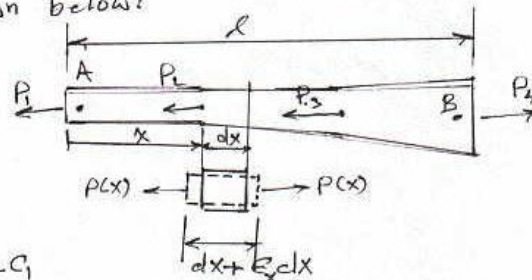


Fig. (4-12)

$$du = E_x dx$$

$$\frac{du}{dx} = E_x$$

$$\therefore u = \int_0^x du + C_1 = \int_0^x E_x dx + C_1$$

$$\text{But } E_x = \frac{\sigma_x}{E}$$

$$\text{and } \sigma_x = \frac{P}{A}$$

$$\therefore u = \int_0^x \frac{\sigma_x}{E} dx + C_1 = \int_0^x \frac{P(x)}{A(x)E} dx + C_1 \quad \text{----- ①}$$

A second relation is based on the equilibrium requirements for the infinitesimal element of an axially loaded bar as shown below:



$$\sum F_x = 0$$

$$P + dP + p(x)dx - P = 0$$

$$dP + p(x)dx = 0$$

$$\text{or } \frac{dP}{dx} = -p(x) \text{ ----- (2)}$$

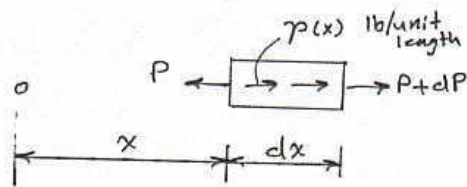


Fig. (4-13)

From Equation (1) : (by differentiation wrt x)

$$\frac{du}{dx} = \frac{P}{AE}$$

$$\text{or } P = AE \frac{du}{dx}$$

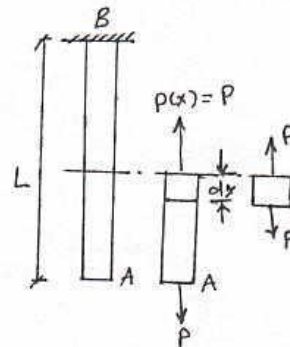
$$\frac{dP}{dx} = AE \frac{d}{dx} \left(\frac{du}{dx} \right) = AE \frac{d^2u}{dx^2} \text{ ----- (3)}$$

Substitute Equation (2) into Equation (3)

$$AE \frac{d^2u}{dx^2} = -p(x)$$

Example 1,

Find the deflection of the free end A with respect to B caused by the application of a concentrated force P. The elastic modulus of the material is E.



Solution:

Since the infinitesimal element has the same area A everywhere and subjected to a constant force P, then:

$$u = \frac{P}{AE} \int_0^x dx + C_1 = \frac{Px}{AE} + C_1$$

$$\text{At } x=0, u=0$$

$$0 = \frac{P(0)}{AE} + C_1 \Rightarrow C_1 = 0$$

$$\therefore u = \frac{Px}{AE}$$

$$u(L) = \Delta = \frac{PL}{AE}$$

- L -



Example 2:

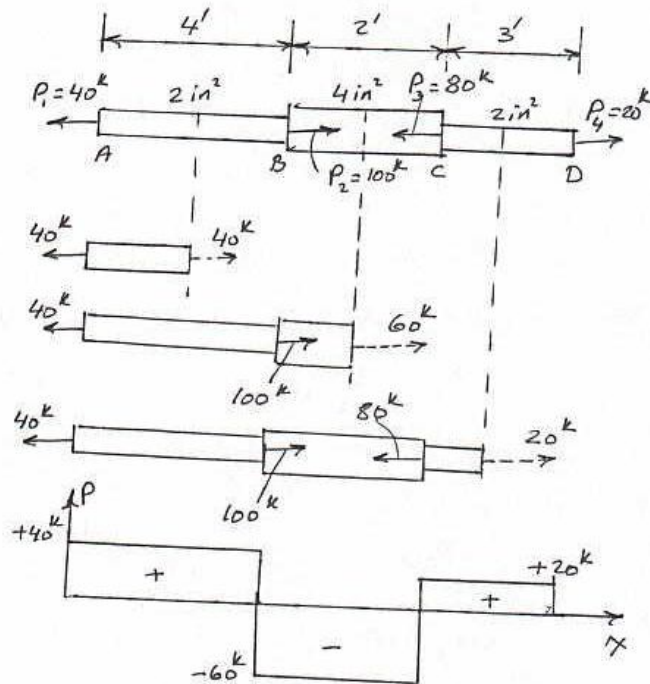
Determine the relative displacement of points A and D of the steel rod of variable cross-sectional area shown below when it is subjected to the four concentrated force P_1, P_2, P_3 and P_4 . Let $E = 30 \times 10^6$ psi.

Solution:

Equilibrium for whole body:

$$\sum F_x = 0$$

$$-40 + 100 - 80 + 20 = 0 \quad \therefore \text{o.k.}$$



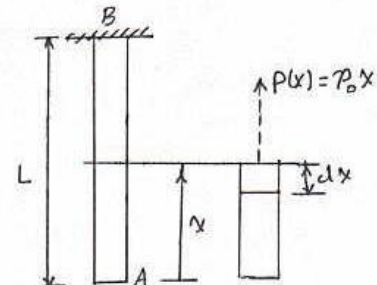
$$u = \int_0^L \frac{P(x)}{A(x)E} dx = \int_A^B \frac{P_{AB}}{A_{AB}E} dx + \int_B^C \frac{P_{BC}}{A_{BC}E} dx + \int_C^D \frac{P_{CD}}{A_{CD}E} dx$$

$$\therefore u = \sum \frac{PL}{AE} = \frac{40000(4 \times 12)}{2 \times 30 \times 10^6} - \frac{60000(2 \times 12)}{4 \times 30 \times 10^6} + \frac{20000(3 \times 12)}{2 \times 30 \times 10^6}$$

$$= +0.032 - 0.012 + 0.012 = +0.032 \text{ in.}$$

Example 3:

Find the deflection caused by its own weight of the free end A of the rod AB having a constant cross-sectional area A and weighing γ_0 lb/in.





Solution:

$$u = \int_0^x \frac{P(x)dx}{A(x)E} + C_1 = \frac{1}{AE} \int_0^x P_0 x dx + C_1$$

$$= \frac{P_0 x^2}{2AE} + C_1$$

At $x = L$, $u(L) = 0$

$$0 = \frac{P_0 L^2}{2AE} + C_1 \Rightarrow C_1 = -\frac{P_0 L^2}{2AE}$$

$$\therefore u = \frac{-P_0(L^2 - x^2)}{2AE}$$

$$u(0) = -\frac{P_0 L^2}{2AE} \text{ is the deflection of free end A.}$$

Another method by application of:

$$AE \frac{d^2 u}{dx^2} = -P(x)$$

$$AE \frac{d^2 u}{dx^2} = -(-P_0)$$

$$AE \frac{d^2 u}{dx^2} = P_0$$

$$AE \frac{du}{dx} = P_0 x + C_1$$

At $x = 0$, $\frac{du}{dx} = \epsilon_x = 0$

$$AE(0) = P_0(0) + C_1 \Rightarrow C_1 = 0$$

$$AE u = P_0 \frac{x^2}{2} + C_2$$

At $x = L$, $u(L) = 0$

$$AE(0) = P_0 \frac{L^2}{2} + C_2 \Rightarrow C_2 = -\frac{P_0 L^2}{2AE}$$

$$\therefore u = -P_0 \frac{(L^2 - x^2)}{2AE}$$

$$u(0) = -\frac{P_0 L^2}{2AE} \text{ (the same result)}$$



Now, If a concentrated force P in addition to the bar's own weight were acting on the bar AB at the end A . The total end deflection due to the two causes by superposition would be:

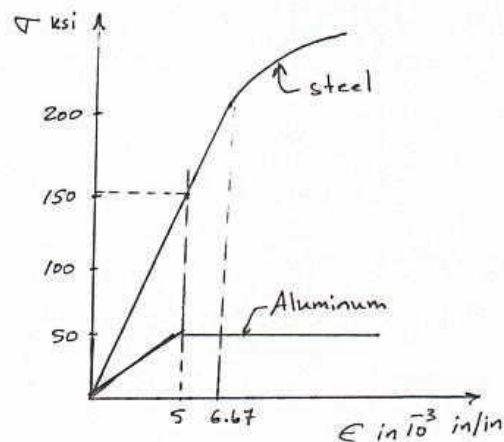
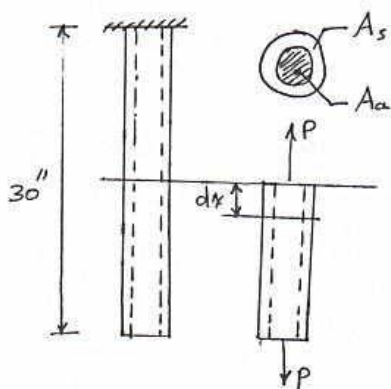
$$u = -\frac{PL}{AE} - \frac{\gamma_0 L^2}{2AE}$$

If $W = \gamma_0 L$ (bar weight)

$$\therefore |u| = \frac{PL}{AE} + \frac{WL}{2AE} = \frac{(P + \frac{W}{2})L}{AE}$$

Example 4:

A 30 in-long aluminum rod is enclosed within a steel-alloy tube as shown. The two materials are bonded together. If the stress-strain diagrams for the two materials can be idealized as shown. What end deflection will occur for $P_1 = 80 \text{ k}$ and $P_2 = 125 \text{ k}$? The cross-sectional area of steel A_s and of aluminum A_a are the same and equal to 0.5 in^2 .



Solution:

Here, the manner in which the resistance to the force P is distributed between the two materials is not known. Thus the problem is internally statically indeterminate.



Additional conditions are necessary to solve problems. This condition comes from the requirements of the compatibility of deformations:

Equilibrium: $P_a + P_s = P_1$ or P_2 ----- ①

Compatibility of deformations: $u_a = u_s$ or $\epsilon_a = \epsilon_s$

From material Properties: $\epsilon_a = \frac{\Delta_a}{E_a}$ and $\epsilon_s = \frac{\Delta_s}{E_s}$ (Hook's Law)

But $\Delta_a = \frac{P_a}{A_a}$ and $\Delta_s = \frac{P_s}{A_s}$

$$\epsilon_a = \epsilon_s$$

$$\frac{\Delta_a}{E_a} = \frac{\Delta_s}{E_s}$$

$$\frac{P_a}{A_a E_a} = \frac{P_s}{A_s E_s}$$

or $P_s = P_a \left(\frac{A_s E_s}{A_a E_a} \right)$

$$P_s = P_a \frac{E_s}{E_a}$$

From stress - strain Curves,

$$E_s = \frac{150,000}{5 \times 10^{-3}} = 30 \times 10^6 \text{ psi}$$

$$E_a = \frac{50,000}{5 \times 10^{-3}} = 10 \times 10^6 \text{ psi}$$

$$\therefore P_s = P_a \left(\frac{30 \times 10^6}{10 \times 10^6} \right) = 3P_a$$

substitute into Equation ①:

1- For $P = P_1 = 80 \text{ k}$

$$P_a + 3P_a = P_1 = 80 \text{ k} \Rightarrow P_a = 20 \text{ k}$$

$$P_s = 80 - 20 = 60 \text{ k}$$

$$u_a = \frac{P_a L}{A_a E_a} = \frac{20 \times 10^3 \times 30}{0.5 \times 10 \times 10^6} = 0.12 \text{ in.}$$

$$u_s = \frac{P_s L}{A_s E_s} = \frac{60 \times 10^3 \times 30}{0.5 \times 30 \times 10^6} = 0.12 \text{ in.}$$

$$u_a = u_s \quad \therefore \text{OK.}$$



$$\epsilon_a = \epsilon_s = \frac{0.12}{30} = 4 \times 10^{-3} \text{ in/in}$$

$\therefore$ OK the strain is within the elastic limit.

2- For $P = P_2 = 125^k$

As shown in the stress-strain curves, the linearly elastic response is limited up to strain of 5×10^{-3} for the aluminum.

$\therefore$ The maximum force can be carried by aluminum rod is:

$$P_a = A_a \sigma_{yp} = 0.5 \times 50 = 25^k$$

The remainder of the applied force must be carried by the steel tube.

$$P_s = 125 - 25 = 100^k$$

$$\sigma_s = \frac{100}{0.5} = 200 \text{ ksi} \quad \text{O.K. within elastic limit.}$$

$$\epsilon_s = \frac{200}{30 \times 10^6} = 6.67 \times 10^{-3} \text{ in/in}$$

$$u = \epsilon_s L = 6.67 \times 10^{-3} \times 30 = 0.2 \text{ in.}$$

Generalized Hooke's Law:

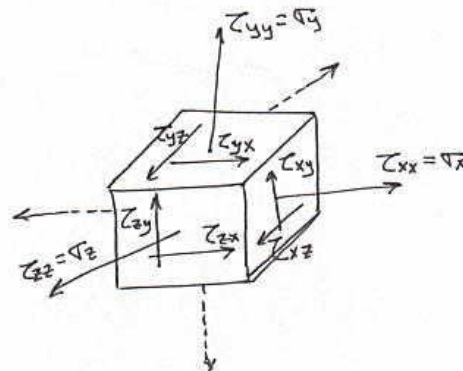
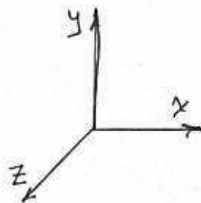


Fig. (4-14)

For Anisotropic material:

Isotropic material: is the material which has same properties in each direction.



$$\begin{aligned}\epsilon_{xx} &= \epsilon_x = A_{11}\tau_{xx} + A_{12}\tau_{yy} + A_{13}\tau_{zz} + A_{14}\tau_{xy} + A_{15}\tau_{yz} + A_{16}\tau_{xz} \\ \epsilon_{yy} &= \epsilon_y = A_{21}\tau_{xx} + A_{22}\tau_{yy} + A_{23}\tau_{zz} + A_{24}\tau_{xy} + A_{25}\tau_{yz} + A_{26}\tau_{xz} \\ \epsilon_{zz} &= \epsilon_z = A_{31}\tau_{xx} + A_{32}\tau_{yy} + A_{33}\tau_{zz} + A_{34}\tau_{xy} + A_{35}\tau_{yz} + A_{36}\tau_{xz} \\ \epsilon_{xy} &= \frac{\gamma_{xy}}{2} = A_{41}\tau_{xx} + A_{42}\tau_{yy} + A_{43}\tau_{zz} + A_{44}\tau_{xy} + A_{45}\tau_{yz} + A_{46}\tau_{xz} \\ \epsilon_{yz} &= \frac{\gamma_{yz}}{2} = A_{51}\tau_{xx} + A_{52}\tau_{yy} + A_{53}\tau_{zz} + A_{54}\tau_{xy} + A_{55}\tau_{yz} + A_{56}\tau_{xz} \\ \epsilon_{xz} &= \frac{\gamma_{xz}}{2} = A_{61}\tau_{xx} + A_{62}\tau_{yy} + A_{63}\tau_{zz} + A_{64}\tau_{xy} + A_{65}\tau_{yz} + A_{66}\tau_{xz}\end{aligned}$$

$$\tau_{xx} \equiv \sigma_x, \quad \tau_{yy} \equiv \sigma_y \quad \text{and} \quad \tau_{zz} = \sigma_z$$

$$\text{Or: } \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{xz} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}$$

For isotropic material:

$$A_{11} = A_{22} = A_{33} = \frac{1}{E}$$

$$A_{44} = A_{55} = A_{66} = \frac{1}{2G}$$

$$A_{12} = A_{13} = A_{23}$$

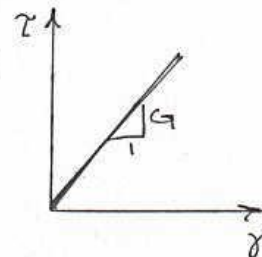
$$A_{12} = A_{21} = -\frac{\nu}{E}, \quad A_{13} = A_{31} = -\frac{\nu}{E} \quad \text{and} \quad A_{23} = A_{32} = -\frac{\nu}{E}$$

$$\text{All other } A'_{ij} = 0$$

ν = Poisson's ratio

G = shear modulus (or modulus of rigidity)

$$\tau = G\gamma$$





Hence:

$$\epsilon_x = \frac{1}{E} \sigma_x - \frac{\nu}{E} \sigma_y - \frac{\nu}{E} \sigma_z$$

$$\epsilon_y = -\frac{\nu}{E} \sigma_x + \frac{1}{E} \sigma_y - \frac{\nu}{E} \sigma_z$$

$$\epsilon_z = -\frac{\nu}{E} \sigma_x - \frac{\nu}{E} \sigma_y + \frac{1}{E} \sigma_z$$

$$\epsilon_{xy} = \frac{\tau_{xy}}{2G}$$

$$\epsilon_{yz} = \frac{\tau_{yz}}{2G}$$

$$\epsilon_{xz} = \frac{\tau_{xz}}{2G}$$

$$G = \frac{E}{2(1+\nu)}$$

Poisson's Ratio (ν): is the ratio of the absolute value of the strain in the lateral direction to the axial direction.

$$\begin{aligned} \nu &= -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x} \\ &= \frac{\text{Lateral strain}}{\text{Axial strain}} \end{aligned}$$

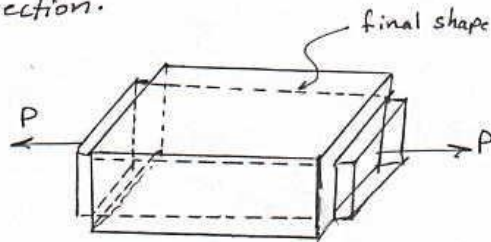


Fig. (4-15)

Example 1:

$$D = 60 \text{ mm}$$

$$P = 150 \text{ kN}$$

$$\Delta l = 0.24 \text{ mm}$$

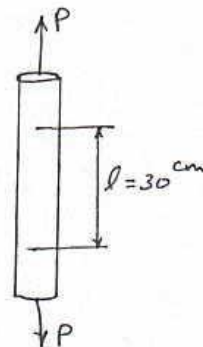
$$\Delta D = -0.015 \text{ mm}$$

Find ν & E

Solution:

$$\epsilon_x = \frac{\Delta l}{l_0} = \frac{0.24}{300} = 0.008 \text{ mm/mm}$$

$$\epsilon_y = \frac{\Delta D}{D} = \frac{-0.015}{60} = -0.00025 \text{ mm/mm}$$





$$\nu = - \frac{\epsilon_y}{\epsilon_x} = - \frac{(-0.00025)}{0.008} = 0.31$$

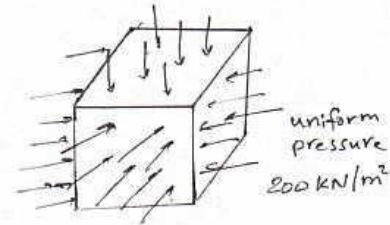
$$\sigma_x = \frac{P}{A} = \frac{150 \times 1000}{60^2 \times \frac{\pi}{4}} = 53.0 \text{ N/mm}^2$$

$$\sigma_x = E \epsilon_x$$

$$E \neq \frac{\sigma_x}{\epsilon_x} = \frac{53}{0.008} = 66250 \text{ N/mm}^2$$

Example 2:

Steel cube $50 \times 50 \times 50 \text{ mm}$, $\nu = \frac{1}{4}$,
 $E = 204 \times 10^6 \text{ kN/m}^2$, uniform pressure on
all faces = 200 kN/m^2 . Find the change in
dimensions between two parallel faces
of the cube?



Solution:

$$\begin{aligned} \epsilon_x &= \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E} \\ &= \frac{-200}{204 \times 10^6} - \frac{1}{4} \left(\frac{-200}{204 \times 10^6} \right) - \frac{1}{4} \left(\frac{-200}{204 \times 10^6} \right) \\ &= \frac{200}{204 \times 10^6} \left(-1 + \frac{1}{4} + \frac{1}{4} \right) = -\frac{100}{204 \times 10^6} \text{ m/m} \end{aligned}$$

$$\begin{aligned} \Delta l_x &= \epsilon_x l \\ &= -\frac{100}{204 \times 10^6} \times \frac{50}{1000} \\ &= \frac{-5}{204 \times 10^6} \text{ m} \end{aligned}$$



Thermal Strain

Changes in temperature cause deformation of materials.

For homogeneous isotropic materials, a change in temperature of ΔT degrees causes uniform linear strain in every direction expressed as thermal strain :-

$$\epsilon_x = \epsilon_y = \epsilon_z = \alpha \Delta T$$

Where, α is the coefficient of linear thermal expansion.

Change in temperature cause no shearing strains.

$$\text{i.e. : } \gamma_{xy} = \gamma_{yz} = \gamma_{xz} = 0$$

Thermal strain can be added directly to the linear strain due to stress :

$$\epsilon_x = \frac{1}{E} \sigma_x - \frac{\nu}{E} \sigma_y - \frac{\nu}{E} \sigma_z + \alpha \Delta T$$

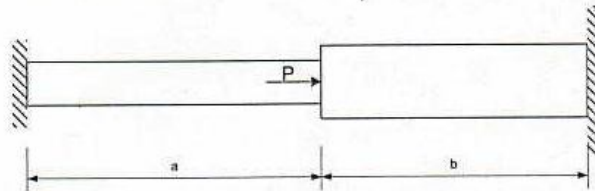
$\Delta T \rightarrow$ positive for increase in temperature.



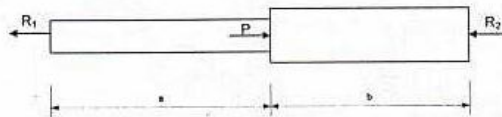
STATICALLY INDETERMINATE PROBLEMS

There are many problems in the stress and strain analysis that can not be solved based on the principles of equilibrium only. Such problems are called "The Statically Indeterminate Problems".

For example, if it's required to compute the reactions of the bar shown below.



We start the solution of the problem by drawing the F.B.D. of the bar, as shown below:



The applying the equilibrium equations for the above F.B.D., led to the following equations:

$$\Sigma F_x = 0 \Rightarrow R_1 + R_2 = P$$

1

$$\Sigma F_y = 0$$

$$\Sigma M = 0$$

Only the first equation can be useful in the solution of the problem, as the other two equations give nothing, then:

The Number of unknowns = 2

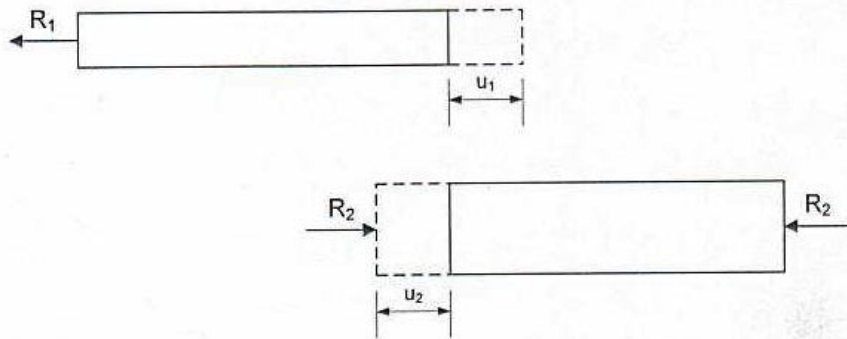
Number of the useful equations = 1

Then the equilibrium only is insufficient for the solution of the problem, and then the problem can be classified as a "Statically Indeterminate Problem".

The additional equation that required for the solution of the problem is based on the relation between the deformations as shown below. From Fig. below, as the ends of the bar are fixed, then:

$$|u_1| = |u_2|$$

Actually this equation is the additional required for the solution of the problem. This type of relation is called as "The Compatibility Equations".



Based on the formula for the deformation of the axially loaded members, the deformations can be written in terms for the axial forces.

$$u_1 = \frac{R_1 \times a}{EA_1}$$

$$u_2 = \frac{R_2 \times b}{EA_2}$$

Then

$$\frac{R_1 \times a}{EA_1} = \frac{R_2 \times b}{EA_2} \Rightarrow R_1 = R_2 \times \frac{b}{a} \times \frac{A_1}{A_2} \quad 2$$

Substitute equation 2 into equation 1, then:

$$R_2 \times \frac{b}{a} \times \frac{A_1}{A_2} + R_2 = F$$

$$R_2 \left(1 + \frac{b}{a} \times \frac{A_1}{A_2} \right) = F$$

$$R_2 = \frac{F}{\left(1 + \frac{b \times A_1}{a \times A_2} \right)} \quad 3$$

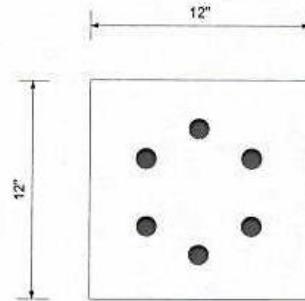
Substitute equation 3 into equation 1, to obtain:

$$R_1 = \frac{F}{\left(1 + \frac{a \times A_2}{b \times A_1} \right)}$$



Example 1.

The short concrete post shown below is reinforced axially with six symmetrically placed steel bars, each 1 in² in section. If the applied load P is 228000lb, compute the stress developed in each material. Assume the following moduli of elasticity; for steel $E_s=30 \times 10^6$ psi; for concrete $E_c=2 \times 10^6$ psi.

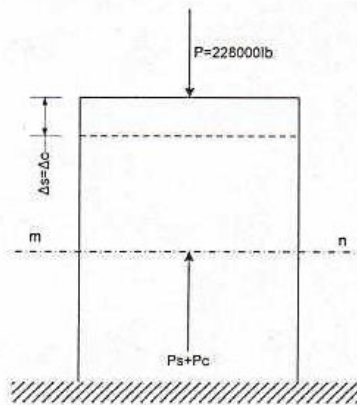


Solution

From the equilibrium equation:

$$\Sigma F_y = 0 \Rightarrow P_s + P_c = P$$

Since there is no other useful equilibrium, then the problem can be classified as a statically indeterminate problem. The addition equation that necessary for the solution of the problem is based on the compatibility equation that based on the equating the deformation in steel to the deformation in the concrete as shown in the Fig. below:





$$\Delta_s = \Delta_c \Rightarrow \frac{\sigma_s L}{E_s} = \frac{\sigma_c L}{E_c}$$

$$\therefore \sigma_s = \frac{E_s}{E_c} \sigma_c$$

Substitute the above equation into the equation of the equilibrium above to

$$\sigma_c \times A_c + \sigma_s \times A_s = 228000$$

$$\sigma_c (A_c + \frac{E_s}{E_c} A_s) = 228000$$

$$\text{Gross Area of Section} = A_g = 12 \times 12 = 144 \text{ in}^2$$

$$A_s = 1 \text{ in}^2 \times 6 = 6 \text{ in}^2$$

$$A_c = A_g - A_s = 144 - 6 = 138 \text{ in}^2$$

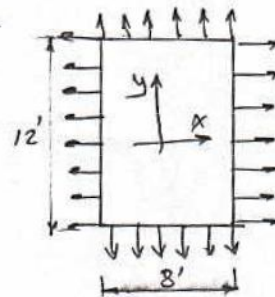
$$\therefore \sigma_c (138 + \frac{30}{2} \times 6) = 228000 \text{ Ans.}$$

$$\sigma_c = 1000 \text{ psi}$$

$$\sigma_s = \frac{E_s}{E_c} \sigma_c = \frac{30}{2} \times 1000 = 15000 \text{ psi Ans.}$$

Solved Problems (Popov)

Prob. 4-6: An 8-ft by 12-ft, $\frac{1}{4}$ -in thick steel plate panel is subjected to a uniformly distributed loading P_x and P_y . If the total change in length from the unstressed condition in the x-direction is +0.0768 in., and in the y-direction +0.0864 in., what are P_x and P_y in kips per foot? Let $E = 30 \times 10^6$ psi, and $G = 12 \times 10^6$ psi.



Solution: $G = \frac{E}{2(1+\nu)} \Rightarrow 12 \times 10^6 = \frac{30 \times 10^6}{2+2\nu}$

$$\nu = 0.25$$

$$\epsilon_y = \frac{\Delta l_y}{l_y} = \frac{0.0864}{12+12} = 6 \times 10^{-4} \text{ in/in.}$$

$$\epsilon_x = \frac{\Delta l_x}{l_x} = \frac{0.0768}{8+12} = 8 \times 10^{-4} \text{ in/in.}$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E}$$

$$8 \times 10^{-4} = \frac{\sigma_x}{30 \times 10^6} - \frac{0.25 \sigma_y}{30 \times 10^6} \quad \text{--- (1)}$$



$$\epsilon_y = -\frac{\nu \sigma_x}{E} + \frac{\sigma_y}{E}$$

$$6 \times 10^{-4} = -\frac{0.25 \sigma_x}{30 \times 10^6} + \frac{\sigma_y}{30 \times 10^6} \quad \text{--- (2)}$$

Solving Eqs. (1) and (2) for σ_x and σ_y

$$P_x = \sigma_x A_x = \sigma_x \times 0.25 \times 10^3 = 91.2 \text{ kips/ft}$$

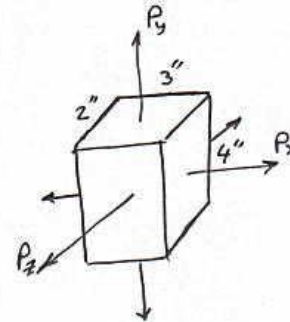
$$P_y = \sigma_y A_y = \sigma_y \times 0.25 \times 10^3 = 76.8 \text{ kips/ft}$$

Prob. 4-7: A rectangular aluminium alloy block has the dimensions shown in the Figure. The resultants of

uniformly distributed stresses are

$P_x = 40 \text{ kips}$, $P_y = 48 \text{ kips}$ and $P_z = 36 \text{ kips}$.

Determine the magnitude of a single system of tensile forces acting only in the x -direction as the initial forces. Let $E = 10 \times 10^6 \text{ psi}$, and $\nu = 0.25$.



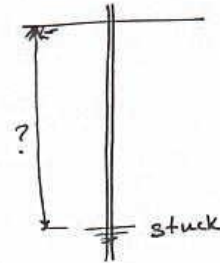
Solution:

$$\begin{aligned} \epsilon_x &= \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) \\ &= \frac{1}{10 \times 10^6} \left(\frac{40 \times 10^3}{2 \times 4} - \frac{0.25 \times 48 \times 10^3}{2 \times 3} - \frac{0.25 \times 36 \times 10^3}{3 \times 4} \right) \\ &= 2.25 \times 10^{-4} \text{ in/in} \end{aligned}$$

$$\begin{aligned} \epsilon_x &= \frac{\sigma_x}{E} \\ 2.25 \times 10^{-4} &= \frac{\frac{P_x}{4 \times 2}}{10 \times 10^6} \\ P_x &= 2.25 \times 8 \times 10^3 \\ &= 18 \times 10^3 \text{ lb} \\ &= 18 \text{ kips} \end{aligned}$$



Prob. 4-18: A very long drill pipe got stuck in hard clay. It was necessary to determine at what depth this occurred. The pipe is subjected to a large upward tensile force. As a result of this operation the pipe came up elastically 2 ft. At the same time the pipe elongated 0.0014 in. in an 8-in gage length. Approximately where was the pipe stuck? Assume that the cross-sectional area of the pipe was constant and that the media surrounding the pipe hindered elastic deformation of the pipe in a static test very little.



Solution:

$$u = 2 \text{ ft} = 2 \times 12 = 24 \text{ in.}$$

$$\Delta y = 0.0014 \text{ in. at } y = 8 \text{ in.}$$

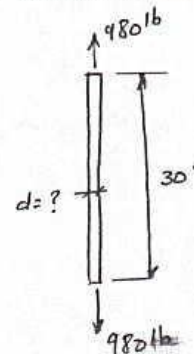
$$\Delta y = E \cdot y = 0.0014$$

$$E = \frac{0.0014}{8} = 1.75 \times 10^{-4} \text{ in/in}$$

$$u = E \cdot l$$

$$l = \frac{u}{E} = \frac{24}{1.75 \times 10^{-4}} = 11428 \text{ ft}$$

Prob. 4-19: A stainless steel rod 30 ft long used in a control mechanism must transmit a tensile force of 980 lb without stretching more than 0.2 in or exceeding an allowable stress of 20000 psi. What must the diameter of the rod be? Give the answer to the nearest eighth of an inch. $E = 28 \times 10^6 \text{ psi}$.





Solution: $\Delta l = \frac{Pl}{EA}$

$$0.2 = \frac{980 \times 30 \times 12}{A \times 28 \times 10^6} \Rightarrow A = 0.063 \text{ in.}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 0.063}{\pi}} = 0.283 \text{ in} \approx \frac{3}{8} \text{ in}$$

Check for tensile stress

$$\begin{aligned} \sigma &= \frac{P}{A} \\ &= \frac{980}{\left(\frac{3}{8}\right)^2 \times \frac{\pi}{4}} = < 20000 \text{ (allowable stress)} \therefore \text{OK.} \end{aligned}$$

Prob. 4-20: A solid cylinder of 2-in diameter and 36 in length is subjected to a tensile force of 30 kips. One part of this cylinder, L_1 long is of steel; the other part, fastened to steel, is aluminum and is L_2 long. (a) Determine the lengths L_1 and L_2 so that the two materials elongate an equal amount. (b) What is the total elongation of the cylinder?

$$E_s = 30 \times 10^6 \text{ psi}; E_a = 10 \times 10^6 \text{ psi}$$

Solution: (a) $L_1 + L_2 = 36$ --- (1)

$$P = P_s = P_a$$
 --- (2)

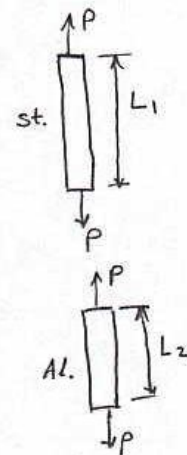
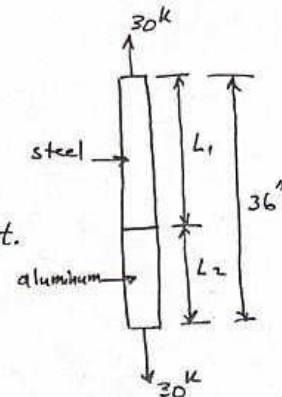
$$\frac{PL_1}{AE_s} = \frac{PL_2}{AE_a}$$

$$\therefore L_1 = \frac{L_2 E_s}{E_a} = \frac{30 \times 10^6}{10 \times 10^6} L_2 = 3L_2 \text{ sub. into (1):}$$

$$3L_2 + L_2 = 36 \Rightarrow L_2 = 9''$$

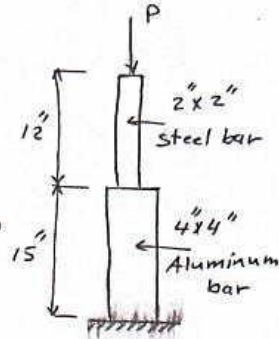
$$\therefore L_1 = 36 - 9 = 27''$$

$$\begin{aligned} \text{(b) } \Delta &= \Delta_s + \Delta_a = \frac{30 \times 10^3 \times 27}{2^2 \times \frac{\pi}{4} \times 30 \times 10^6} + \frac{30 \times 10^3 \times 9}{2^2 \times \frac{\pi}{4} \times 10 \times 10^6} \\ &= \end{aligned}$$





Prob. 4-23: A bar of steel and a bar of aluminum have the dimensions shown in the figure. Calculate the magnitude of the force P that will cause the total length of the two bars to decrease 0.010 in. Assume that the normal stress distribution over all cross sections of both bars is uniform and that the bars are prevented from buckling sideways. Plot the axial deflection diagram. Let $E_s = 30 \times 10^6$ psi and $E_a = 10 \times 10^6$ psi.



Solution: $P_s = P_a = P$ ----- (1)

$$u_s + u_a = 0.01 \text{ ----- (2)}$$

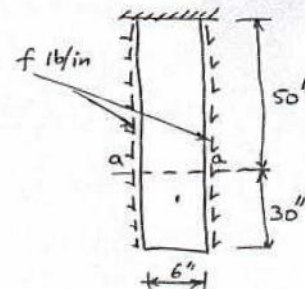
$$-\frac{P_s L_s}{A_s E_s} - \frac{P_a L_a}{A_a E_a} = -0.01$$

substitute equation (1) into (2):

$$\frac{P \times 12}{2 \times 2 \times 30 \times 10^6} + \frac{P \times 15}{4 \times 4 \times 10 \times 10^6} = 0.01$$

$$P = 51613 \text{ lb} = 51.6 \text{ kips.}$$

Prob. 4-25: A $\frac{1}{2}$ in-thick steel plate 6 in wide and 80 in. long is subjected to a set of uniformly distributed frictional forces along its two edges as shown in the figure. If, due to these forces, the total change in the transverse 6-in dimension at level a-a is 600×10^{-6} in, determine the total elongation





of the bar in the longitudinal direction. Let $E = 30 \times 10^6$ psi and $\nu = 0.25$.

Solution: $\Delta l_x = \epsilon_x l_x$

$$\epsilon_x = \frac{\Delta l_x}{l_x} = \frac{-600 \times 10^{-6}}{6} = -10^{-4} \text{ in/in}$$

$$\nu = -\frac{\epsilon_x}{\epsilon_y}$$

$$0.25 = -\frac{(-10^{-4})}{\epsilon_y}$$

$$\epsilon_y = \frac{10^{-4}}{0.25} = 4 \times 10^{-4} \text{ in/in}$$

$$\epsilon_y = \frac{P}{AE}$$

$$4 \times 10^{-4} = \frac{P}{6 \times 0.5 \times 30 \times 10^6}$$

$$P = 36 \times 10^3 \text{ lb}$$

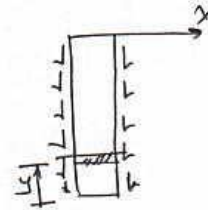
$$f = \frac{P}{L} = \frac{36 \times 10^3}{30} = 1.2 \times 10^3 \text{ lb/in}$$

$$\Delta L = \int_0^{80} \frac{P}{AE} dy = \int_0^{80} \frac{fy}{AE} dy$$

$$= \frac{1.2 \times 10^3}{6 \times 0.5 \times 30 \times 10^6} \left[\frac{y^2}{2} \right]_0^{80}$$

$$= \frac{1.2 \times 10^3}{6 \times 0.5 \times 30 \times 10^6} (3200)$$

$$= 0.0427 \text{ in.}$$





Prob. 4-31: Two wires are connected to a rigid bar as shown in the figure. The wire on the left is of steel having $A = 0.10 \text{ in}^2$, and $E = 30 \times 10^6 \text{ psi}$. The Aluminum-alloy wire on the right has $A = 0.20 \text{ in}^2$ and $E = 10 \times 10^6 \text{ psi}$. If a weight $W = 2000 \text{ lb}$ is applied as shown, how much will it deflect due to the stretch in the wires?

Solution:

$$\sum M_A = 0$$

$$T_s(30) - 2000(20) = 0$$

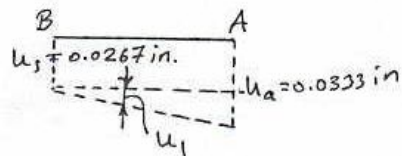
$$T_s = \frac{400000}{30} = 1333.3 \text{ lb}$$

$$\sum F_y = 0$$

$$T_a = 2000 - 1333.3 = 666.7 \text{ lb}$$

$$u_s = \frac{PL}{AE} = \frac{1333.3 \times 60}{0.1 \times 30 \times 10^6} = 0.0267 \text{ in.}$$

$$u_a = \frac{PL}{AE} = \frac{666.7 \times 100}{0.2 \times 10 \times 10^6} = 0.0333 \text{ in.}$$



$$\frac{u}{10} = \frac{u_a - u_s}{30}$$

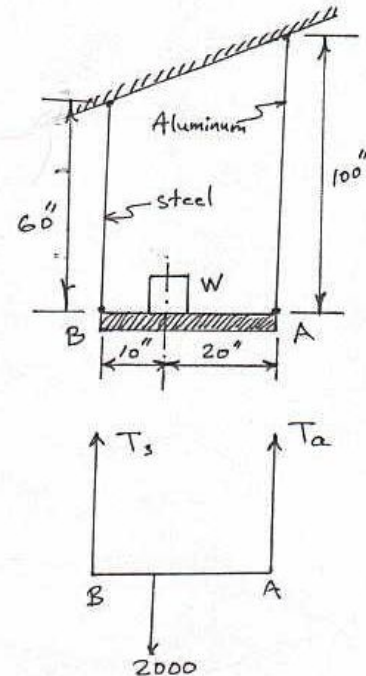
$$u_1 = \frac{10}{30} (0.0333 - 0.0267)$$

$$= \frac{1}{3} (0.0066) = 0.0022 \text{ in}$$

$$\therefore u_w = u_s + u_1$$

$$= 0.0267 + 0.0022$$

$$= 0.0289 \text{ in.}$$





Prob. 4-34: If a load of 1 Kip is applied to a rigid bar suspended by three wires as shown in the figure, what force will be resisted by each wire? The out side wires are aluminum ($E = 10^7$ psi). The inside wire is steel ($E = 30 \times 10^6$ psi). Initially there is no slack in the wire. (Hint: This is a statically indeterminate problem. A supplementary equation based on deformation must be formulated as in Example 4-10).

Solution:

$$\sum F_y = 0$$

$$1 = 2P_A + P_S \quad \text{--- (1)}$$

$$\epsilon_s = \epsilon_A$$

$$\frac{P_S L_S}{A_S E_S} = \frac{P_A L_A}{A_A E_A} \quad \text{--- (2)}$$

$$P_S = \frac{A_S E_S}{L_S} \cdot \frac{L_A}{A_A E_A} = \frac{0.2 \times 30 \times 10^6 \times 25 \times 12}{0.3 \times 10^7} P_A$$

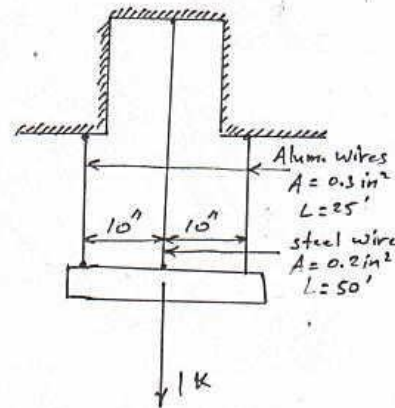
$$P_S = P_A$$

substitute into equation (1):

$$1 = 2P_A + P_A = 3P_A$$

$$P_A = \frac{1}{3} \text{ Kip}$$

$$P_S = \frac{1}{3} \text{ Kip}$$





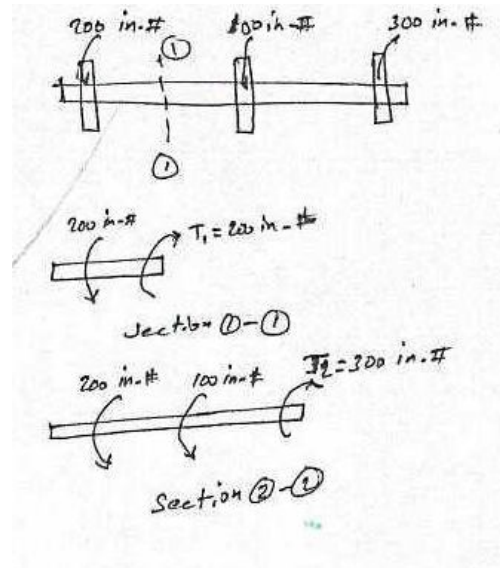
Chapter Five Torsion

Syllabus:

1. Computing the internal torque from static requirements.
2. Derivation the formulas for
 - i. Shear stress in the circular solid and tube sections.
 - ii. Angle of twist of the circular and tube sections.
3. Applications of the formulas. Four types of problem have been considered in the application of the torsion formulas:
 - i. Parametric Problem. Discussed the most economical shape for circular shafts. (Problems 5 -1, 5 - 8).
 - ii. Design Problems. Dealing with the computing the shaft dimensions based on the strength and / or deformations. (Problems 5 - 10).
 - iii. Checking Problems. Deals with the assessment of circular shaft based on the strength and / or deformations requirements.
 - iv. Computation problems that focusing on the computations tricks of the torsions formulas. (Problems 5 - 2, 5 - 12, 5 - 13, 5 - 14, 5 - 15)



5.1 Application of Method of Sections



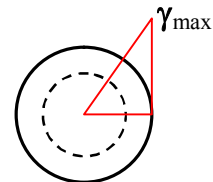
5.2 Basic Assumptions

I. Equilibrium:

- (a) The shaft is in equilibrium.
- (b) The resultant is a couple, the vector representation of which is parallel to the axis of the shaft is: $\longrightarrow$ (+ve) and $\longleftarrow$ (-ve).

II. Geometry:

- (a) The axis of the shaft is straight.
- (b) The shaft is circular in cross-section.
- (c) Plane section perpendicular to axis before twisting remains plane after twisting (No warping).
- (d) Shearing strain vary linearly from the central axis.



III. Properties of the Material:

- (a) The material is homogenous and isotropic.
- (b) Hook's law is applied.

5.3 The Torsion Formula

γ = shearing strain.

ϕ = Angle of twist

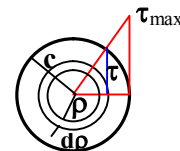
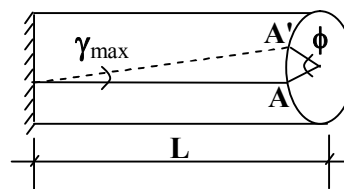
$$\gamma_{max} L = \phi c$$

$$\gamma_{max} = \frac{\phi c}{L}$$

$$\frac{\gamma_{max}}{c} = \frac{\gamma}{\rho}$$

$$\frac{\gamma_{max}}{\gamma} = \frac{c}{\rho}$$

$$\frac{\gamma_{max} G}{\gamma G} = \frac{c}{\rho}$$





$$\frac{\tau_c}{\tau} = \frac{\tau_{max}}{\tau} = \frac{c}{\rho}$$

$$\text{Or } \tau = \frac{\rho}{c} \tau_{max}$$

$$T = \int_A \tau dA \rho = \int_A \frac{\rho}{c} \tau_{max} \rho dA$$

$$= \frac{\tau_{max}}{c} J$$

$$\therefore T = \frac{\tau_{max}}{c} J$$

Where, J = Polar moment of inertia = $\int_A \rho^2 dA$

$$\therefore \tau_{max} = \frac{Tc}{J}$$

$$\tau = \frac{\rho}{c} \tau_{max}$$

$$\text{Or } \tau = \frac{T\rho}{J}$$

Polar moment of Inertia:

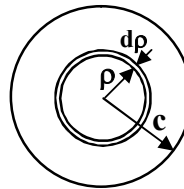
a. Solid Circular Section

$$J_1 = \int_A \rho^2 dA$$

$$dA = 2\pi\rho d\rho$$

$$J_1 = \int_0^c \rho^2 \times 2\pi\rho d\rho$$

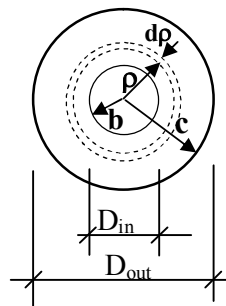
$$\therefore J_1 = 2\pi \int_0^c \rho^3 d\rho = \frac{2\pi}{4} \rho^4 \Big|_0^c = \frac{\pi}{2} c^4 = \frac{\pi}{32} D^4$$



b. Hollow circular section

$$J_2 = 2\pi \int_b^c \rho^3 d\rho = \frac{2\pi}{4} \rho^4 \Big|_b^c$$

$$= \frac{\pi}{2} (c^4 - b^4) = \frac{\pi}{32} (D_{out}^4 - D_{in}^4)$$





Example S-2

Find τ_{max} .
Solution:

$$\tau_{max} = \frac{T C}{J}$$

$$J = \frac{\pi}{32} D^4$$

Sec. ① - ①

$$J_1 = \frac{\pi}{32} (1)^4 = \frac{\pi}{32} \text{ in}^4$$

$$T_1 = 500 \text{ in-}\#$$

$$\tau_{max,1} = \frac{500 \times 0.5}{\pi/32} = 2547 \text{ psi max.}$$

$$\text{Sec. ② - ② } J_2 = \frac{\pi}{32} (4)^4 = \frac{256\pi}{32} \text{ in}^4$$

$$T_2 = 8000 - 500 = 7500 \text{ in-}\#$$

$$\tau_{max,2} = \frac{7500 \times 2}{256\pi/32} = 597 \text{ psi}$$

Sec. ③ - ③

$$T_3 = 5000 \text{ in-}\#$$

$$J_3 = \frac{\pi}{32} (3)^4 = \frac{81\pi}{32} \text{ in}^4$$

$$\tau_{max,3} = \frac{5000 \times 1.5}{81\pi/32} = 943 \text{ psi}$$

Sec. ④ - ④

$$T_4 = 1000 \text{ in-}\#$$

$$J_4 = \frac{81\pi}{32} \text{ in}^4$$

$$\tau_{max,4} = \frac{1000 \times 1.5}{81\pi/32} = 189 \text{ psi}$$

The solid cylindrical shaft of variable size shown in the figure is acted upon by the torques indicated. What is the maximum torsional stress in the shaft and between what two bulleys does it occur?

5.4: Design of Circular Members in Torsion

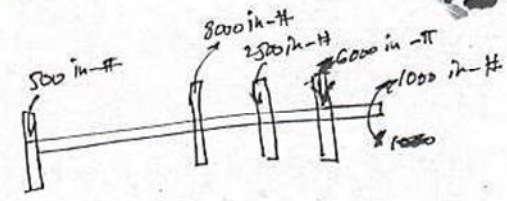
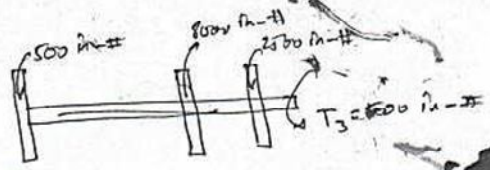
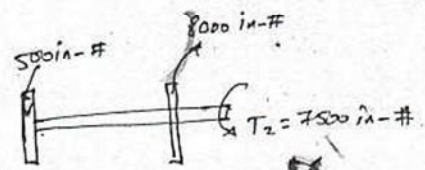
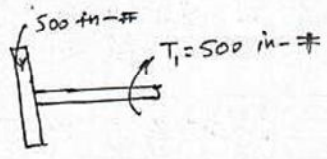
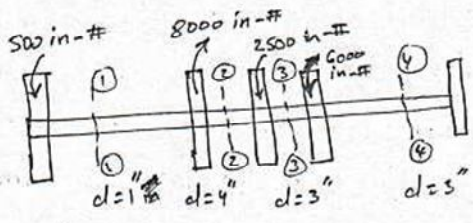
$$\tau_{max} = \frac{T C}{J}$$

$$\therefore \frac{J}{C} = \frac{T}{\tau}$$

$\tau_{max} \rightarrow \tau_{all}$. (from laboratory testing)

$$\frac{J}{C} = \frac{\pi}{2c} c^4 = \frac{\pi}{2} c^3$$

to resist the applied torque T.



③



Prob. 5-3: A motor, through a set of gears, drives a shaft as shown in the figure, at 630 rpm. 30 hp are delivered to a machine on the right; 90 hp on the left. Set a solid round shaft of the same size throughout. The allowable shearing stress is 5750 psi.

Solution:

An equation to predict the applied torque (T) from horse power (hp) and no. of revolution (N) is:

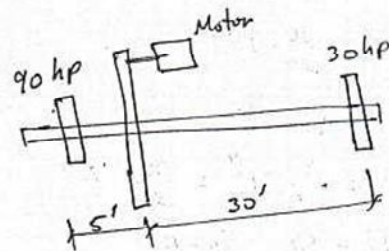
$$T = \frac{63000 \text{ hp}}{N} \quad (\text{in} \cdot \#)$$

$$T_1 = \frac{63000 + 90}{630} = 9000 \text{ in} \cdot \#$$

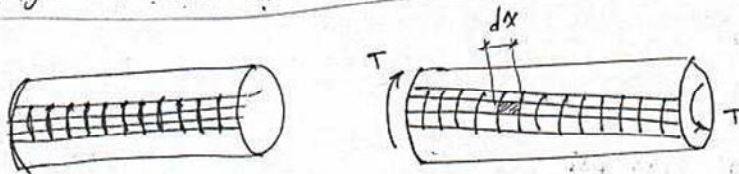
$$\frac{T}{C} = \frac{T}{r_{\max}}$$

$$\frac{\pi}{32} d^4 = \frac{9000}{5750}$$

$$\therefore d = \sqrt[3]{\frac{9000}{5750} \cdot \frac{16}{\pi}} = 2 \text{ in.}$$



5.5 Angle of Twist of Circular Members



before torque is applied

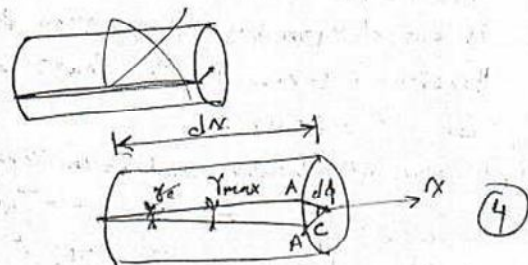
after torque is applied

$$AA' = \gamma_{\max} dx = d\phi \quad \text{--- (1)}$$

$$\text{But } \gamma_{\max} = \frac{\tau_{\max}}{G} \quad (\text{Hook's Law})$$

$$\text{Anal } \tau_{\max} = \frac{TC}{J} \quad (\text{Torsion Formula})$$

$$\therefore \gamma_{\max} = \frac{TC}{JG}$$





Substitute into equation (1):

$$\frac{T_r}{JG} dx = d\phi$$

$$d\phi = \frac{T}{GJ} dx$$

$$\phi = \int \frac{T(x)}{GJ(x)} dx + C$$

If T is constant and J is constant

$$\text{Then } \phi = \frac{TL}{GJ}$$

Now consider an element subjected to end torques T and $T+dT$ and to an applied distributed $t(x)$ (in-lb/in)

$$\phi = \int \frac{T(x) dx}{GJ(x)} + C$$

$$\frac{d\phi}{dx} = \frac{T}{GJ} \quad (\text{for constant } J) \quad \frac{dT}{dx} = -t(x)$$

$$\sum F_x = 0 \quad dT + T + t(x) - T = 0$$

$$\frac{dT}{dx} = -t(x)$$

$$\frac{d^2\phi}{dx^2} = \frac{1}{GJ} \frac{dT}{dx} \quad \text{or} \quad GJ \frac{d^2\phi}{dx^2} = -t(x)$$

$$\text{or } \frac{dT}{dx} = GJ \frac{d^2\phi}{dx^2} \quad \text{or } EA \frac{d^2u}{dx^2} = -p(x)$$

Prob. 5-4: (a) Determine the maximum shearing stress in the shaft subjected to the torques shown in the figure. (b) Find the angle of twist in degrees between the two ends. Let $G = 12 \times 10^6$ psi

Solution:

The member is divided into parts when J and T are constant to apply the formula $\phi = \frac{TL}{GJ}$ and

$$\tau_{max} = \frac{TC}{J} \quad \text{for each part}$$

$$J_1 = J_2 = \frac{\pi}{32} (2^4 - 1^4) = \frac{15\pi}{32} \text{ in}^4$$

$$J_3 = J_4 = \frac{\pi}{32} 2^4 = \frac{\pi}{2} \text{ in}^4$$

$$T_1 = 375\pi \text{ in-lb}$$

$$T_2 = 375\pi - 135\pi = 240\pi \text{ in-lb}$$

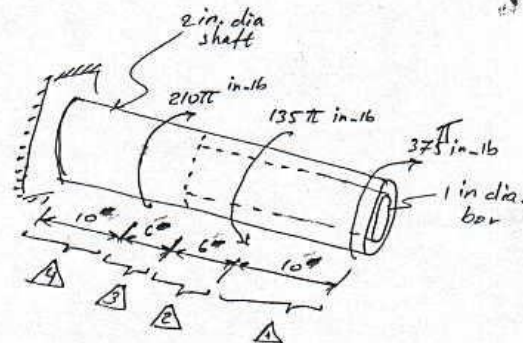
$$T_3 = T_2 = 240\pi \text{ in-lb}$$

$$T_4 = 240\pi + 210\pi = 450\pi \text{ in-lb}$$

Part (a):

$$\tau_{max,1} = \frac{375\pi \times 1}{\frac{15\pi}{32}} = \frac{375 \times 32}{15} = 800 \text{ psi}$$

$$\phi_1 = \frac{375\pi \times 10}{12 \times 10^6 \times \frac{15\pi}{32}} = \frac{375 \times 32}{15 \times 10^6} = 8 \times 10^{-3} \text{ rad.} = 0.458^\circ$$





Part 2

$$\tau_{max2} = \frac{240\pi \times 1}{15\pi/32} = 512 \text{ psi}$$

$$\phi_2 = \frac{240\pi \times (6 \times 10^{-6})}{15\pi/32 + (12 \times 10^6)} = \frac{0.256}{3.075} \times 10^{-3} \text{ rad} = 0.0147^\circ$$

part 3

$$\tau_{max3} = \frac{240\pi \times 1}{\pi/2} = 480 \text{ psi}$$

$$\phi_3 = \frac{240\pi \times (6 \times 10^{-6})}{\pi/2 + (62 \times 10^6)} = \frac{0.24}{250} \times 10^{-3} \text{ rad} = 0.0138^\circ$$

Part 4

$$\tau_{max4} = \frac{450\pi \times 1}{\pi/2} = 900 \text{ psi}$$

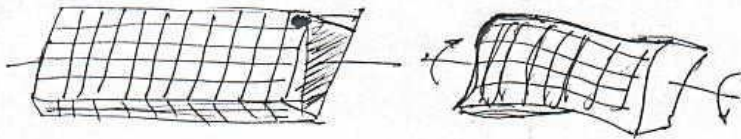
$$\phi_4 = \frac{450\pi \times 10}{12 \times 10^6 + \frac{\pi}{2}} = 0.75 \times 10^{-3} \text{ rad} = 0.043^\circ$$

$$\tau_{max} = 900 \text{ psi}$$

$$\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4 = 0.0382 + 0.0147 + 0.0138 + 0.043 = 0.11^\circ$$



Solid noncircular Members



Mathematically, the problem is complicated

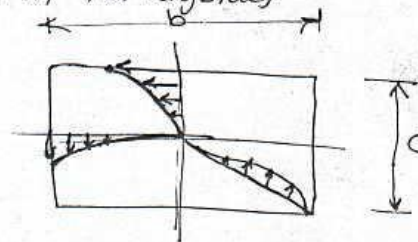
~~Corner element~~ The first two assumptions do not apply
Sections $\perp$ to the axis of a member warp when a torque is applied

No distortion in corner elements $\Rightarrow$ no shearing stress there.

Max. shear stress ~~at~~ at the midpoints of the long sides

Compared with circular sections.

In circular sections, the stress is
max. at most remote point, but
for the noncircular sections is zero
at most remote points



Analytical solution made by

Traashenko & Goodier results in:

$$\tau_{\max} = \frac{T}{\alpha b c^3} \quad \text{and} \quad \phi = \frac{TL}{\beta b c^3 G}$$

b = long side, c = short side

α & β depend on b/c

b/c	1.00	1.5	2.0	3.0	6.0	10.0	∞
α	0.208	0.231	0.246	0.267	0.299	0.312	0.333
β	0.141	0.196	0.229	0.263	0.289	0.312	0.333

2/1/05



Thin-walled Hollow Sections

$$T = \int r q ds$$

$$T = 2 A q$$

$$q = \frac{T}{2A}$$

$$\tau = \frac{q}{t}$$

$$\therefore \tau = \frac{T}{2At}$$

$$\text{external work} = \frac{T\theta}{2}$$

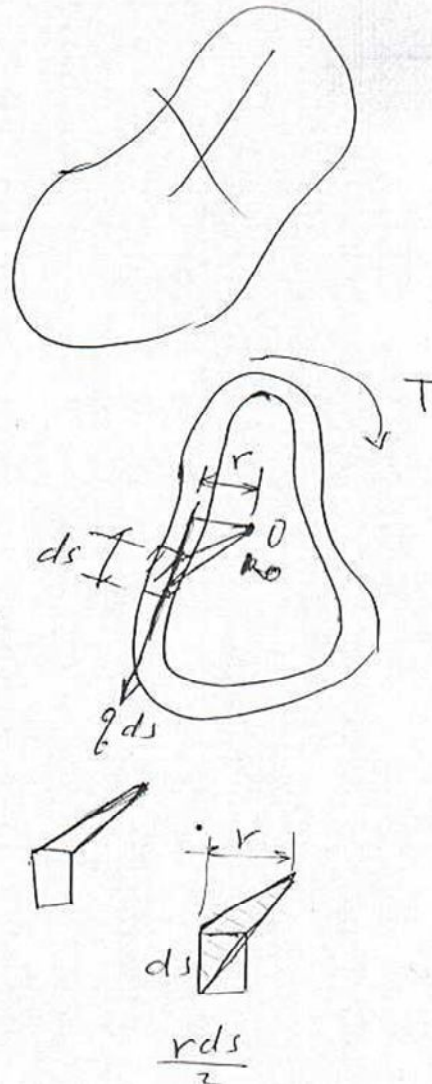
θ = angle of twist in a unit distan

$$\theta = \frac{d\phi}{dx}$$

Internal strain energy:

$$\frac{T\theta}{2} = \frac{T}{2} \frac{d\phi}{dx} = \int_{Vol} \frac{\tau^2}{2G} dVol.$$

$$\theta = \frac{d\phi}{dx} = \frac{T}{4A^2G} \int \frac{ds}{t}$$





Prob. 5-9: Two gears are attached to two 2-in-diameter steel shafts as shown in the figure. The gear at B has an 8-in pitch diameter. The gear at C has a 16-in pitch diameter. Through what angle will the end A turn if at A a torque of 5000 in-lb is applied and the end D of the second shaft is prevented from rotating?

$$G = 12 \times 10^6 \text{ psi}$$

Solution:

$$J = \frac{\pi}{32} (2)^4 = \frac{\pi}{2}$$

$$\phi_{A1} = \frac{T_1 L}{GJ} = \frac{5000 \times 20 \times 12}{12 \times 10^6 \times \frac{\pi}{2}} = \frac{2}{10\pi}$$

$$F = \frac{5000}{4} = 1250 \text{ lb}$$

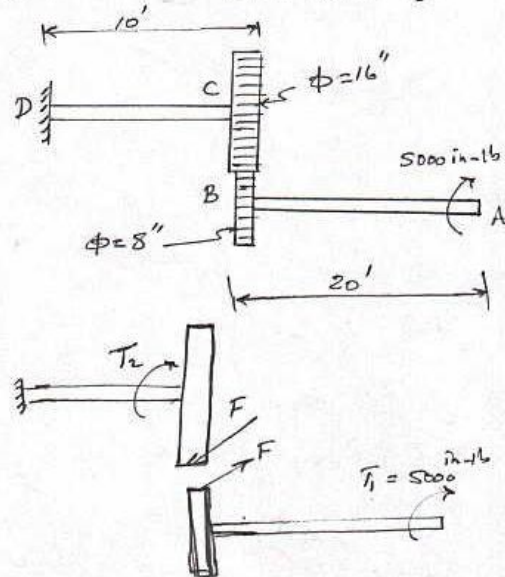
$$T_2 = 1250 \times 8 = 10000 \text{ in-lb}$$

$$\phi_{A2} = \frac{T_2 L}{GJ} = \frac{10000 \times 10 \times 12}{12 \times 10^6 \times \frac{\pi}{2}} = \frac{2}{10\pi}$$

$$\phi_A = \phi_{A1} + 2\phi_{A2}$$

$$= \frac{2}{10\pi} + \frac{2 \times 2}{10\pi} = \frac{6}{10\pi} \text{ rad}$$

$$= \frac{6}{10\pi} \times \frac{180}{\pi} = 10.9^\circ \approx 11^\circ$$



Prob. 5-12: A solid tapered steel shaft is rigidly fastened to a fixed support at one end and is subjected to a torque T at the other end (see figure). Find the angular rotation of the free end if $d_1 = 6 \text{ in}$, $d_2 = 2 \text{ in}$, $L = 20 \text{ in}$, and $T = 27000 \text{ in-lb}$. Assume that the usual assumption of strain in prismatic circular shafts subjected to torque apply and let $G = 12 \times 10^6$.



Solution:

$$\frac{y}{x} = \frac{2}{20}$$

$$y = \frac{x}{10}$$

$$d = (1 + \frac{x}{10}) * 2 = 2 + \frac{x}{5}$$

$$J = \frac{\pi}{32} d^4$$

$$J(x) = \frac{\pi}{32} (2 + \frac{x}{5})^4$$

$$\bullet \phi = \int_0^x \frac{T dx}{G J(x)} + C$$

$$= \frac{27000}{12 * 10^6 * \frac{\pi}{32}} \int_0^x \frac{dx}{(2 + \frac{x}{5})^4} + C$$

$$\text{Let } 2 + \frac{x}{5} = u \Rightarrow dx = 5 du$$

$$\therefore \phi = \frac{27000 * 32}{12 * 10^6 * \pi} \int_u \frac{5 du}{u^4} + C_1$$

$$= \frac{72 * 5}{10^3 \pi * 3} (u^{-3}) + C_1$$

$$\bullet = - \frac{360}{3 * 10^3 \pi} \left[(2 + \frac{x}{5})^{-3} \right]_0^x + C_1$$

$$\text{at } x = l, \phi = 0$$

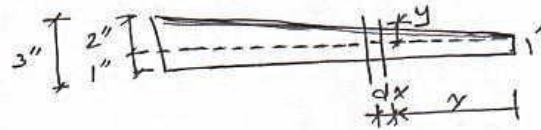
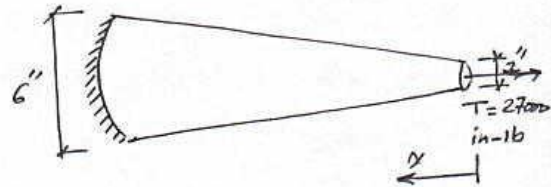
$$0 = - \frac{120}{10^3 \pi} \left[(2 + \frac{20}{5})^{-3} - (2)^{-3} \right] + C_1$$

$$\therefore C_1 = \frac{1}{1800 \pi}$$

$$\therefore \phi = - \frac{120}{10^3 \pi} \left(\frac{5}{10+x} \right)^3 + \frac{1}{1800 \pi}$$

$$\phi_0 = - \frac{120}{10^3 \pi} \left(\frac{1}{8} \right) + \frac{1}{1800 \pi} \text{ rad.}$$

$$\phi_0 = -0.263^\circ$$





Prob. 5-15: The loading on a control torque-tube for an aileron of an airplane may be idealized by a uniformly varying torque $t(x) = kx$ in-lb per inch, where k is a constant (see figure). Determine the angle of twist of the free end. Assume GJ to be constant. Solve the problem using Eq. 5-11 and 5-12 as directed.

Solution:

$$GJ \frac{d^2 \phi}{dx^2} = -t(x) = -kx$$

$$GJ \frac{d\phi}{dx} = -\frac{kx^2}{2} + c_1$$

$$\text{at } x=0, \frac{d\phi}{dx} = 0 \Rightarrow c_1 = 0$$

$$GJ \phi = -\frac{kx^3}{6} + c_2$$

$$\text{at } x=l, \phi = 0$$

$$0 = -\frac{kl^3}{6} + c_2 \Rightarrow c_2 = \frac{kl^3}{6}$$

$$GJ \phi = -\frac{kx^3}{6} + \frac{kl^3}{6}$$

$$GJ \phi = \frac{k}{6}(l^3 - x^3)$$

$$\text{at } x=0 \text{ (free end)}$$

$$GJ \phi = \frac{k}{6}(l^3 - 0)$$

$$\text{or } \phi = \frac{k}{6GJ} l^3$$

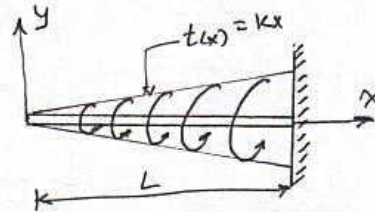
Another method:

$$\phi = \int_0^l \frac{T(x) dx}{GJ(x)} + c$$

$$\frac{dT}{dx} = -kx$$

$$T = -\frac{kx^2}{2} + c_1$$

$$\text{at } x=0, T=0 \Rightarrow c_1 = 0$$





$$\therefore T(x) = -\frac{kx^2}{2}$$

$$\therefore \phi = \int -\frac{kx^2}{2GJ} dx + C_2$$

$$= -\frac{kx^3}{6GJ} + C_2$$

$$\text{at } x = l, \phi = 0$$

$$0 = -\frac{kl^3}{6GJ} + C_2 \Rightarrow C_2 = \frac{kl^3}{6GJ}$$

$$\phi = \frac{kl^3}{6GJ} - \frac{kx^3}{6GJ}$$

$$\phi = \frac{k}{6GJ} (l^3 - x^3)$$

$$\text{at } x = 0 \text{ (free end)}$$

$$\phi = \frac{k}{6GJ} l^3$$

Prob. 5-16: Assume that during a drilling operation a shaft of constant torsional rigidity JG is loaded by a concentrated torque $T_1 = 1000$ in-lb and a distributed torque $t(x) = 100$ in-lb per inch as shown in the figure. Find the angular rotation of the free end. Use singularity functions and Eq.

● S-12. plot the torque $T(x)$ and the angle of twist $\phi(x)$ diagrams.

Solution:

Part A:

$$JG \frac{d^2\phi}{dx^2} = -100$$

$$JG \frac{d\phi}{dx} = -100x + C_1$$

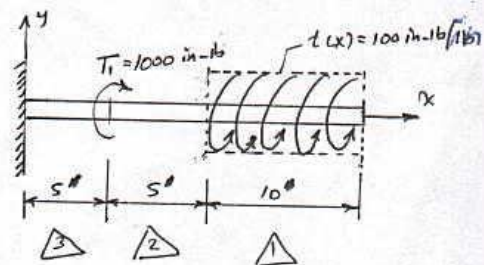
$$\text{at } x = 240'', \frac{d\phi}{dx} = 0$$

$$0 = -100 \times 240 + C_1 \Rightarrow C_1 = 24000$$

$$JG \phi = -50x^2 + 24000x + C_2$$

$$\text{at } x = 0, \phi = 0$$

$$0 = 0 + 0 + C_2 \Rightarrow C_2 = 0$$





$$\therefore JG\phi_1 = -50x^2 + 24000x$$

$$\text{at } x = 240'' \Rightarrow \phi_1 = \frac{1}{JG} (-50 \times 240^2 + 24000 \times 240)$$

$$\phi_1 = \frac{2880000}{JG}$$

Part 2:

$$T_2 = 100 \times 10 \times 12 = 12000 \text{ in-lb}$$

$$\phi_2 = \frac{T_2 L}{JG} = \frac{12000 \times 5 \times 12}{JG} = \frac{720000}{JG}$$

Part 3:

$$T_3 = 12000 - 1000 = 11000 \text{ in-lb}$$

$$\phi_3 = \frac{T_3 L}{JG} = \frac{11000 \times 5 \times 12}{JG} = \frac{660000}{JG}$$

$$\therefore \phi = \phi_1 + \phi_2 + \phi_3 = \frac{1}{JG} (2880000 + 720000 + 660000)$$

$$= \frac{4260000}{JG}$$



Parametric Problems

Prob. 5.1

Given



The two sections are from the same linear elastic material

Required

Compute the thickness t to have the same maximum stress in two sections. Determine the weight ratio

$$\tau_{\max \text{ Solid}} = \frac{T \cdot R}{J_{\text{Solid}}}, \tau_{\max \text{ Tube}} = \frac{T \cdot R}{J_{\text{Tube}}}$$

$$\therefore \frac{1}{\frac{\pi \cdot 2^4}{32}} = \frac{1.5}{\frac{\pi \cdot (3^4 - 1.5^4)}{32}} \rightarrow J_{\text{Tube}} = 2.36 \text{ in}^4$$

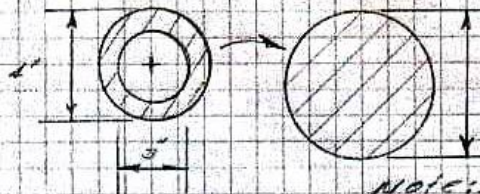
$$2.36 = \frac{\pi}{32} [3^4 - R_{\text{in}}^4] \rightarrow R_{\text{in}} = 1.37 \text{ in}$$

$$\therefore R_{\text{in}} = 1.37 \text{ in} \quad \therefore t = 0.126 \text{ in}$$

$$\text{Weight ratio} = \frac{\text{Weight of tube shaft}}{\text{Weight of solid shaft}}$$

$$= \frac{(\frac{\pi}{4} \cdot 3^2 \cdot \pi) \cdot L \cdot \rho}{\frac{\pi}{4} \cdot 2^2 \cdot \pi \cdot L \cdot \rho} = \frac{0.373}{1} = 0.373$$

Prob. 5.8



Determine the minimum diameter d for the solid shaft to replace the hollow shaft so that in the new shaft neither the maximum stress nor the angle of twist exceed the same quantities in the hollow tube.

Note: the two shaft have been made from the same material.

Solution

1.0 based on Stress

$$J_{\text{Hollow}} = (\frac{\pi}{32} \cdot (4^4 - 1.5^4)) = 17.2 \text{ in}^4$$

$$\tau_{\max} = \frac{T \cdot R}{J} = \frac{T \cdot d/2}{\frac{\pi \cdot d^4}{32}} \rightarrow \frac{1}{17.2} = \frac{16}{d^3} \rightarrow d^3 = 43.8 \text{ in}^3$$

$$d = 3.52 \text{ in}$$



2. Based on Angle of Twist

$$\phi = \frac{T \cdot L}{JG} \Rightarrow \frac{1}{17.2} = \frac{1}{d^4 \cdot \pi}{32} \Rightarrow d^4 \cdot \pi = 17.2 \Rightarrow d^4 = \frac{17.2 \times 32}{\pi}$$

$$\therefore d = 175.3 \text{ mm}^4, d = 3.64 \text{ m}$$

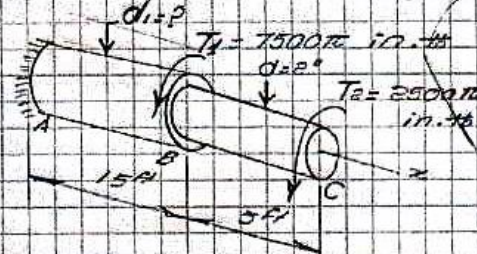
$$\therefore D_{min.} = 100\% [d_1, d_2]$$

$d_{min.} = 3.64 \text{ m}$
 لا يمكن القول بفرق الاختلاف في المقام
 معادلات الدوائر في الموضع الرابع في المقام
 معادلات الدوائر، على أن تكون معادلات الدوائر في المقام
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Design Problems

5.10 For the Stepped Steel Shaft ($E = 29 \times 10^6$ psi, $G = 12 \times 10^6$ psi) shown, Find the minimum permissible diameter d_1 if the allowable Shearing Stress is 6000 psi and the total twist between A to C is limited to 3° .



Solution

1. d_1 based on Stress:

$$\sum T_{AB} = 0$$

$$-T_{AB} + 7500\pi + 2500\pi = 0 \Rightarrow T_{AB} = 10000\pi$$

$$T_{max} = \frac{T \cdot d/2}{J} \Rightarrow 6000 = \frac{10000\pi \cdot d/2}{\frac{\pi d^4}{32}} \Rightarrow 6 = \frac{10 \times 16}{d^3}$$

$$\therefore d^3 = 26.7 \Rightarrow d_1 = 2.98$$

2. d_2 based on angle of twist

$$\phi = \phi_{A/B} + \phi_{B/C} \quad \phi_{C/A} = 3^\circ = 3^\circ \times \frac{\pi}{180} = 0.0523 \text{ rad.}$$

$$0.0523 = \frac{10000\pi \times (15 \times 12)}{12 \times 10^6 J_{AB}} + \frac{2500\pi \times (5 \times 12)}{12 \times 10^6 \times \frac{\pi d_2^4}{32}}$$

$$0.0523 = \frac{471}{J_{AB}} \Rightarrow J_{AB} = 17.25 \text{ in}^4$$

$$17.25 = \frac{d_2^4 \cdot \pi}{32} \Rightarrow d_2^4 = 175.8 \Rightarrow d_2 = 3.64 \text{ in}$$

$$\therefore d_1 = \text{Max.} [2.98", 3.64"]$$

$$\therefore d_1 = 3.64"$$



Computation Problems

Problem 5.12
For the shaft shown,
compute the angular
rotation of the free
end.

Solution

$$\phi = \int_0^{20} \frac{T \cdot dx}{GJ}$$

but $J = \frac{\pi d^4}{32}$

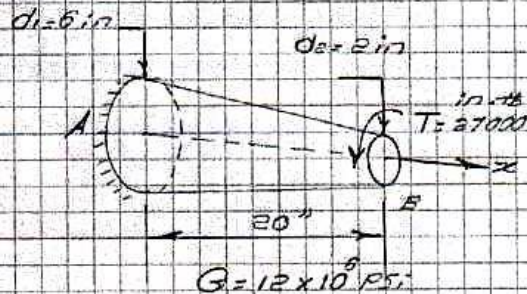
$$d(x) = 6 - 0.2x$$

$$J(x) = \frac{\pi (6 - 0.2x)^4}{32}$$

$$\phi = \int_0^{20} \frac{T \cdot dx}{\frac{\pi (6 - 0.2x)^4}{32} \times G}$$

$$= \frac{32T}{\pi G} \int_0^{20} \frac{dx}{(6 - 0.2x)^4} = \frac{32T}{\pi G} \left[\frac{5/3}{(6 - 0.2x)^3} \right]_0^{20} = \frac{32T}{\pi G} \left(\frac{5}{324} \right)$$

$$= 4.6 \times 10^{-4} \text{ rad} = 0.264^\circ$$



Prob. 5.13

$$\phi_{C/A} = \phi_{C/B} + \phi_{B/A}$$

From B to C, the J can be
written as a function of
 x :

$$d_0 = 6"$$

$$d(x) = \frac{4}{24} x = 0.167x$$

$$J = \frac{\pi (6 - 0.167x)^4}{32}$$

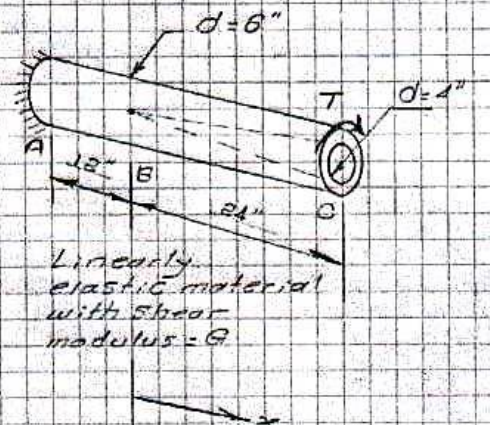
$$= \frac{\pi (1.296 - 7.77 \times 10^{-4} x^4)}{32}$$

$$\phi_{C/B} = \int_0^{24} \frac{T \cdot dx}{\frac{\pi (1.296 - 7.77 \times 10^{-4} x^4)}{32} G}$$

$$= \frac{32T}{\pi G} \int_0^{24} \frac{dx}{(1.296 - 7.77 \times 10^{-4} x^4)} = \frac{32T}{\pi G} \times 0.0193 = \frac{0.196T}{G}$$

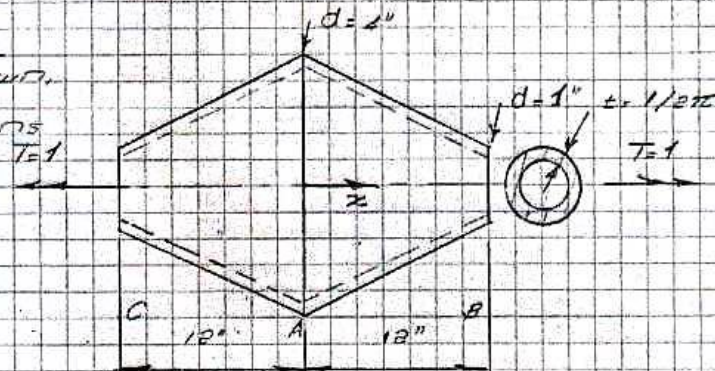
$$\phi_{B/A} = \frac{T \times 12}{G \times 54\pi} = \frac{0.09136T}{G} \Rightarrow \phi_{C/A} = \frac{0.1137T}{G}$$

Computation Problems





Problem 5.14 -
For the Shaft Shown,
determine the
relative rotations
of ends.



$$d(x) = 2 - 0.25x$$

$$J(x) = \frac{\pi}{32} \frac{d^4}{8} = \frac{\pi}{32} d^4$$

$$\therefore J(x) = \frac{\pi}{32} (2 - 0.25x)^4$$

$$\begin{aligned} \phi_{B/A} &= \int_0^L \frac{T dx}{GJ} = \frac{1 \times 12}{G \times \frac{\pi}{32}} \int_0^{12} \frac{dx}{(2 - 0.25x)^4} \\ &= \frac{384}{G} \int_0^{12} \frac{dx}{(2 - 0.25x)^4} = \frac{384 \times 1.875}{G} = \frac{720}{G} \end{aligned}$$

$$\phi_{B/A} = \frac{720}{G}$$

$$\therefore \phi_{B/C} = 2\phi_{B/A} = \frac{1440}{G}$$

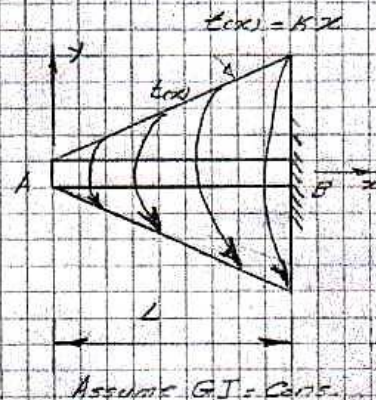
Problem 5.15
For the Shaft Shown, determine
the angle of twist of the
free end.

Solution

$$\begin{aligned} \phi_{B/A} &= \int_0^L \frac{T dx}{GJ}, \quad T(x) = \frac{E(x)}{2} \cdot x = K \cdot \frac{x^2}{2} \\ &= \frac{K}{2GJ} \int_0^L x^2 dx = \frac{KL^3}{6GJ} \end{aligned}$$

$$\phi_{B/A} = \phi_B - \phi_A = \frac{KL^3}{6GJ}$$

$$\Rightarrow \phi_A = \frac{KL^3}{6GJ}$$





Problem 5.16

For the shaft shown

1. Find the angular rotation of the free end.
2. Plot the torque $T(x)$ and angle of twist $\phi(x)$ diagrams.

Solution

$$1. \phi = \phi_A + \phi_B + \phi_C$$

$$\phi_A = \phi_B = \phi_C = 0$$

$$\sum M_x = 0 \rightarrow \phi$$

$$+T_R - 1000 + 100 \times 10 = 0$$

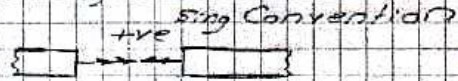
$$\therefore T_R = 0$$

$$\phi = \int \frac{T(x) dx}{GJ}$$

$$\begin{aligned} \phi_{\text{free}} &= 0 - \frac{1000 \times 5}{GJ} + \int_0^{10} \frac{(-1000 + 100x) dx}{GJ} \\ &= 0 - \frac{5000}{GJ} + \frac{1}{GJ} \left[-1000x + 50x^2 \right]_0^{10} \\ &= -\frac{1000}{GJ} \end{aligned}$$

$$\therefore \phi_{\text{free}} = \frac{1000}{GJ} \text{ rad}$$

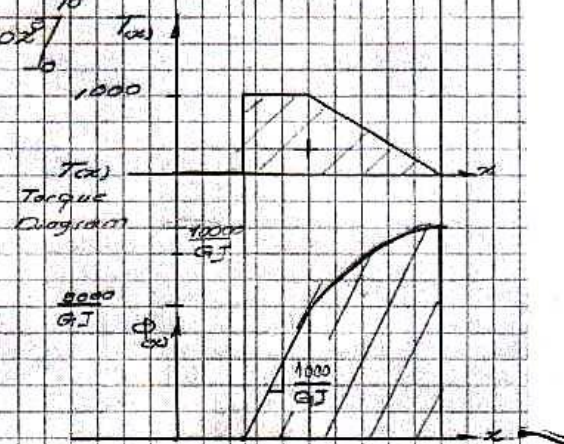
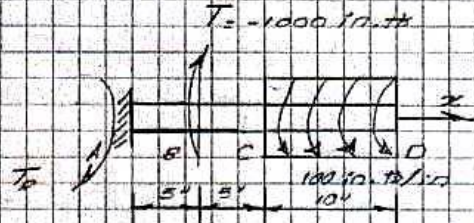
2. Diagrams



$$\phi = \int \frac{T(x) dx}{GJ}$$

$$\therefore d\phi = \frac{T(x) dx}{GJ}$$

$$\therefore \frac{d\phi}{dx} = \frac{T(x)}{GJ}$$





Chapter Two

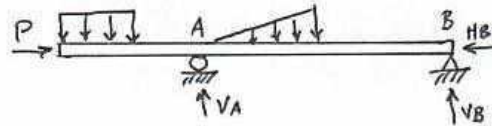
Axial, Shear Force and Bending Moment Diagrams

(1) Direct Method:

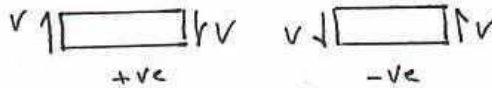
$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_z = 0$$

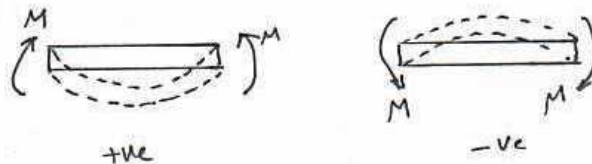


Sign Conventions for shear force



Note: This sign convention was mentioned in most references except Popov, he adopted another one which is opposite.

Sign Conventions for Bending moment



Example 1: Draw Axial, shear force & B.M. Diagrams:

Sol:

$$\sum F_x = 0$$

$$H_A = 50 \text{ kN (ten.)}$$

$$\sum M_B = 0$$

$$R_{Ay} \times 10 - 100 \times 5 = 0$$

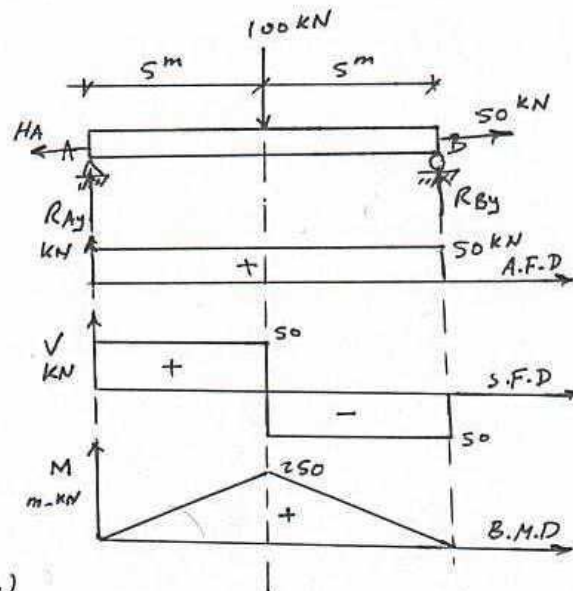
$$R_{Ay} = 50 \text{ kN}$$

$$\sum F_y = 0$$

$$50 + R_{By} - 100 = 0$$

$$R_{By} = 50 \text{ kN}$$

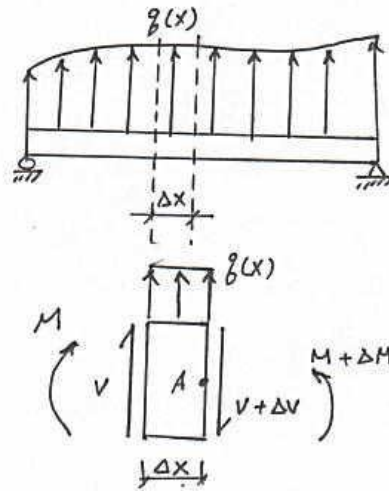
$$M_{\max} = 50 \times 5 = 250 \text{ m-kN (at middle of span)}$$





(2) Summation Method:

For the beam shown which is loaded by a general loading. Applying the equilibrium on the infinitesimal segment Δx yields into:



$$\sum F_y = 0 \uparrow$$

$$V + q(x) \Delta x - (V + \Delta V) = 0$$

$$\frac{\Delta V}{\Delta x} = -q$$

$$\text{Take } \lim_{\Delta x \rightarrow 0} \frac{\Delta V}{\Delta x}$$

$$\boxed{\therefore \frac{dV}{dx} = -q} \quad \text{----- (1)}$$

$$\sum M_A = 0$$

$$M + V \Delta x + q \frac{(\Delta x)^2}{2} - (M + \Delta M) = 0$$

$$\frac{\Delta M}{\Delta x} = -V$$

$$\text{Take } \lim_{\Delta x \rightarrow 0} \frac{\Delta M}{\Delta x}$$

$$\boxed{\therefore \frac{dM}{dx} = -V} \quad \text{----- (2)}$$

Differentiating eq(2) wrt x :

$$\frac{d^2 M}{dx^2} = \frac{dV}{dx} = -q$$

$$\boxed{\frac{d^2 M}{dx^2} = -q} \quad \text{----- (3)}$$

Hence:

$$\boxed{V = \int q(x) dx + C_1} \quad \text{----- (4)}$$

$$\boxed{M = \int V(x) dx + C_2} \quad \text{----- (5)}$$



Example (2): Draw shear force and bending moment diagrams for the beam shown.

Solution:

$$\sum M_B = 0$$

$$R_{Ay} \times 20 - 1 \times 10 \times 10 - 200 \times 10 = 0$$

$$R_{Ay} = 115 \text{ K}$$

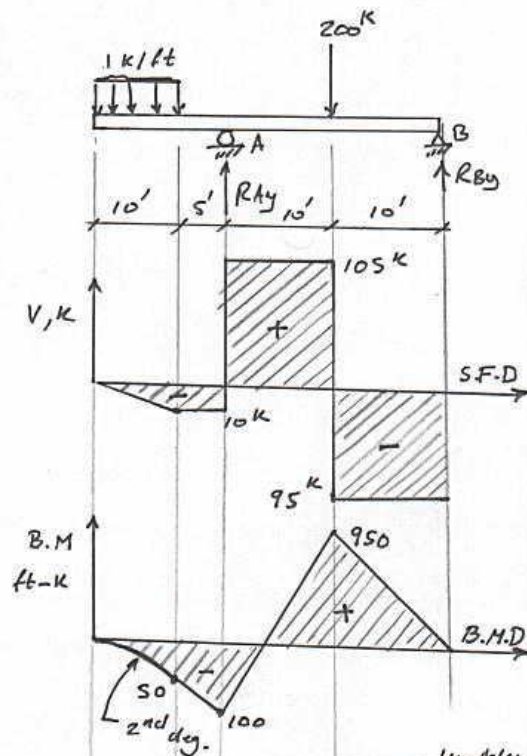
$$\sum F_y = 0$$

$$115 - 1 \times 10 - 200 + R_{By} = 0$$

$$R_{By} = 95 \text{ K}$$

$$V = \int_0^{10} (-1) dx = -x \Big|_0^{10} = -10 \text{ K}$$

$$M = \int_0^{10} -x dx = -\frac{x^2}{2} \Big|_0^{10} = -50 \text{ ft-K}$$



Example (3): Prob. (2-3) Popov.

Draw S.F & B.M. Diagrams:

Solution:

$$\sum M_B = 0$$

$$R_{Ay} \times 10 - 100 \times 10 \times 5 - 300 \times 10 \times 0.5 + \frac{10}{3} = 0$$

$$R_{Ay} = 1000 \text{ lb}$$

$$\frac{y}{x} = \frac{300}{10} \Rightarrow y = 30x$$

$$\therefore q = 100 + 30x \text{ lb/ft}$$

$$V = \int_0^{10} -(100 + 30x) dx + 1000$$

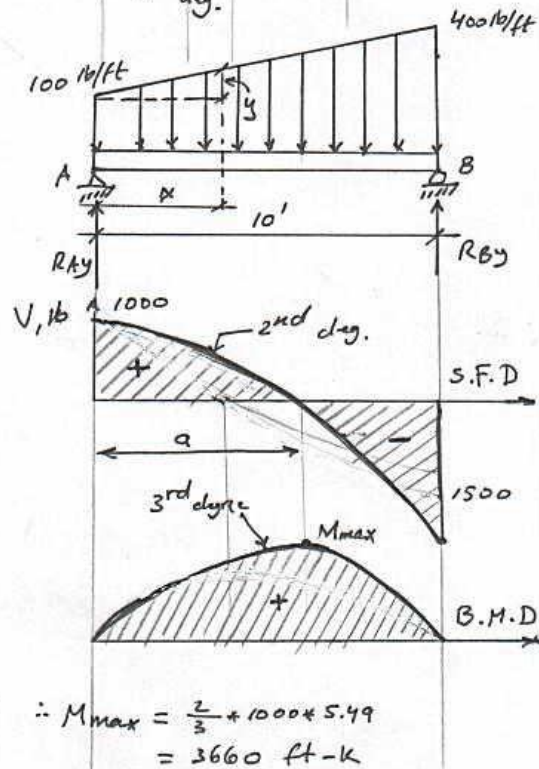
$$= -100x - 15x^2 \Big|_0^{10} + 1000 = -1500 \text{ lb}$$

The distance a can be found from:

$$1000 = (100a + \frac{30a^2}{2})$$

$$15a^2 + 100a - 1000 = 0$$

$$a = \frac{-100 \pm \sqrt{100^2 + 4 \times 15 \times 1000}}{2 \times 15} = 5.49'$$



$$\therefore M_{max} = \frac{2}{3} \times 1000 \times 5.49$$

$$= 3660 \text{ ft-K}$$



Example 4:

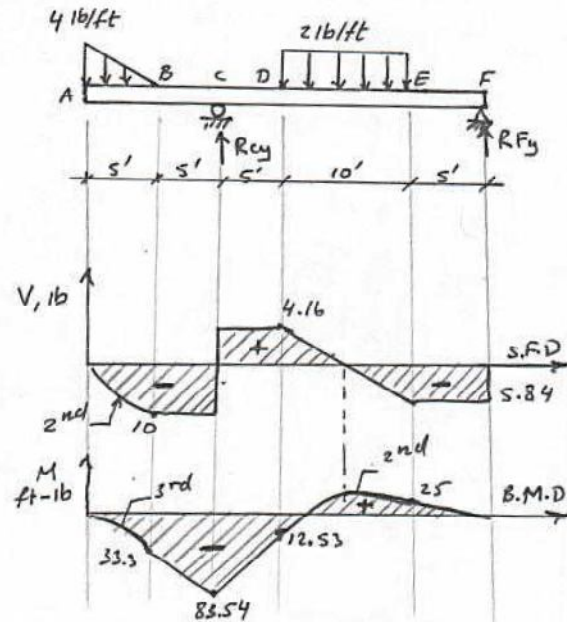
Draw shear force & B.M.

Diagrams -

Solution: Find reactions

$$R_{cy} = 24.16 \text{ lb}$$

$$R_{fy} = 5.84 \text{ lb}$$



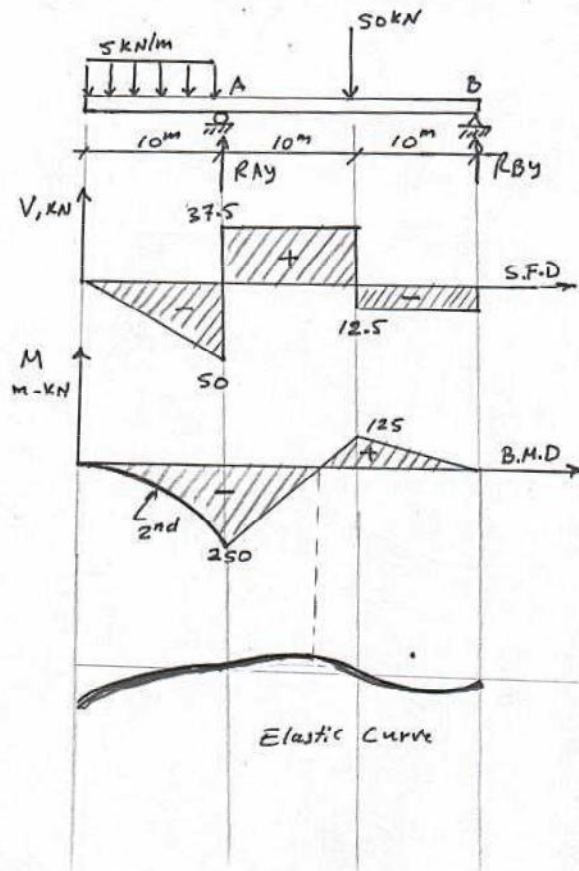
Example 5:

Draw shear force and Bending moment diagrams

Solutions: Find reactions:

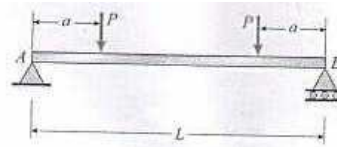
$$R_{Ay} = 87.5 \text{ kN}$$

$$R_{By} = 12.5 \text{ kN}$$

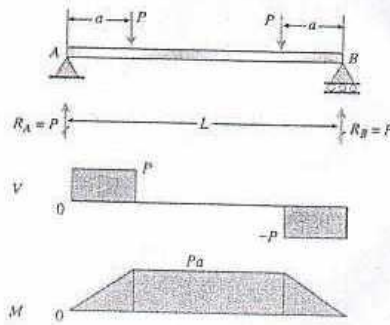




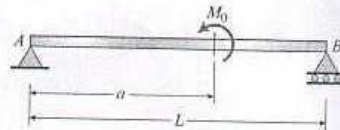
Problem 1 Draw the shear-force and bending-moment diagrams for a simple beam AB supporting two equal concentrated loads P (see figure).



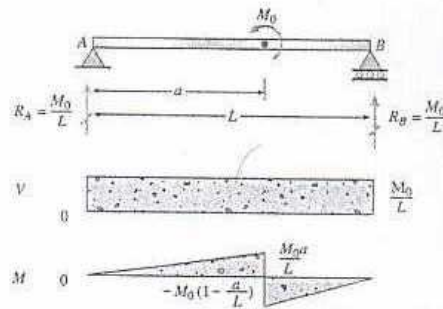
Solution Simple beam



Problem 2 A simple beam AB is subjected to a counterclockwise couple of moment M_0 acting at distance a from the left-hand support (see figure). Draw the shear-force and bending-moment diagrams for this beam.



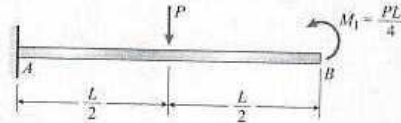
Solution Simple beam



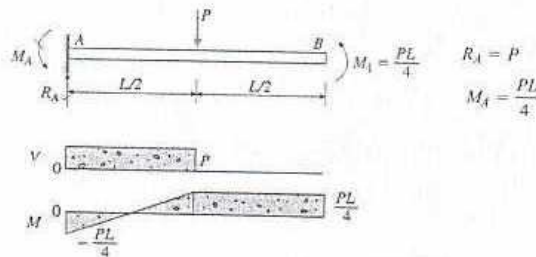


Problem 3 The cantilever beam AB shown in the figure is subjected to a concentrated load P at the midpoint and a counterclockwise couple of moment $M_1 = PL/4$ at the free end.

Draw the shear-force and bending-moment diagrams for this beam.

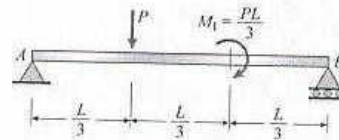


Solution Cantilever beam

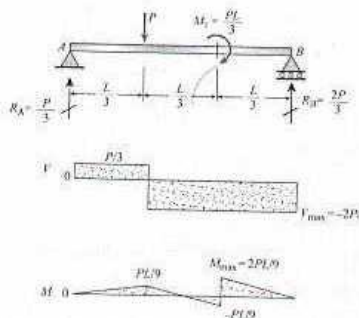


Problem 4 The simple beam AB shown in the figure is subjected to a concentrated load P and a clockwise couple $M_1 = PL/3$ acting at the third points.

Draw the shear-force and bending-moment diagrams for this beam.

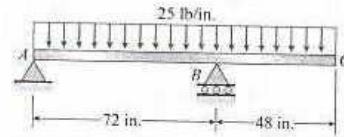


Solution

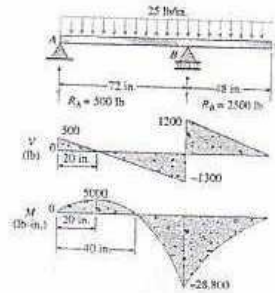




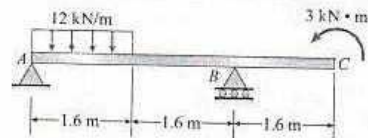
Problem 7 The uniformly loaded beam ABC has simple supports at A and B and an overhang BC (see figure). Draw the shear-force and bending-moment diagrams for this beam.



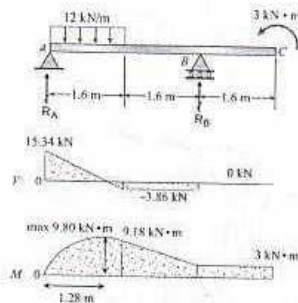
Solution Beam with an overhang



Problem 8 A beam ABC with an overhang at one end supports a uniform load of intensity 12 kN/m and a concentrated moment of magnitude $3 \text{ kN} \cdot \text{m}$ at C (see figure). Draw the shear-force and bending-moment diagrams for this beam.



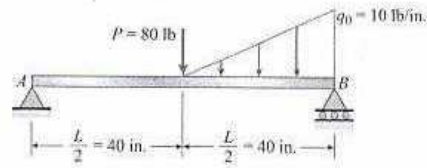
Solution Beam with an overhang





Problem 9 The simple beam AB supports a triangular load of maximum intensity $q_0 = 10 \text{ lb/in.}$ acting over one-half of the span and a concentrated load $P = 80 \text{ lb}$ acting at midspan (see figure).

Draw the shear-force and bending-moment diagrams for this beam.



Solution Simple beam

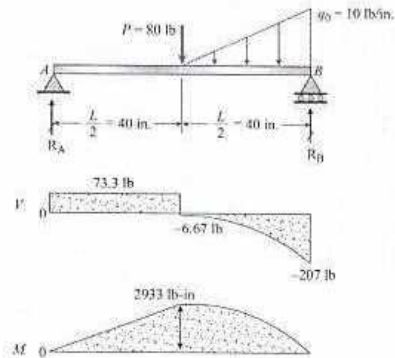
$$\sum M_A = 0; R_B(80 \text{ in.}) - (80 \text{ lb})(40 \text{ in.})$$

$$- (10 \text{ lb/in.})\left(\frac{1}{2}\right)(40 \text{ in.})(40 + 40\frac{2}{3} \text{ in.}) = 0$$

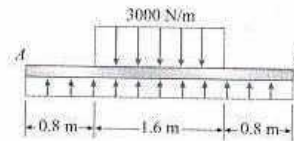
$$R_B = 206.7 \text{ lb}$$

$$\sum F_{\text{VERT}} = 0; R_A + R_B - 80 \text{ lb} - \left(10 \text{ lb/in.}\right)\left(\frac{1}{2}\right)(40 \text{ in.}) = 0$$

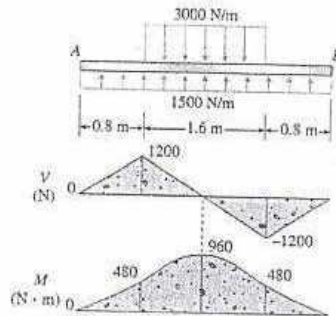
$$R_A = 73.3 \text{ lb}$$



Problem 10 The beam AB shown in the figure supports a uniform load of intensity 3000 N/m acting over half the length of the beam. The beam rests on a foundation that produces a uniformly distributed load over the entire length. Draw the shear-force and bending-moment diagrams for this beam.



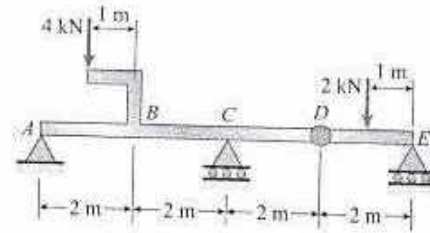
Solution Beam with distributed loads



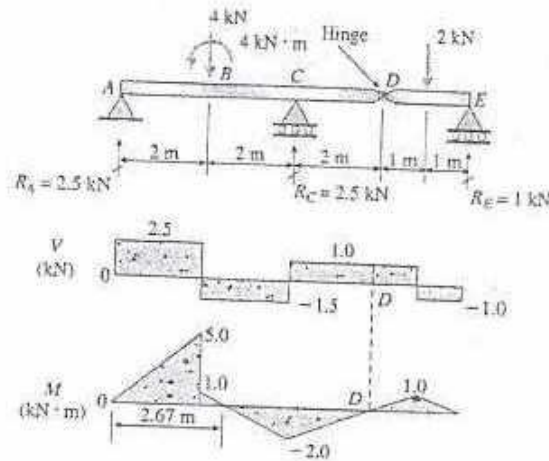


Problem 11 The compound beam $ABCDE$ shown in the figure consists of two beams (AD and DE) joined by a hinged connection at D . The hinge can transmit a shear force but not a bending moment. The loads on the beam consist of a 4-kN force at the end of a bracket attached at point B and a 2-kN force at the midpoint of beam DE .

Draw the shear-force and bending-moment diagrams for this compound beam.

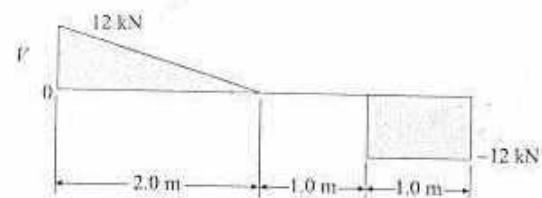


Solution Compound beam

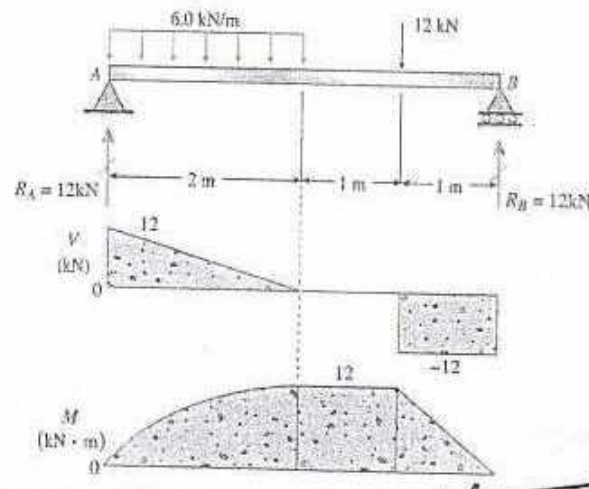


Problem 12 The shear-force diagram for a simple beam is shown in the figure.

Determine the loading on the beam and draw the bending-moment diagram, assuming that no couples act as loads on the beam.

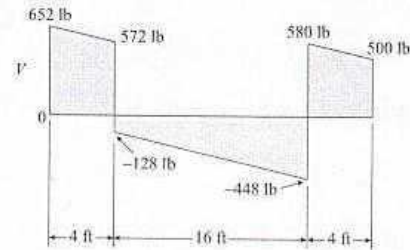


Solution Simple beam (V is given)



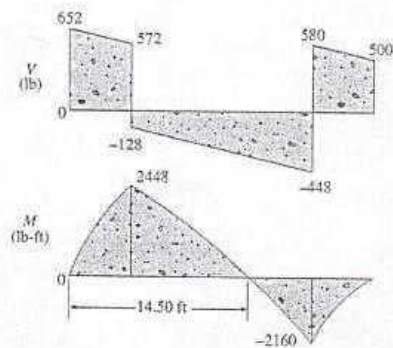
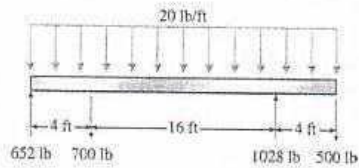


Problem 13 The shear-force diagram for a beam is shown in the figure. Assuming that no couples act as loads on the beam, determine the forces acting on the beam and draw the bending-moment diagram.



Solution Forces on a beam (V is given)

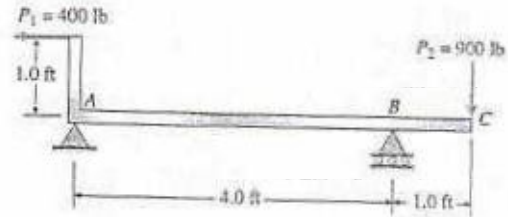
FORCE DIAGRAM



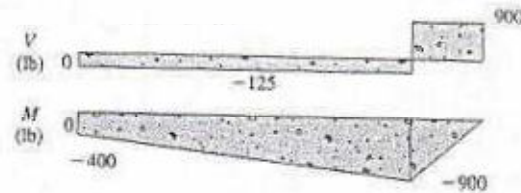
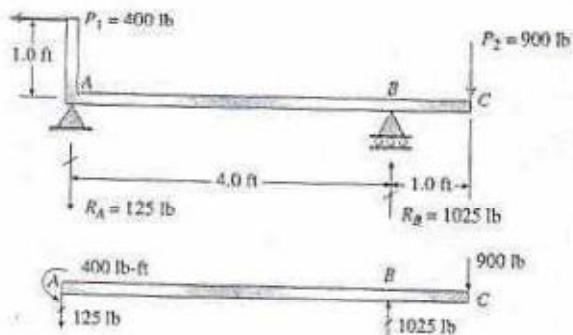


Problem 14 A beam $ABCD$ shown in the figure is simply supported at A and B and has an overhang from B to C . The loads consist of a horizontal force $P_1 = 400$ lb acting at the end of the vertical arm and a vertical force $P_2 = 900$ lb acting at the end of the overhang.

Draw the shear-force and bending-moment diagrams for this beam. (Note: Disregard the widths of the beam and vertical arm and use centerline dimensions when making calculations.)

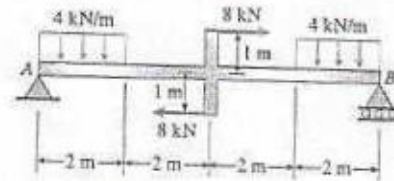


Solution Beam with vertical arm

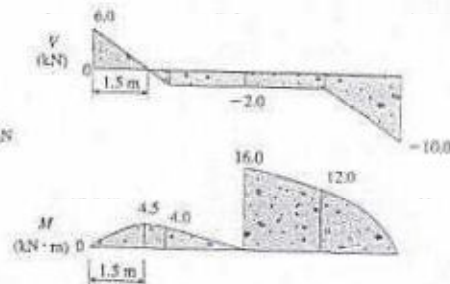
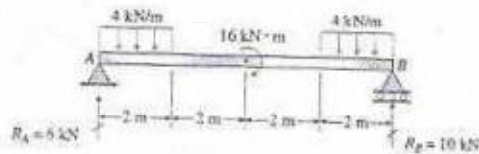


Problem 15 A simple beam AB is loaded by two segments of uniform load and two horizontal forces acting at the ends of a vertical arm (see figure).

Draw the shear-force and bending-moment diagrams for this beam.

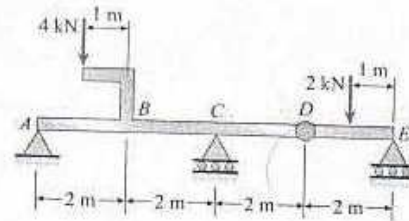


Solution Simple beam

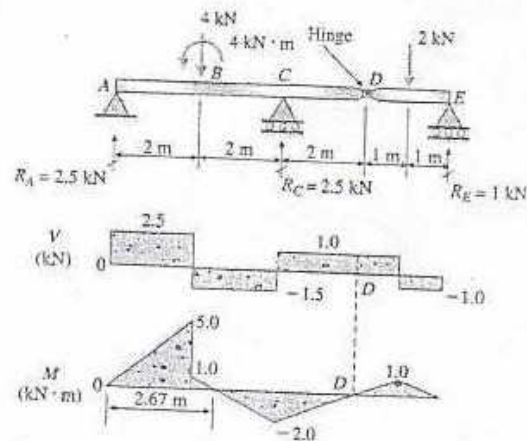




Problem #1 The compound beam $ABCDE$ shown in the figure consists of two beams (AD and DE) joined by a hinged connection at D . The hinge can transmit a shear force but not a bending moment. The loads on the beam consist of a 4-kN force at the end of a bracket attached at point B and a 2-kN force at the midpoint of beam DE . Draw the shear-force and bending-moment diagrams for this compound beam.

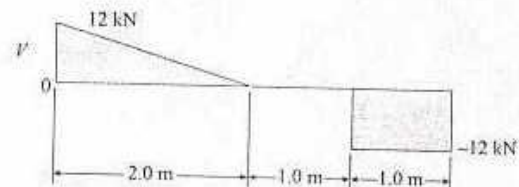


Solution Compound beam

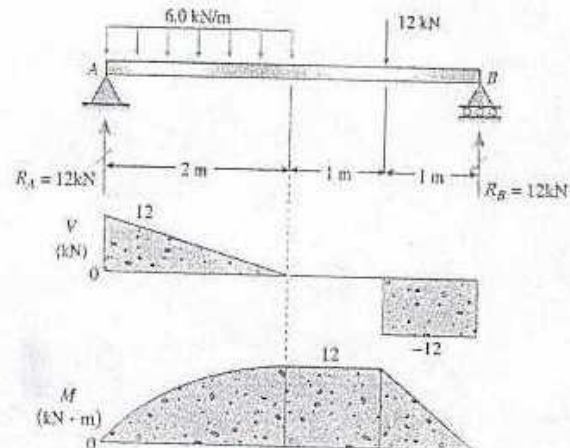


Problem #2 The shear-force diagram for a simple beam is shown in the figure.

Determine the loading on the beam and draw the bending-moment diagram, assuming that no couples act as loads on the beam.

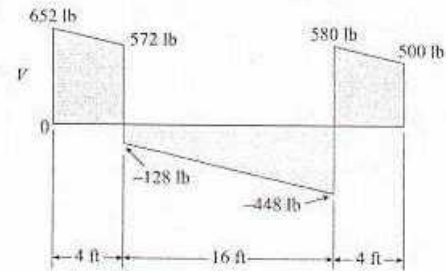


Solution Simple beam (V is given)



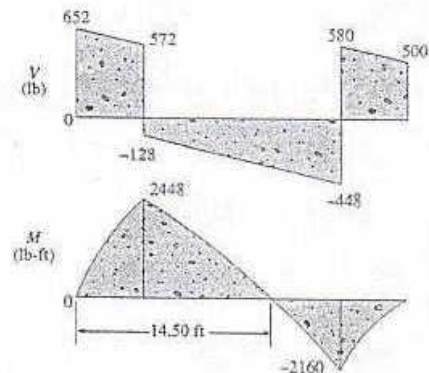
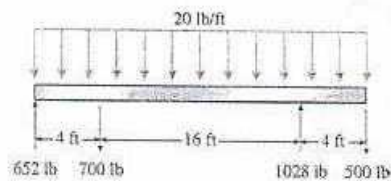


Problem 13 The shear-force diagram for a beam is shown in the figure. Assuming that no couples act as loads on the beam, determine the forces acting on the beam and draw the bending-moment diagram.



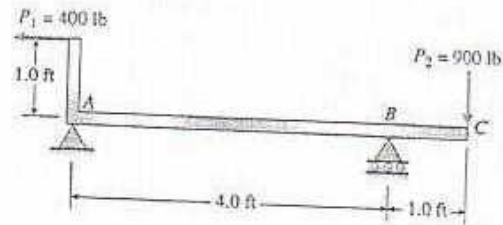
Solution Forces on a beam (V is given)

FORCE DIAGRAM

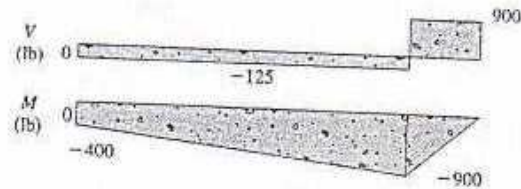
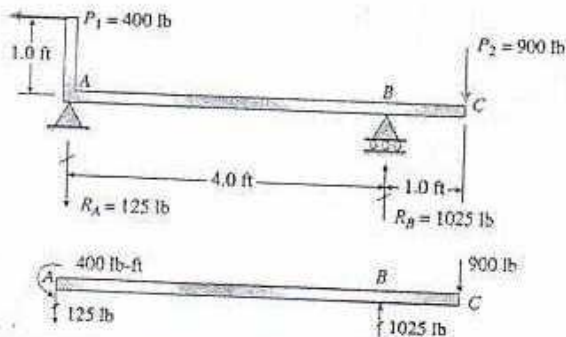


Problem 14 A beam $ABCD$ shown in the figure is simply supported at A and B and has an overhang from B to C . The loads consist of a horizontal force $P_1 = 400$ lb acting at the end of the vertical arm and a vertical force $P_2 = 900$ lb acting at the end of the overhang.

Draw the shear-force and bending-moment diagrams for this beam. (Note: Disregard the widths of the beam and vertical arm and use centerline dimensions when making calculations.)

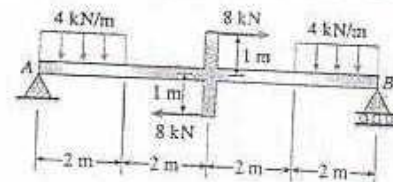


Solution *a.* Beam with vertical arm

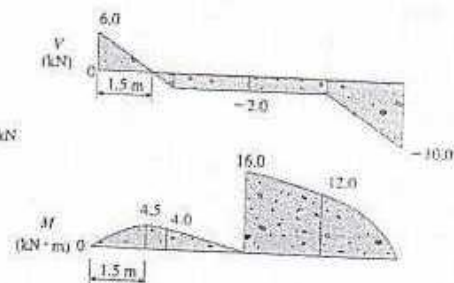
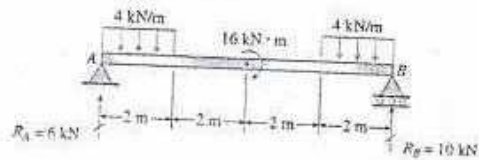


Problem 15 - A simple beam AB is loaded by two segments of uniform load and two horizontal forces acting at the ends of a vertical arm (see figure).

Draw the shear-force and bending-moment diagrams for this beam.

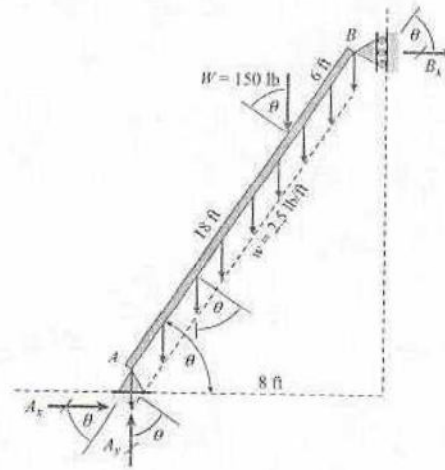


Solution **Simple beam**

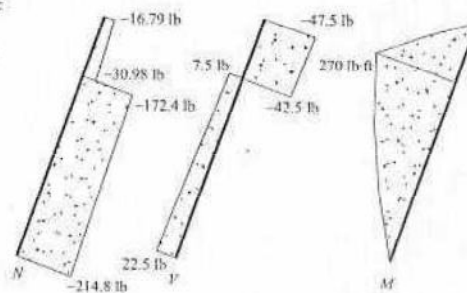
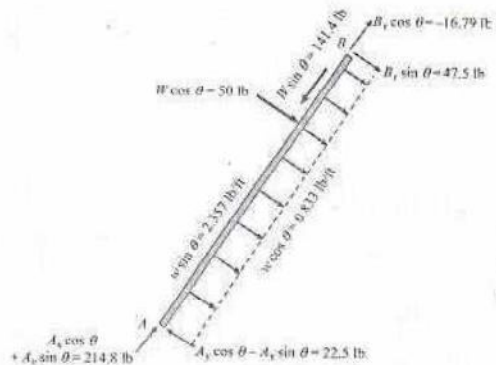




Problem 16 The inclined beam below represents the loads applied to a ladder: the weight (W) of the house painter and the self weight (w) of the ladder itself. Find support reactions at A and B , then plot axial force (N), shear (V) and moment (M) diagrams. Label all critical N , V & M values and also the distance to points where any critical ordinates are zero. Plot N , V & M diagrams normal to the inclined ladder.



Solution

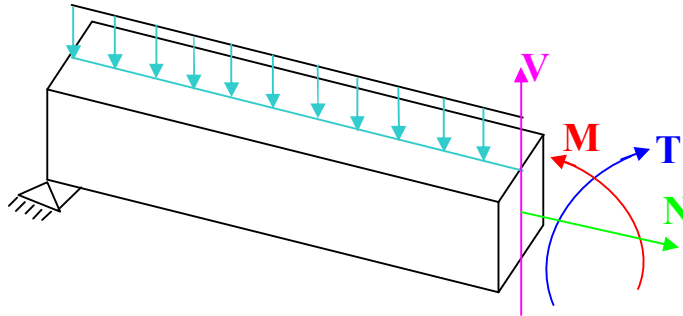




Chapter Six Bending Stresses in Beams

6.1 Derivation of Flextural Formula

As Shown previously in "Chapter 2", in any section of beam shown in Fig.(6-1) there is:



Figure(6-1)

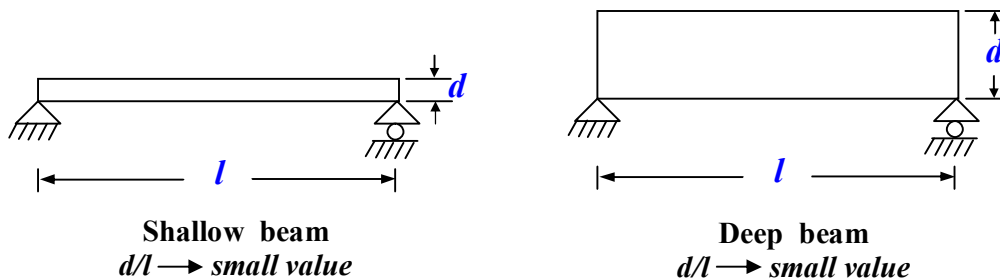
1. Normal (or axial), **N** force (as considered in Chapters 3&4)
2. Torque, **T** (as considered in Chapter 5)
3. Bending moment, **M** (as will be as considered in this chapter)
4. Shear force, **V** (as will be as considered in Chapter 7)

The Basic Kinematic Assumptions and Flextural Formula

1. Equations of Compatibility
2. Constitutive relation (Hook's Law)
3. Equilibrium Equations

1. Basic Assumption:

Plane section perpendicular to its axis before bending remains plane after bending. This assumption is true for shallow (or thin) beams and it is not for deep beams. See Figure(6-2).

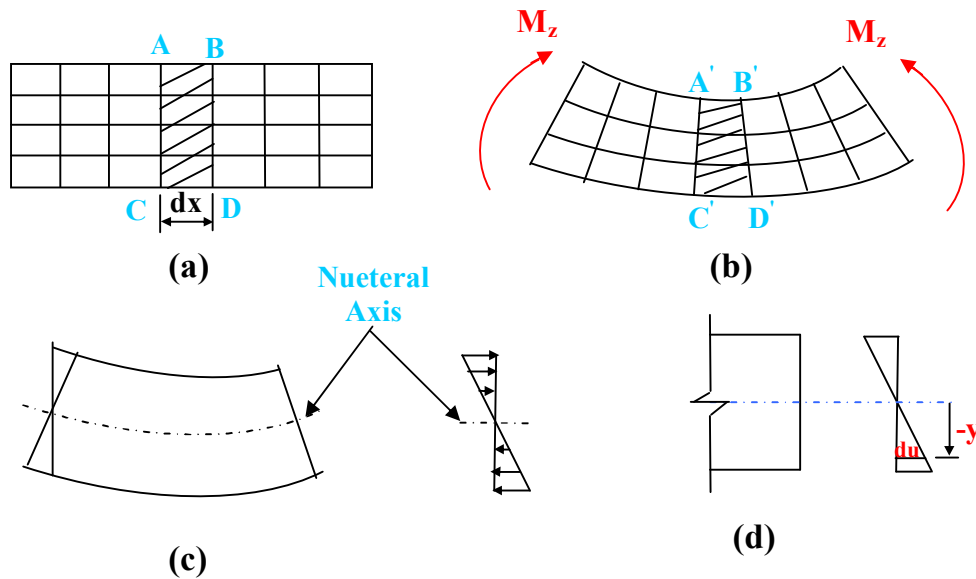


Figure(6-2)

In Fig.(6-3):

$$\epsilon_x = \frac{du}{dx} \quad , \quad du : \text{vary linearly} \\ dx : \text{constant}$$

$\therefore \epsilon_x : \text{vary linearly or directly as its distance from the neutral surface (axis).}$



Figure(6-3)

$$\therefore \epsilon_x = by \quad (\text{See Fig. (6-3)d})$$

Where, $b = \text{constant}$

2. Hook's Law:

According to Hook's Law:

$$\therefore \epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

σ_y & σ_z are very small

$$\therefore \epsilon_x = \frac{\sigma_x}{E}$$

$$\text{or } \sigma_x = E\epsilon_x \\ = Eby$$

$\therefore \sigma_x = By$ i.e. the stress varies linearly also.

3. Equilibrium Equations

In Fig.(6-4):

$$\int_A \sigma_x dA = 0$$

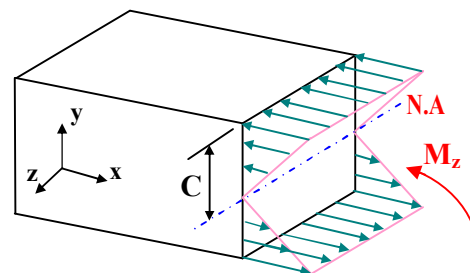
But $\sigma_x = By$

$$\therefore \int_A By dA = 0$$

$$\therefore B \int_A y dA = 0$$

$$\because B \neq 0, \therefore \int_A y dA = 0$$

$$\text{But } \int_A y dA = \bar{y}A$$



Figure(6-4)



Where, $\bar{y}$ = is the distance from N.A. to the centroid of the area.

$\therefore \bar{y} = 0$, i.e. the N.A. must pass through the centroid of the cross-sectional area of the beam.

$$\sum M_z = 0$$

$$M_z + \int_A (\sigma_x dA) y = 0$$

But $\sigma_x = By$

$$\begin{aligned} \therefore M_z &= - \int_A [(By) dA] y \\ &= - \int_A By^2 dA = - B \int_A y^2 dA \end{aligned}$$

But $\int_A y^2 dA = I$ = moment of inertia of the cross – sectional area about Centroidal axis

$$M_z = -BI \quad \text{or} \quad B = -\frac{M_y}{I}$$

$$\therefore \sigma_x = -\frac{My}{I}$$

Where, M = internal resisting moment

y = the distance from the neutral axis (N.A.) of the beam to the point on the section where the normal stress (σ_x) is wanted.

I = moment of inertia.

$$\therefore \sigma_x)_{\max} = -\frac{Mc}{I}$$

Where, $c = y_{\max}$

$$\text{Or } \sigma_x)_{\max} = -\frac{M}{I/c}$$

I/c : are constant for a given section of beam.

$I/c = S$ = elastic section modulus.

$$\therefore \sigma_x)_{\max} = -\frac{M}{S}$$

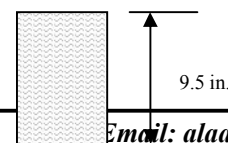
Examples:

Example(1): Problem(6-1), Popov (Analysis Problem)

According to the National Lumber Manufacturers Association the desired size of a 6-in.-by-10-in. timber is 5.5 in. by 9.5 in. If the allowable stress for a certain grade of wood is 1200 psi, determine the capacity in bending.

Solution:

$$\sigma_x)_{\max} = -\frac{Mc}{I}$$

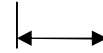




$$1200 = -\frac{M \times \frac{9.5}{2}}{\frac{9.5^3 \times 5.5}{12}}$$

$$\therefore M_{all} = 99200 \text{ in.-lb}$$

$$= 8.27 \text{ ft-kip} \quad (\text{ans.})$$



Example(2): Problem(6-2), Popov (Analysis Problem)

Consider the lightest member listed in Table 4 of the Appendix, which is 8WF17, and verify the moment of inertia around the x-axis and the y-axis for it. If the allowable stress is 24 ksi, find the flextural capacity of this member around each of the two axes.

Solution:

$$I_x = \frac{8^3 \times 5.25}{12} - \frac{(8 - 2 \times 0.308)^3 \times (5.25 - 0.23)}{12} = 55.6 \text{ in.}^4$$

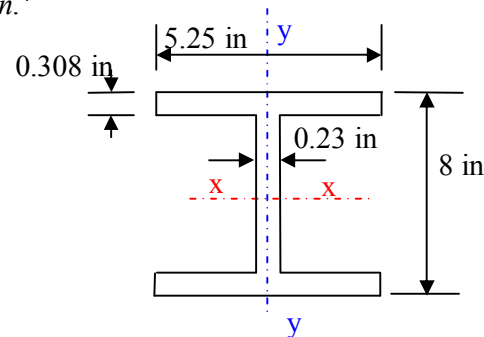
$$I_y = \frac{5.25^3 \times 8}{12} - \frac{(5.25 - 0.23)^3 \times (8 - 2 \times 0.308)}{12} = 7.5 \text{ in.}^4$$

$$\sigma_x)_{\max} = -\frac{Mc}{I}$$

$$M = -\frac{\sigma_{all} \times I}{c}$$

$$M_x = -\frac{24 \times 55.6}{4} = 330 \text{ in.-k} = 28 \text{ ft-k}$$

$$M_y = -\frac{24 \times 7.5}{4} = 45 \text{ in.-k} = 3.75 \text{ ft-k}$$



Example(3): Problem(6-9), Popov (Design Problem)

Select a standard steel I beam based on the the maximum bending moment. Assume $\sigma_{allow} = 24 \text{ ksi}$ and neglect the weight of the beam.

Solution:

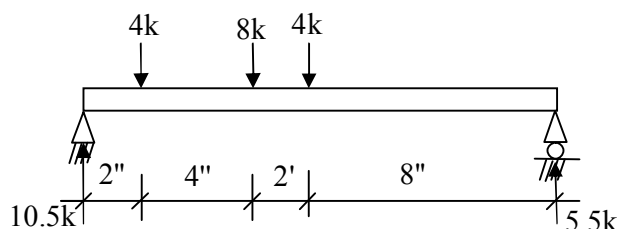
From B.M.D:

$$M_{\max} = 47 \text{ ft-k}$$

$$\sigma_{all} = \frac{M.c}{I} = \frac{M}{S}$$

$$24 = \frac{47 \times 12}{S}$$

$$S = 23.5 \text{ in}^3$$

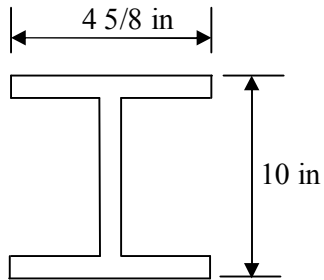




From Table(3): Try I $10 \times 4 \frac{5}{8}$

$$S = 24.4 \text{ in}^3$$

O.K.





Prob. 6-14

Req'd : $P = ?$

Solution:

$$\bar{y} = \frac{2 \times 5.16 \times 1 + (1 \times 3.16) \left(\frac{3.16}{2} + 2 \right) \times 2}{5.16 \times 5 - 3 \times 3.16}$$

$$= 2 \text{ in.}$$

$$I = \frac{2^3 \times 5}{3} + \frac{3.16^3 \times 1}{3} \times 2$$

$$= 34.4 \text{ in}^4$$

$$\sigma_B = 900 \times 10^6 \times 30 \times 10^6 = 27000 \text{ psi}$$

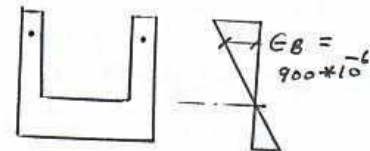
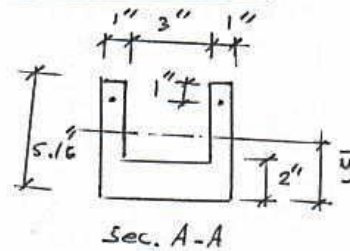
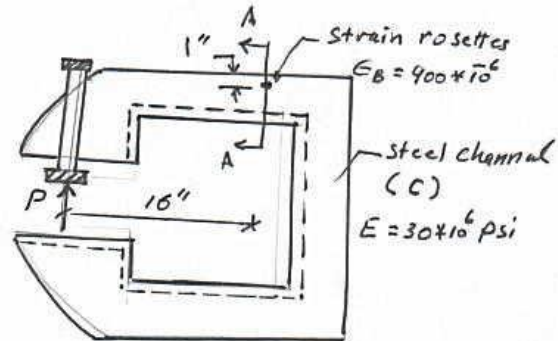
$$\sigma_B = \frac{M \cdot y_B}{I}$$

$$27000 = \frac{M \times 2.16}{34.4}$$

$$M = 430000 \text{ in-lb}$$

$$M = 16P$$

$$P = 26.9 \text{ kips}$$

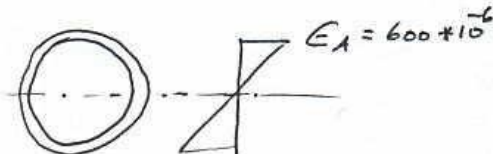


Prob. 6-15 : $E = 29 \times 10^6 \text{ psi}$

Req'd $P = ?$

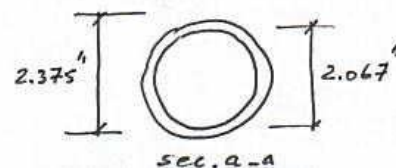
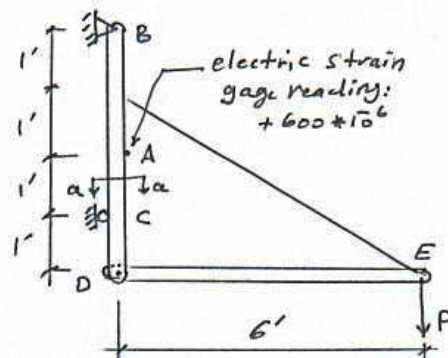
Solution:

$$I = \frac{(2.375^4 - 2.067^4) \pi}{64} = 0.666 \text{ in}^4$$



$$\sigma_A = 600 \times 10^6 \times 29 \times 10^6 = 17400 \text{ psi}$$

$$6 = 7$$





$$\sigma_A = \frac{M y_A}{I}$$

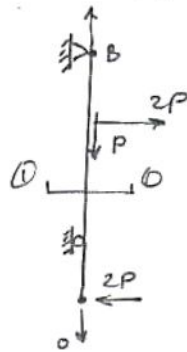
$$17400 = \frac{M * \frac{2.375}{2}}{0.666} \Rightarrow M = 9759 \text{ in-lb}$$

$$= 813 \text{ ft-lb}$$



$$\sum M_D = 0$$

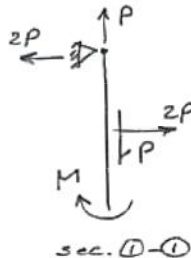
$$P * 6 = \frac{1}{\sqrt{5}} T * 4 \Rightarrow T = \sqrt{5} P$$



$$\sum F_y = 0 \Rightarrow R_{By} = P$$

$$\sum M_B = 0 \Rightarrow R_{Cx} = 2P \rightarrow$$

$$\sum F_x = 0 \Rightarrow R_{Bx} = 2P \leftarrow$$



$$M = 2P * 1 = 2P$$

$$2P = 813$$

$$\therefore P = 407 \text{ lb}$$

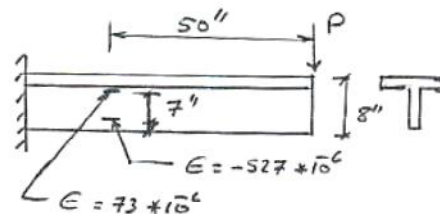
sec. 1-1

Prob. 6-16

Given: $I = 48.3 \text{ in}^4$

$E = 30 * 10^6 \text{ psi}$

Req'd: $P = ?$



Solution:



$$\frac{C_1}{527} = \frac{6}{(527 + 73)} \Rightarrow C_1 = 5.27 \text{ in.}$$

$$\sigma_A = 527 * 10^6 * 30 * 10^6 = 15810 \text{ psi}$$



$$\sigma_A = \frac{My}{I}$$

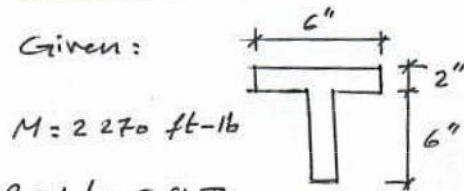
$$15810 = \frac{M \times 5.27}{48.3} \Rightarrow M = 144900 \text{ in-lb}$$

$$SO P = 144900$$

$$P = 2.89 \text{ kips.}$$

Prob. 6-19 (Resultant Force)

Given:



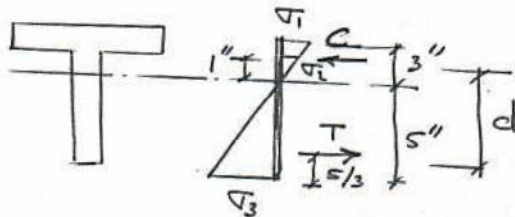
$$M = 2270 \text{ ft-lb}$$

Req'd: C & T

Solution:

$$\bar{y} = \frac{6 \times 2 \times 7 + 6 \times 2 \times 3}{6 \times 2 \times 2} = 5 \text{ in.}$$

$$I = \frac{6 \times 3^3}{3} + \frac{5^3 \times 2}{3} - \frac{1^3 \times 4}{3} = 136 \text{ in}^4$$



$$M = 2270 \times 12 = 27240 \text{ in-lb}$$

$$\sigma_1 = \frac{27240 \times 3}{136} = 602 \text{ psi}$$

$$\sigma_2 = \frac{27240 \times 1}{136} = 200 \text{ psi}$$

$$\sigma_3 = \frac{27240 \times 5}{136} = 1001 \text{ psi}$$

$$T = \frac{1001}{2} \times 5 \times 2 = 5005 \text{ lb}$$

$$C = \left(\frac{601 + 200}{2} \right) \times 6 \times 2 + \frac{200}{2} \times 1 \times 2 = 5005 \text{ lb}$$

$$\sum F_x = 0$$

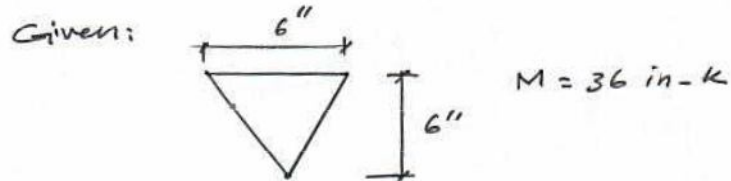
$$C = T \quad \therefore \text{o.k.}$$

$$C \cdot d = M$$

$$d = \frac{27240}{5005} = 5.5 \text{ in.}$$

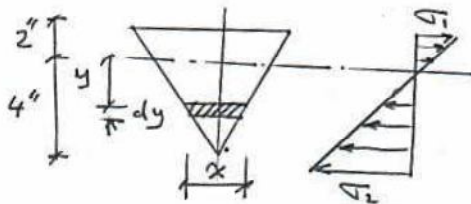


Prob. 6-20 (Resultant Force)



Req'd: C & T

Solution:



$$I = \frac{h^3 b}{36} = \frac{6^3 \times 6}{36} = 36 \text{ in}^4$$

$$\sigma = \frac{M \cdot y}{I} = \frac{36 \cdot y}{36} = y$$

$$C = \int_0^4 \sigma dA = \int_0^4 y \cdot (x dy)$$

$$\frac{x}{4-y} = \frac{y}{4} \Rightarrow x = 4-y$$

$$C = \int_0^4 y(4-y) dy$$

$$= \int_0^4 (4y - y^2) dy$$

$$= 2y^2 - \frac{y^3}{3} \Big|_0^4$$

$$= 2 \cdot 4^2 - \frac{4^3}{3}$$

$$C = 10.67 \text{ kips}$$

$$\sum F_x = 0$$

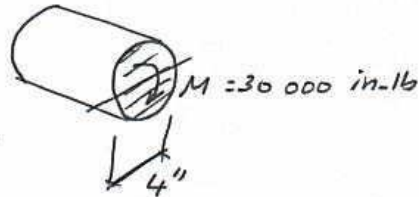
$$T = C = 10.67 \text{ kips}$$



Prob. 6-21 (Resultant Force)

Given:

Req'd: C & T.



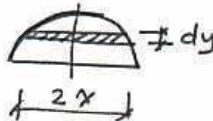
Solution:

$$I = \frac{4^4 \pi}{64} = 12.6 \text{ in}^4$$

$$\sigma = \frac{M \cdot y}{I}$$

$$= \frac{30000 \cdot y}{12.6} = 2381 y \text{ psi}$$

$$C = \int_0^2 2381 y \, dA$$



$$dA = 2x \, dy$$

$$x^2 + y^2 = 2^2$$

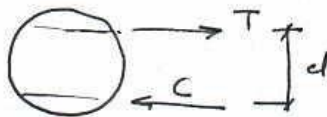
$$x = \sqrt{4 - y^2}$$

$$C = \int_0^2 (2381 y) (2 \sqrt{4 - y^2}) \, dy$$

$$= \int_0^2 4762 y \sqrt{4 - y^2} \, dy$$

$$= \frac{-2 \times 4762}{3} (4 - y^2)^{3/2} \left(\frac{1}{2} \right) \Big|_0^2$$

$$= 12688 \text{ lb} = 12.7 \text{ K}$$



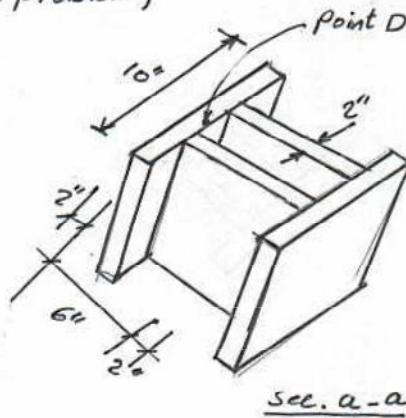
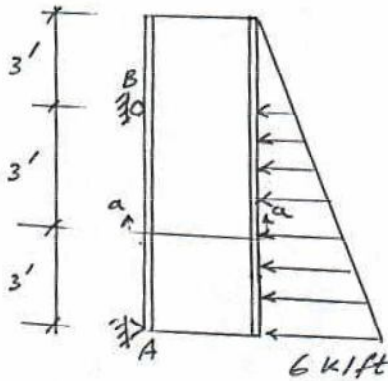
$$C \cdot d = M$$

$$d = \frac{30000}{12688} = 2.36 \text{ in.}$$



Prob. 6-8 (check problem)

Given:



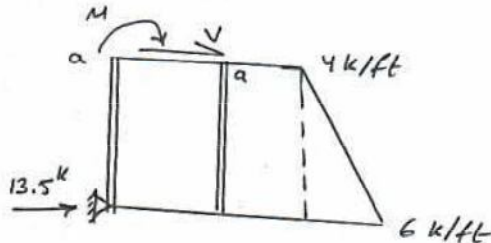
Req'd = $\tau_D = ?$

Solution:

$$\tau_D = \frac{M \cdot y}{I}$$

$$\sum M_B = 0$$

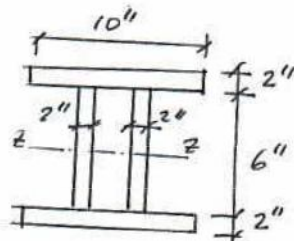
$$R_A \times 6 - \frac{6}{2} \times 9 \times 3 = 0 \Rightarrow R_A = 13.5 \text{ k}$$



$$\sum M_a = 0 \quad \uparrow$$

$$M_a + 4 \times 3 \times 1.5 + \frac{2}{2} \times 3 \times 2 - 13.5 \times 3 = 0$$

$$M_a = 16.5 \text{ ft-k}$$



$$I_{zz} = \frac{10 \times 10^3}{12} - \frac{6 \times 6^3}{12} = 725 \text{ in}^4$$

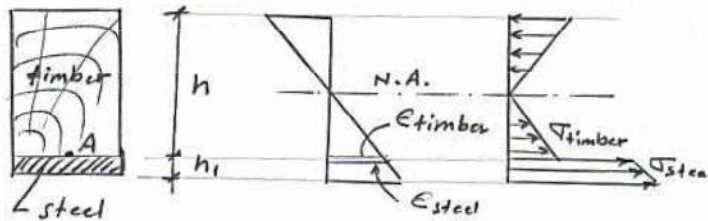
$$y_D = 3''$$

$$\tau_D = \frac{16.5 \times 12 \times 3}{725}$$

$$= 0.819 \text{ ksi}$$



Beams of Different Materials



At point A:

$$E_{\text{timber}} = E_{\text{steel}}$$

$$\frac{\sigma_{\text{timber}}}{E_{\text{timber}}} = \frac{\sigma_{\text{steel}}}{E_{\text{steel}}} \quad \text{----- (1)}$$

$$P_{\text{steel}} = P_{\text{timber}}$$

$$\sigma_s A_s = \sigma_t A_t \quad \text{----- (2)}$$

$$\text{But } \sigma_s = \frac{E_s}{E_t} \sigma_t \quad (\text{From eqn. (1)})$$

$$\text{Let } \frac{E_s}{E_t} = n$$

$$\therefore \sigma_s = n \sigma_t$$

Sub. into eqn. (2):

$$\therefore n \sigma_t A_s = \sigma_t A_t)_{\text{equivalent}}$$

$$\therefore A_t)_{\text{equivalent}} = n A_s$$



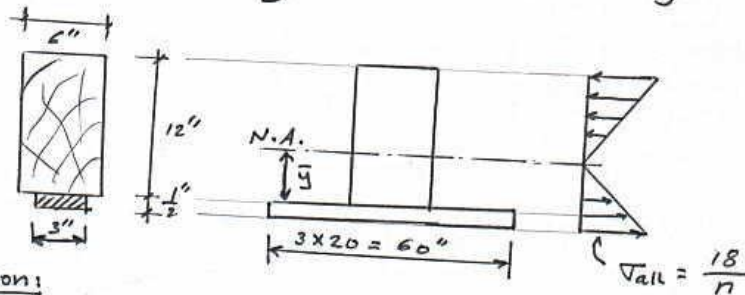
Example 1: (over-reinforced sec.)

let $\sigma_{steel} = 18 \text{ Ksi}$

$\sigma_{timber} = 1.2 \text{ Ksi}$

$\& \frac{E_s}{E_t} = n = 20$

Req'd: The max. resisting moment for the following section:



Solution:

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{6 \times 12 \times 6.5 + 60 \times 0.5 \times 0.25}{6 \times 12 + 60 \times 0.5}$$

$$\bar{y} = 4.67 \text{ in.} = y_{\text{tension}}$$

$$y_{\text{comp.}} = 12.5 - 4.67 = 7.83 \text{ in.}$$

$$I = \frac{7.83^3 \times 6}{3} + \frac{4.67^3 \times 6}{3} + \left(\frac{0.5^3 \times 60}{12} + 0.5 \times 60 \times \left(4.67 - \frac{1}{4} \right)^2 \right)$$

$$= 1685 \text{ in}^4$$

$$M = \frac{\sigma \cdot I}{c}$$

$$M_{\text{timber}} = \frac{1.2 \times 1685}{7.83} = 258 \text{ in-K}$$

$$M_{\text{steel}} = \frac{\frac{18}{20} \times 1685}{4.67} = 324 \text{ in-K}$$

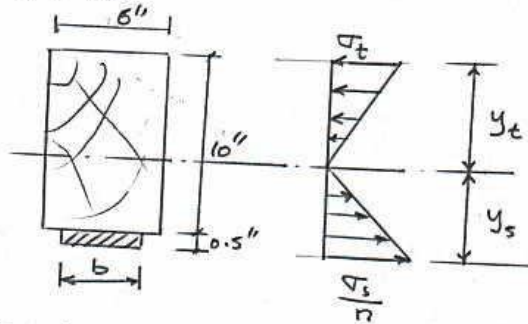
Take the smaller: $M = 258 \text{ in-K}$.

There is an excess of steel area, hence the beam may be said to be over reinforced.



Example 2: For the Fig. shown below, determine the width b of the steel plate that will simultaneously stress the wood and the steel to their permissible limits of $\sigma_t = 1.2 \text{ ksi}$ and $\sigma_s = 18 \text{ ksi}$

$$n = 20$$



Solution:

$$\frac{\sigma_s}{n} = \frac{M y_s}{I} \quad \text{----- (1)}$$

$$\sigma_t = \frac{M y_t}{I} \quad \text{----- (2)}$$

By dividing Eqn. (1) / Eqn. (2):

$$\frac{y_s}{y_t} = \frac{\sigma_s}{n \sigma_t} = \frac{18}{20 \times 1.2}$$

$$y_s = 0.75 y_t$$

$$\text{But } y_s + y_t = 10.5$$

$$0.75 y_t + y_t = 10.5 \Rightarrow y_t = 6 \text{ in} \\ y_s = 4.5 \text{ in.}$$

$$y_s = \bar{y} = \frac{\sum a_i y_i}{\sum a_i}$$

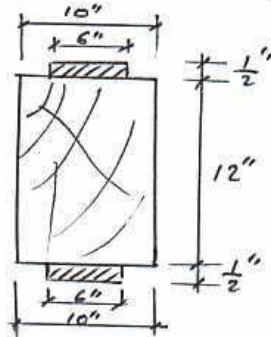
$$4.5 = \frac{6 \times 10 \times 5.5 + 20b \times 0.5 \times \frac{1}{4}}{6 \times 10 + 20b \times 0.5}$$

Solve for b :

$$b = 1.41 \text{ in.}$$



Example 3: Find $\Delta M/M$ due to reinforcing the beam below:



$$\sigma_{st} = 18 \text{ ksi}$$

$$\sigma_t = 1 \text{ ksi}$$

$$n = 15$$

Solution:

(a) without reinforcing:

$$M = \frac{\sigma_t \cdot I}{C} = \frac{1 \times \frac{12^3 \times 10}{12}}{6} = 240 \text{ in-K}$$

(b) with reinforcement:

$$M = \frac{\sigma_s}{n} \cdot I$$

$$I = \frac{12^3 \times 10}{12} + 2 \times [6 \times 15 \times 0.5 \times (6 + 0.25)^2] = 4956 \text{ in}^4$$

1- Assuming failure at steel:

$$M = \frac{\frac{18}{15} \times 4956}{6.5} = 914 \text{ in-K}$$

2- Assuming failure at timber:

$$M = \frac{1 \times 4956}{6} = 826 \text{ in-K}$$

Take the smaller value, $M = 826 \text{ in-K}$

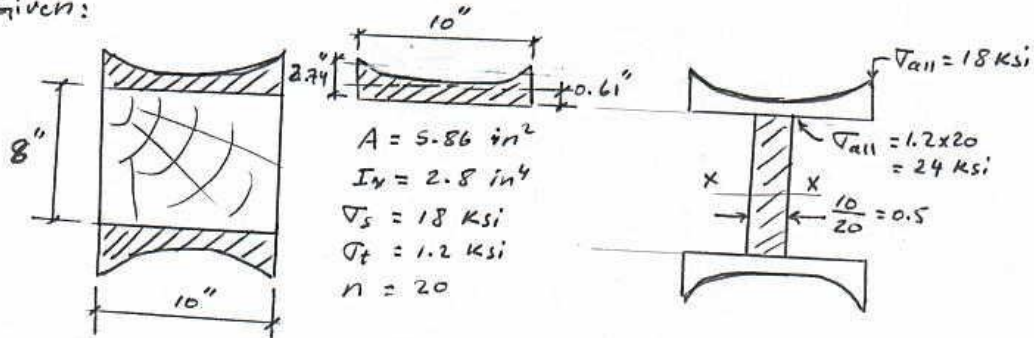
$$\Delta M = 826 - 240 = 586 \text{ in-K}$$

$$\frac{\Delta M}{M} = \frac{586}{240} = 2.44$$



Example 4:

Given:



Solution:

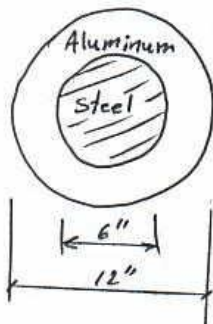
$$I_x = [2.8 + 5.86 * (4 + 0.61)^2] * 2 + \frac{0.5 * 8^3}{12} = 276.3 \text{ in}^4$$

$$M = \frac{\tau_{all} * I}{y}$$

$$= \frac{18 * 276.3}{4 + 2.74} = 737.8 \text{ in-k}$$

$$= 61.5 \text{ ft-k}$$

Example 5: (Prob. 6-34)



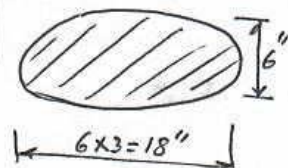
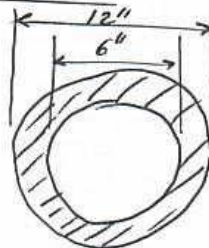
Given:

$$E_s = 30 * 10^6 \text{ psi}$$

$$E_{al} = 10 * 10^6 \text{ psi}$$

$$M = 60 \text{ ft-k}$$

Req'd: τ_{st} & τ_{al} .



Solution: $n = \frac{E_{st}}{E_{al}} = \frac{30 * 10^6}{10 * 10^6} = 3$

$$I = \frac{\pi}{64} [12^4 - 6^4] + \frac{(9 * 3^3) \pi}{4} = 1144.8 \text{ in}^4$$

$$\tau_{st} = \frac{60 * 12 * 3}{1144.8} * 3 = 5.67 \text{ Ksi}$$

$$\tau_{al} = \frac{60 * 12 * 6}{1144.8} = 3.77 \text{ Ksi}$$

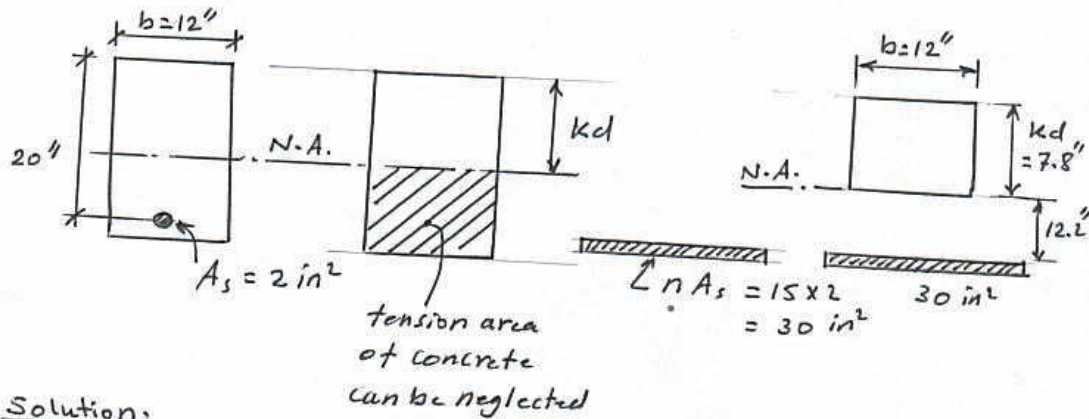


Reinforced Concrete Beams

Example 1: (Check Problem)

Determine the max. stresses in the concrete and reinforcing steel

$$M = 50000 \text{ ft-lb} \quad \& \quad n = 15$$



Solution:

$$12 \times kd \left(\frac{kd}{2} \right) = 30 \times (20 - kd)$$

$$6kd^2 + 30kd - 600 = 0$$

$$kd = \frac{-30 \pm \sqrt{30^2 - 4 \times 6 \times (-600)}}{2 \times 6} = 7.8 \text{ in.}$$

1st method:

$$I_{N.A.} = \frac{7.8 \times 12}{3} + 30 \times (20 - 7.8)^2 - \left[\frac{\pi d b^4}{64} \times n \text{ of bars} \right]$$

$$= 6363.4 \text{ in}^4$$

$$\sigma_c = \frac{(50 \times 12) \times 7.8}{6363.4} = 0.735 \text{ ksi}$$

$$\sigma_s = \frac{50 \times 12 \times (20 - 7.8)}{6363.4} \times 15 = 17.3 \text{ ksi}$$

2nd method:

$$M = (\sigma_s A_s) \times jd, \quad j = 1 - \frac{k}{3}$$

$$50 \times 12 = (\sigma_s \times 2) \left(20 - \frac{7.8}{3} \right) = 17.3 \text{ ksi}$$

$$\sum F_x = 0$$

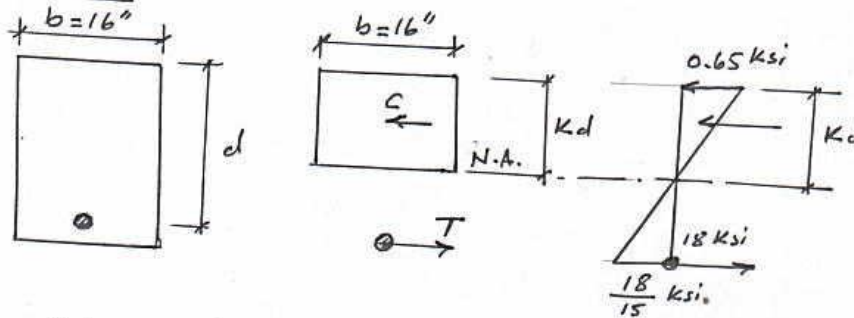
$$\frac{\sigma_c}{2} \times 10 \times kd = \sigma_s \times 2 \Rightarrow \sigma_c = 0.735 \quad 6-18$$



Example (3): Design of reinforced concrete beam.

Design a concrete beam with balanced reinforcement to resist a bending moment of 70 K-ft, $f_c)_{all.} = 650 \text{ psi}$, $f_s = 18000 \text{ psi}$, $n = 15$ and $b = 16 \text{ in.}$

solution:



$$\frac{Kd}{0.65} = \frac{d}{0.65 + \frac{18}{n}}$$

$$\frac{Kd}{d} = \frac{0.65}{0.65 + \frac{18}{15}}$$

$$K = 0.351$$

$$\sum M \oplus = 0$$

$$\frac{0.65}{2} \times b d \times 0.351 \times \left(1 - \frac{0.351}{2}\right) d = 70 \times 12$$

$$b d^2 = 8340$$

$$d = \sqrt{\frac{8340}{16}} = 22.8 \text{ in.}$$

$$\sum F_x = 0$$

$$\frac{0.65}{2} \times (22.8 \times 0.351) \times 16 = 18 \times A_s$$

$$\therefore A_s = 2.31 \text{ in}^2$$



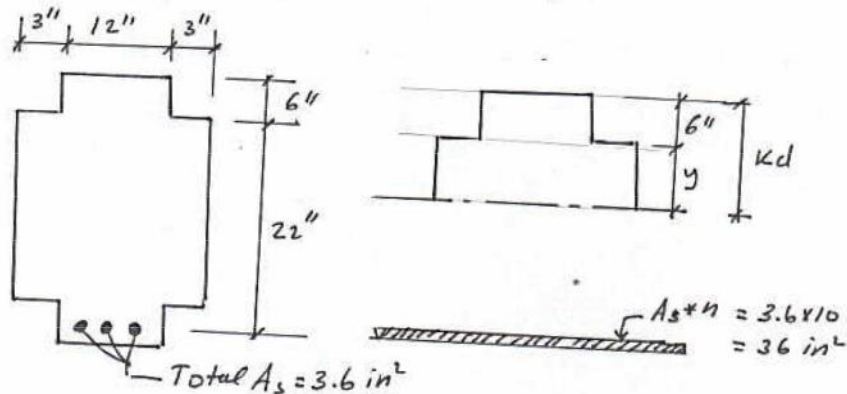
Example (5): Prob. 6-41

Given: $n = 10$

$$\tau_s)_{all.} = 50 \text{ ksi}$$

$$\tau_c)_{all.} = 4 \text{ ksi}$$

Req'd: Final Mall.



Solution:

$$12 \times 6 \times (3 + y) + 18 \times y \times \frac{y}{2} = 36 \times (22 - y)$$

$$9y^2 + 108y - 576 = 0$$

$$y = \frac{-108 \pm \sqrt{108^2 + 4 \times 9 \times 576}}{2 \times 9} = 4 \text{ in}$$

$$\therefore kd = 6 + y = 6 + 4 = 10 \text{ in.}$$

$$I = \frac{10^3 \times 12}{3} + \frac{4^3 \times 3}{3} \times 2 + 36 \times 18^2 = 15792 \text{ in}^4$$

$$M_{all.} = \frac{\tau_{all.} \times I}{C}$$

$$M_c = \frac{4 \times 15792}{10} = 6317 \text{ in-k}$$

$$M_s = \frac{\frac{50}{10} \times 15792}{18} = 4387 \text{ in-k}$$

$$\therefore M_{all.} = 4387 \text{ in-k} \quad \text{Under Reinforced section.}$$



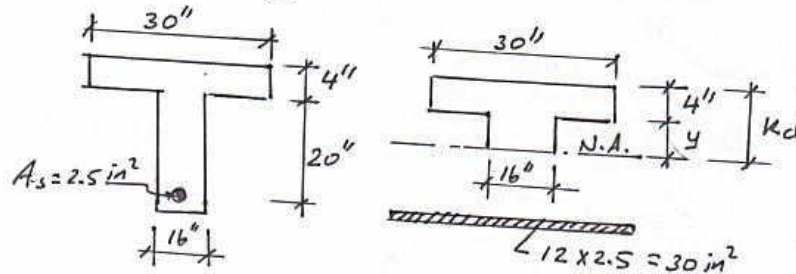
Example (4): (Analysis Prob.)

Given: $n = 12$

$$f'_c = 0.6 \text{ ksi}$$

$$f_y = 18 \text{ ksi}$$

Req'd: $M_{resisting}$



Solution:

$$30 \times kd \times \frac{kd}{2} - 14 \times (kd - 4) \times \frac{kd - 4}{2} = 30 \times (20 - kd)$$

Or: let $kd - 4 = y$

$$30 \times 4 \times (y + 2) + 16 \times y \times \frac{y}{2} = 30 \times (20 - y)$$

$$8y^2 + 150y - 360 = 0$$

$$y = \frac{-150 \pm \sqrt{150^2 + 4 \times 8 \times 360}}{2 \times 8} = 2.16 \text{ in}$$

$$\therefore kd = 2.16 + 4 = 6.16 \text{ in.}$$

$$I = \frac{30 \times 6.16^3}{3} - \frac{14 \times 2.16^3}{3} + 30 \times (20 - 2.16)^2 = 11840 \text{ in}^4$$

$$M = \frac{\sigma_{all} \times I}{c}$$

$$M_c = \frac{0.6 \times 11840}{6.16} = 1153.2 \text{ in-K}$$

$$M_s = \frac{\frac{18}{12} \times 11840}{(20 - 6.16)} = 996 \text{ in-K}$$

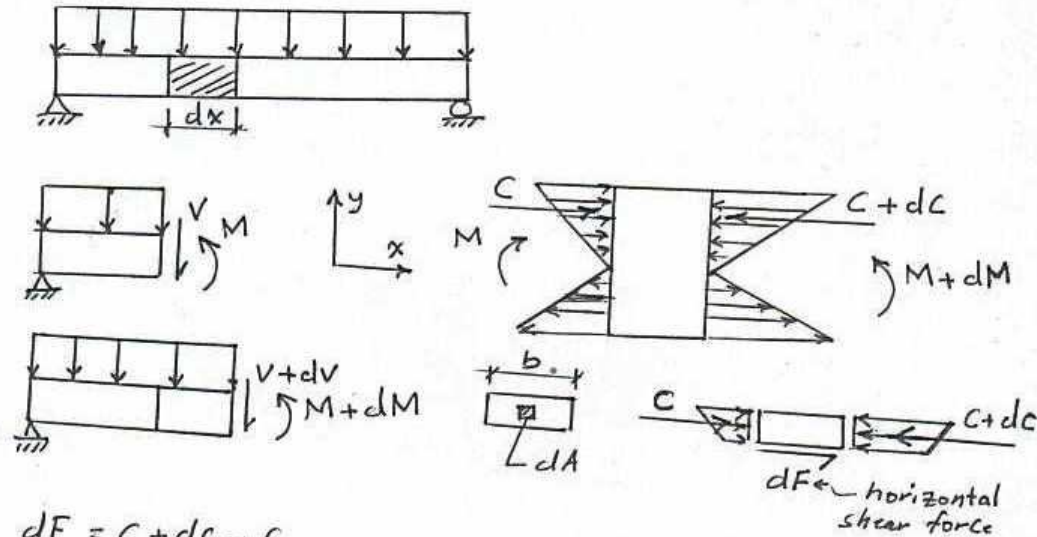
$$\therefore M_{allowable} = 996 \text{ in-K}$$

The section is under reinforced section



Chapter Seven Shearing Stresses in Beams

Horizontal Shearing Stress Formula



$$dF = C + dc - C$$

$$dF = dc = \int_A d\sigma dA$$

$$\text{but } d\sigma = d\left(\frac{M \cdot y}{I}\right)$$

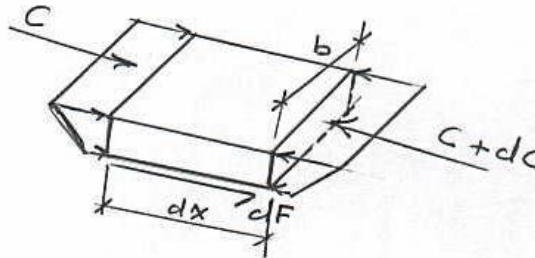
$$d\sigma = \frac{dM \cdot y}{I}$$

$$dF = \int_A \frac{dM \cdot y}{I} dA$$

$$dF = \frac{dM}{I} \int_A y dA$$

$\int_A y dA$ = the sum of moments of the area about the neutral axis = Q

$$dF = \frac{dM Q}{I}, \quad dF: \text{Horizontal shearing force}$$



$$\tau_h = \frac{dF}{b dy}$$

$$\tau_h = \tau_{yx} = \frac{dMQ}{I b dx}$$

$$\text{But } \frac{dM}{dx} = V$$

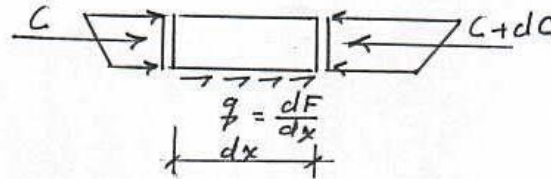
$$\therefore \tau_{yx} = \frac{VQ}{I b}$$

q = shear Flow

$$q = \frac{dF}{dx}$$

$$q = \frac{dMQ}{I dx}$$

$$q = \frac{VQ}{I}$$

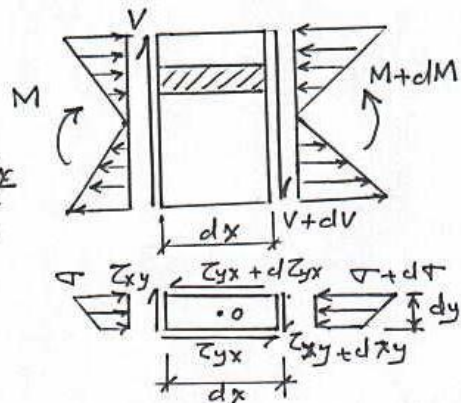


Relation Between Horizontal and Vertical Shear stresses

$$\begin{aligned} \sum M_o &= 0 \rightarrow \\ (\tau_{yx} * b * dx) * dx + (d\tau_{yx} * b * dx) * \frac{dx}{2} &= (\tau_{xy} * b * dx) * dx + (d\tau_{xy} * b * dx) * \frac{dx}{2} \\ \therefore (\tau_{yx} * b * dx) * dx &= (\tau_{xy} * b * dx) * dx \\ \therefore \tau_{yx} &= \tau_{xy} \end{aligned}$$

high order term

high order term

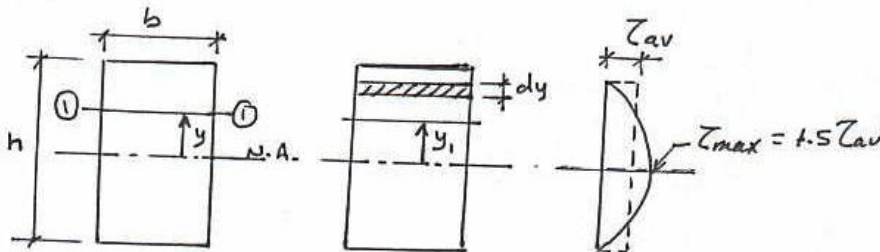




Application to Monolithic Section

Example 1:

Derive an expression for the shearing stress distribution in a beam of solid rectangular section transmitting a vertical shear force V .



$$\begin{aligned}\tau_{xy} = \tau_{yx} &= \frac{V \cdot Q}{I b} \\ &= \frac{V \cdot \int_{y_1}^{h/2} (b dy) \cdot y}{I b} \\ &= \frac{V \cdot b}{I \cdot b} \left[\frac{y^2}{2} \right]_{y_1}^{h/2}\end{aligned}$$

$$\therefore \tau_{xy} = \tau_{yx} = \frac{V}{2I} \left[\left(\frac{h}{2} \right)^2 - y_1^2 \right] \quad \text{parabolic distribution}$$

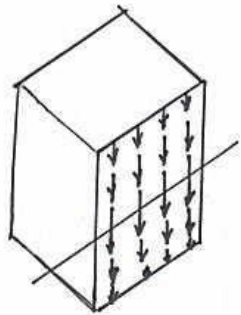
τ_{max} occur at $y_1 = 0$ (i.e.) at N.A.

$$\boxed{\tau_{max} = \frac{V h^2}{8I}}$$

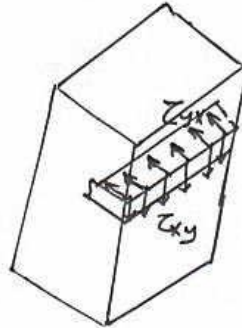
τ_{min} occur at top & bottom faces.

$$\tau_{min} = \frac{V}{2I} \left[\left(\frac{h}{2} \right)^2 - \left(\frac{h}{2} \right)^2 \right] = 0$$

$$\begin{aligned}\tau_{max} &= \frac{V h^2}{8 \cdot \frac{h^3 b}{12}} = \frac{3}{2} \cdot \left(\frac{V}{h \cdot b} \right) \quad \text{average shear stress} \\ &= \frac{3}{2} \tau_{av}.\end{aligned}$$

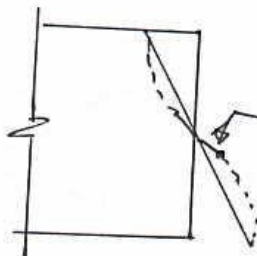
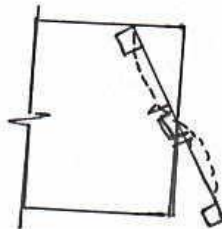
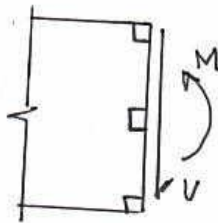


Distribution of
shearing stress



Horizontal & Vertical
shearing stress

Shear Deformation



shear deformation

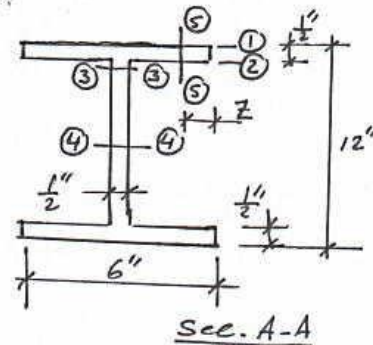
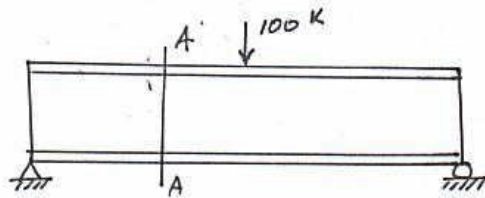
It is clear that the error increase
in equation $\tau = \frac{M \cdot y}{I}$ which is
derived based on assumption:

(plane section before bending remain
plane after bending), when the
ratio of beam depth to its length
is increased i.e. when the beam
become deeper.



Example 2:

For the beam shown in the Fig. below, determine the shear flow & shear stress at the levels indicated.

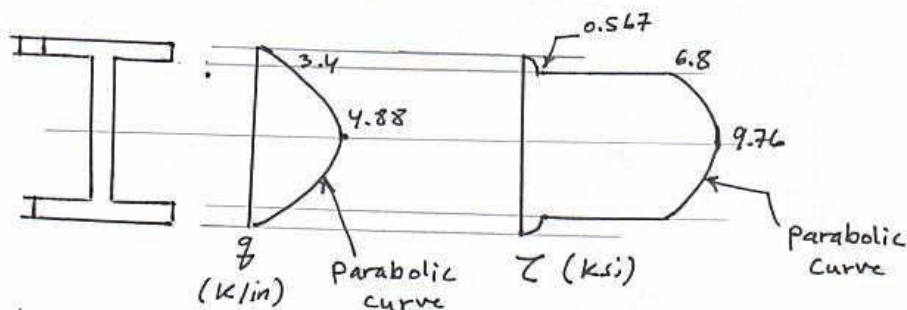


Solution:

$$\tau = \frac{VQ}{Ib}, \quad q = \frac{VQ}{I}$$

$$V = 50 \text{ k}, \quad I = \frac{12^3 \times 6}{12} - \frac{5.5^3 \times 1}{12} = 254 \text{ in}^4$$

level	Q (in ³)	q (k/in)	τ (k/in ²)
1-1	0	0	0
2-2	$3 \times 5.75 = 17.25$	3.4	$3.4/6 = 0.567$
3-3	17.25	3.4	$3.4/0.5 = 6.8$
4-4	$= 17.25 + 5.5 \times \frac{1}{2} \times \frac{5.5}{2} = 24.8$	$\frac{50 \times 24.8}{254} = 4.88$	$\frac{4.88}{0.5} = 9.76$



Notes:

- 1- The lower part of curves was drawn by symmetry
- 2- From practical point of view, since the shear capacity of flanges is very small it is neglected and the shear stress is calculated based on web area only



$$\tau_{max} = \frac{V}{A_{web}}$$

$$= \frac{50}{11 \times \frac{1}{2}} = 9.09 \text{ ksi} < 9.76 \text{ ksi}$$

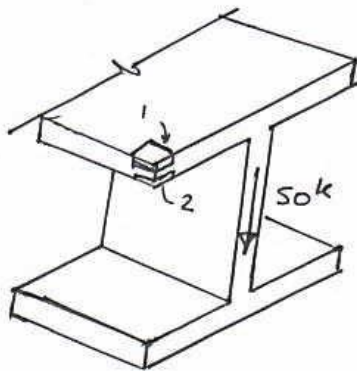
$$\text{error} = \frac{9.09 - 9.76}{9.76} = 6.8\%$$

Hence, the shearing stress cannot be calculated from:

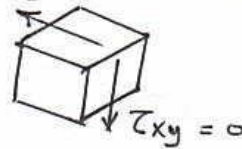
$$V/A_{total}$$

3- Limitation & contradiction:

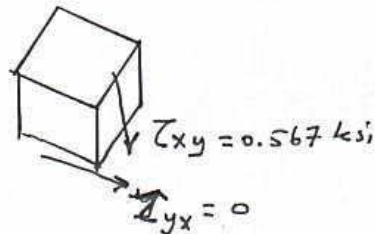
All assumptions related with bending formula $\sigma = \frac{My}{I}$ should be satisfied here.



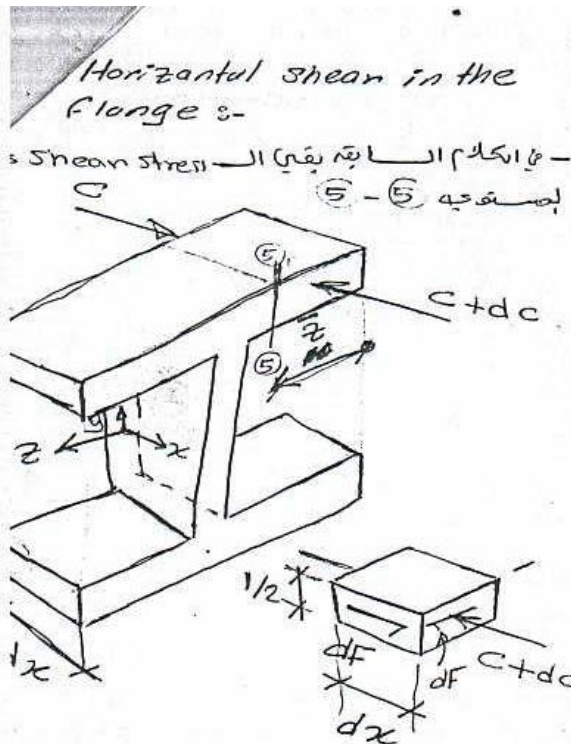
$\tau_{yx} = 0$ from B.C. (top surface)



$\therefore \tau_{xy} = \tau_{yx} \therefore \text{o.k.}$



$\tau_{yx} \neq \tau_{xy} \therefore \text{not o.k.}$



$$\frac{F}{x} = q = \frac{V \times Q}{I}$$

$$F = 50 \times \left[\frac{1}{2} \times \bar{x} \times 5.75 \right]$$

$$254$$

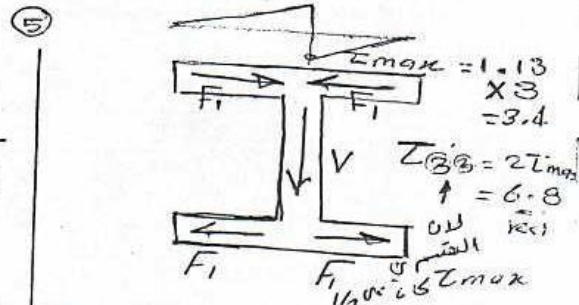
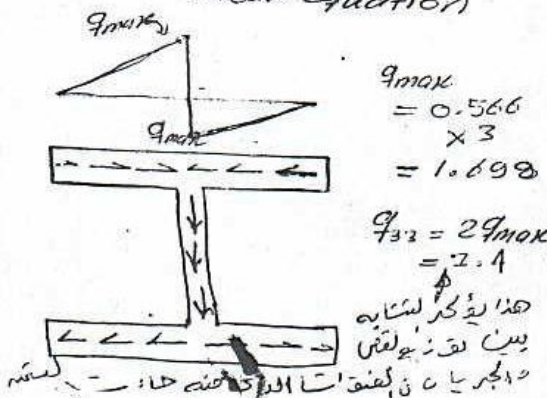
$$q = 0.566 \bar{x} \leftarrow \text{linear equation}$$

$$\tau = \frac{q}{1/2} = \frac{dF}{1/2 dx}$$

$$\tau \bar{x} = \tau x_2$$

$$= 1.13 \bar{x}$$

linear equation



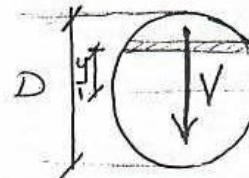
$$F_1 = \frac{Z_{max}}{2} \times (6 \times 1/2)$$

average Stress = $\frac{3.4}{2} \times 6 \times 1/2 = 5.1 \text{ Kips}$

Flow direction -
التي هي في اتجاه الـ Flow direction
الـ Shear Stress في الـ Web
الـ Flow direction في الـ Web

Prob. 7-13

EX3 Find the max. Shearing Stress.



$$\tau = \frac{V \times Q}{I \times b} \leftarrow \text{Vary through the section.}$$

constants through the section

$$\tau = \frac{V}{I \times b} \int_0^{D/2} dA \times y$$

$$dA = 2x \times dy \quad x = \sqrt{D^2/4 - y^2}$$

$$b = \frac{2}{\sqrt{D^2/4 - y^2}}$$

$$\tau = \frac{V}{I \times \sqrt{D^2/4 - y^2}} \left[\int_0^{D/2} \frac{2x}{\sqrt{D^2/4 - y^2}} dy \right]$$

$$\tau = \frac{V}{I \times \sqrt{D^2/4 - y^2}} \times \left[\frac{2}{3} x^2 \left(\frac{D^2}{4} - y^2 \right)^{3/2} \right]$$



$$\tau = \frac{V}{3I} \sqrt{D^2/4 - y_1^2} \times (D^2/4 - y_1^2)^{3/2}$$

$$\tau = \frac{V \times (D^2/4 - y_1^2)}{3I}$$

$$\tau_{min} = 0 \text{ at } y_1 = D/4$$

$$\tau_{max} = \frac{VD^2}{12I} \text{ at } y_1 = 0$$

$$\tau_{max} = \frac{V \times D^2}{3 \times \frac{1}{2} \times \frac{\pi D^4}{64} \times 16}$$

$$= \frac{16V}{3\pi D^2}$$

$$= \frac{4V}{3 \times \frac{\pi D^2}{4}}$$

$$= \frac{4}{3} \frac{V}{A}$$

$$= \frac{4}{3} V_{avg}$$

Prob. 7-13

EX4 Find τ at N.A.

$$\tau = \frac{V \times Q}{I \times b}$$

$$Q = \frac{D^2 \cdot \pi}{8} \times \frac{4R}{3\pi}$$

$$= \frac{D^2 \cdot \pi}{8} \times \frac{2D}{3\pi}$$

$$= \frac{D^3}{12}$$

$$\tau = \frac{V \times D^3/12}{\frac{D^4 \cdot \pi}{64} \times \frac{D}{4}} = \frac{4}{3} \frac{V}{\frac{\pi D^2}{4}}$$

$$= \frac{4}{3} \frac{V}{A}$$

$$= \frac{4}{3} \times V_{avg}$$

Prob. 7-14

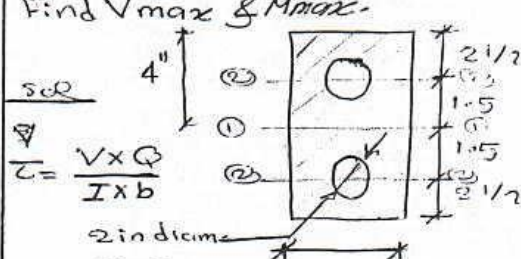
x5

Pure shear

$$\tau_{max} = 2 \frac{V}{A}$$

say N.A. of joint τ_{max} is at N.A. of joint

SEE EX 6
Gall/ten = 7ksi
Gall/comp = 8ksi
Zall = 8ksi
Find V_{max} & M_{max} .



sec ①-①

$$Q = 4 \times 3 \times 2 - \frac{2^2 \cdot \pi}{4} \times 1.5$$

$$= 19.3 \text{ in}^3$$

$$b = 3$$

$$\frac{Q}{b} = \frac{19.3}{3}$$

$$\frac{Q}{b} = 6.43$$

sec ②-②

$$Q = 2.5 \times 3 \times (1.25 + 1.5)$$

$$- \frac{2^2 \cdot \pi}{2 \times 4} \times \left[\frac{4 \times \frac{2}{2}}{3\pi} + 1.5 \right]$$

$$= 17.6 \text{ in}^3$$

$$b = 3 - 2 = 1$$

$$\frac{Q}{b} = 17.6$$

$$I = \frac{8^3 \times 3}{12}$$

$$- 2 \left[\frac{2^2 \cdot \pi}{4} \times 1.5^2 + \frac{2^4 \cdot \pi}{64} \right]$$

$$= 112 \text{ in}^4$$

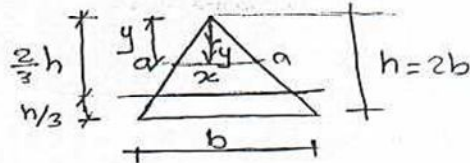
$$\tau_{all} = 8 = \frac{V \times 17.6}{112}$$

$$V \neq 15 \text{ K}$$

$$V = 51.04 \text{ K}$$



Ex 6 Prop. 7-16 Find $\bar{Z}_{max}$



$$\bar{Z}_{a-a} = \frac{V \times Q}{I \times b}$$

$$Q = \frac{yx}{2} \times \left[\frac{y}{3} + \left(\frac{2}{3}h - y \right) \right]$$

$$b = x$$

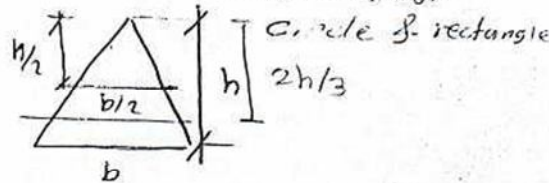
$$\bar{Z}_{a-a} = \frac{V}{I} \times \left[\frac{y}{2} \left(\frac{y}{3} + \left(\frac{2}{3}h - y \right) \right) \right]$$

$$\bar{Z}_{a-a} = \frac{V}{I} \times \left[\frac{hy}{3} - \frac{1}{3}y^2 \right]$$

$$\frac{\partial \bar{Z}}{\partial y} = \frac{V}{I} \left[\frac{h}{3} - \frac{2y}{3} \right] = 0$$

$$\boxed{y = \frac{h}{2}}_{max}$$

← هذا هو
القيمة لـ $\bar{Z}_{max}$ في
هذا الحالة
بما اننا وجدنا
الحد الأقصى



Q 4

$$\bar{Z}_{max} = \frac{V}{I} \left[\frac{h}{3} \times \frac{h}{2} - \frac{1}{3} \times \frac{h^2}{4} \right]$$

$$= \frac{V}{I} \left[\frac{h^2}{12} - \frac{h^2}{12} \right]$$

$$= \frac{Vh^2}{12I} = \frac{Vh^2}{12 \times \frac{h^3 \times b}{36}}$$

$$= \frac{3V}{h \times b}$$

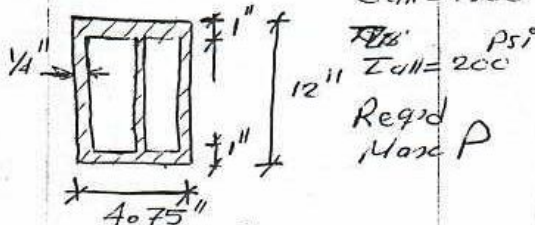
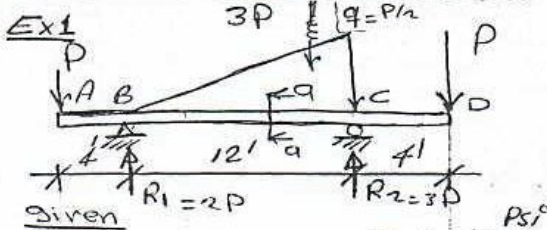
$$\frac{\sigma_{I_2}}{y} = \frac{3}{2} \frac{V}{\frac{h \times b}{2}}$$

$$\boxed{\bar{Z}_{max} = \frac{3}{2} \frac{V}{A}}$$

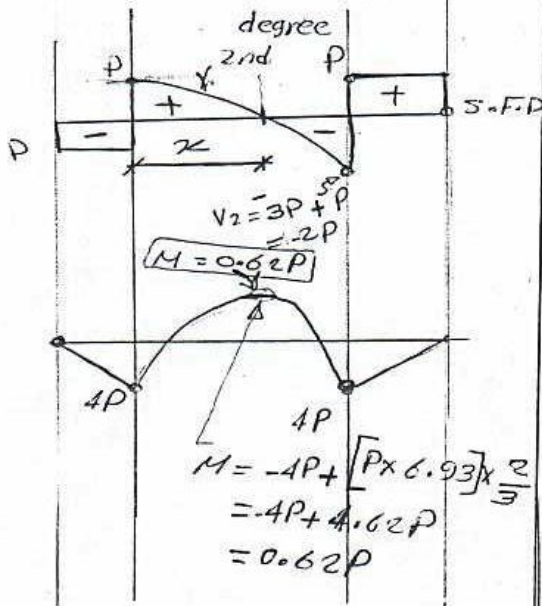


Chapter 7 [C]

Design for flexure and Shear 8-



$$\begin{aligned} \text{Sol: } 3P &= \frac{q \times 12}{2} \\ 3P &= 7 \times 6 \\ \sum M_C &= 0 \\ R_1 &= 2P \uparrow \quad R_2 = 3P \uparrow \end{aligned}$$



Note $\frac{dV}{dx} = -q$ & $\frac{dM}{dx} = V$

$$V_2 = \int q dx + V_1 \quad M_2 = \int V dx + M_1$$

$$\begin{aligned} P &= [P \times x] \times \frac{1}{x} \\ P &= \left[\frac{P}{24} \times x \right] \times \frac{x}{2} \\ 1 &= \frac{x^2}{48} \quad x = 6.93^m \end{aligned}$$

P for bending

$$\begin{aligned} M_{max} &= \sigma_{all} \times S \\ I &= \frac{12^3 \times 4.75}{12} - \frac{4 \times 1^3}{12} \\ &= 351 \text{ in}^4 \\ M_{max} &= 1200 \times \frac{351 \text{ in}^4}{6 \text{ in}} \\ &= 70,200 \text{ lb} \cdot \text{ft} \\ &= 5,850 \text{ lb} \cdot \text{ft} \\ &= 4P \end{aligned}$$

$$P = 1462.5 \text{ lb}$$

P for Shear

$$\tau = \frac{VQ}{Ib}$$

$$\begin{aligned} \tau_{max} &\text{ occurs at the N.A.} \\ Q &= (6 \times 1/4 \times 3) + 4 \times 1 \times 5.5 \\ &= 18.75 \text{ in}^3 \\ \tau_{max} &= 200 = \frac{V \times 18.75}{351 \times (1/4 \times 3)} \\ V &= 1483 \text{ lb} = 2P \end{aligned}$$

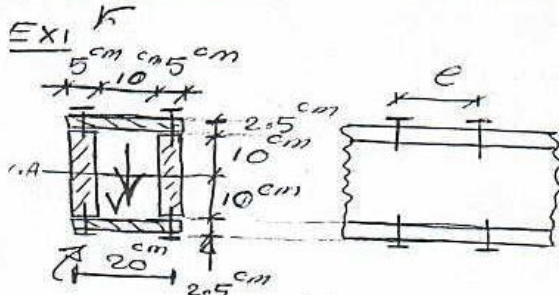
$$P = 742 \text{ lb}$$

take the min

$$\therefore P_{all} = 742 \text{ lb}$$



Application to the Compound Sections 8-

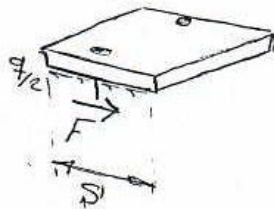


each screw
can transmit
a force $F = 2.8 \text{ kN}$

$V = 12 \text{ kN}$

Req'd Find
the spacing
between screws
"e".

Solution



$$q = \frac{V \times Q}{I}$$

$$Q = 20 \times 2.5 \times (10 + 1.25) = 562.5 \text{ cm}^3$$

$$I = \frac{25^3 \times 20}{12} - \frac{20^3 \times 10}{12} = 19,375 \text{ cm}^4$$

$$q = \frac{12 \times 562.5}{19,375} = 0.348 \frac{\text{kN}}{\text{cm}}$$

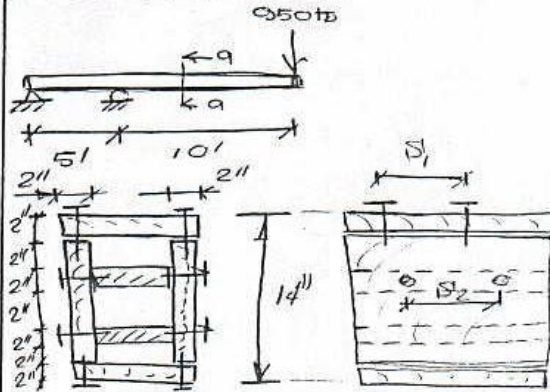
$$\frac{q}{2} = \frac{0.348}{2}$$

$$\frac{q}{2} s = F$$

$$\frac{0.348}{2} \times s = 1/2 \times 2.8$$

$$s = 16 \text{ cm}$$

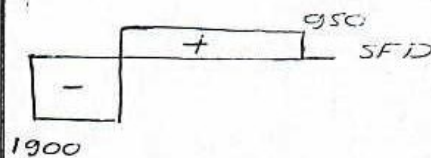
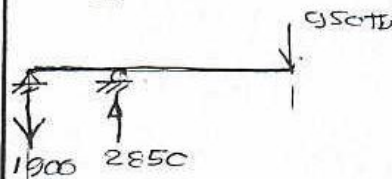
EX 2 Prob. 7-2



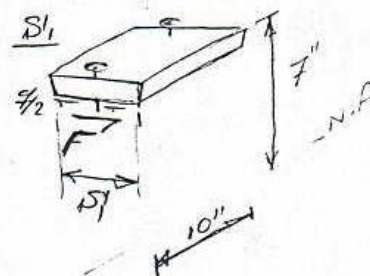
Given
- each nail has a shear resistance
of 96 lb
- $I = 1,900 \text{ in}^4$

Req'd s_1 & s_2

$$q = \frac{V \times Q}{I}$$



$$\therefore V_{max} = 1900 \text{ lb}$$





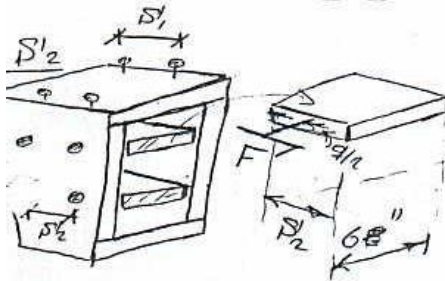
$$I = \frac{V \times Q}{I}$$

$$I = 10 \times 2 \times (7-1) = 120 \text{ in}^3$$

$$I = \frac{1900 \text{ lb} \times 120}{1,900} = 120 \frac{\text{lb}}{\text{in}}$$

$$I \times S_1 = F_{all}$$

$$10 \times S_1 = 96 \text{ lb} \quad (S_1 = 1.6 \text{ in})$$



$$I = \frac{V \times Q}{I}$$

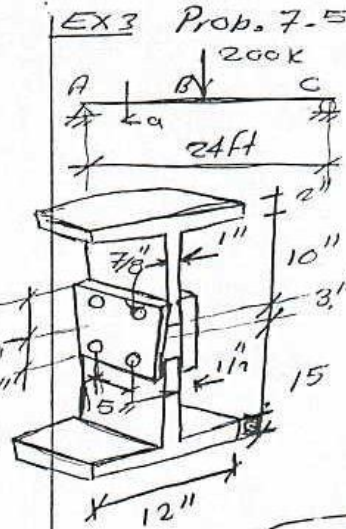
$$Q = 6 \times 2 \times 10 = 120 \text{ in}^2$$

$$I_2 = \frac{1900 \times 120}{1900} = 120 \frac{\text{lb}}{\text{in}}$$

$$I_2 \times S_2 = 96 \text{ lb}$$

$$I_2 \times S_2 = 96 \quad (S_2 = 8 \text{ in})$$

Home work of Part C
Probs 7.1, 7.3, 7.4, 7.8,



Analysis on
checked Prob.

$I = 11,000 \text{ in}^4$
Rec'd
shear stress
in the bolt

$$q = \frac{V \times Q}{I}$$

$$Q = 2 \times 12 \times (1+10+3) + 1 \times 10 \times (5+3) = 416 \text{ in}^3$$

$$q = \frac{K \times 100 \times 416}{11,000 \text{ in}^4} = 3.78 \frac{\text{K}}{\text{in}}$$

$$F = \frac{q}{2} \times 85$$

$$= \frac{3.78}{2} \times 85 = 9.45 \text{ K}$$

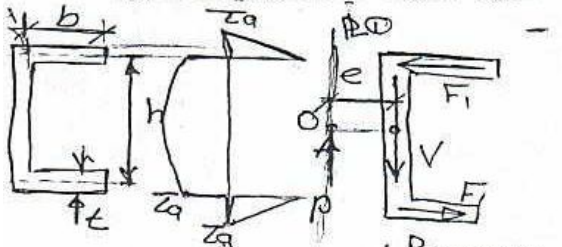
$$\tau_{bolt} = \frac{9.45}{(\frac{7}{8})^2 \cdot \frac{\pi}{4}} = 15.7 \text{ ksi}$$



Chapter 7 D

The Shear Center

كل الكتل التي لها شكل غير متماثل في المقاطع العرضية لها مركز قص. هذا المركز هو النقطة في المقاطع العرضية التي يمر بها محاور القص الناتجة عن القوى القصية المتعامدة. هذا المركز هو النقطة التي يمر بها محاور القص الناتجة عن القوى القصية المتعامدة.



$$P \cdot e = F_1 h$$

$$F_1 = \frac{Z_a}{2} \times b \times t$$

$$Z_a = \frac{V \times Q}{I \times t}$$

$$= \frac{V \times b \times h \times h/2}{I \times t}$$

$$Z_a = \frac{V b h^2}{2I}$$

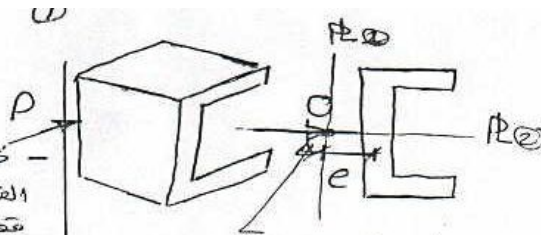
note that $V = P$

$$P \cdot e = \frac{P b h^2}{4I} \times b t$$

$$P e = \left(\frac{P b h^2}{4I} \times b t \right) h$$

$$e = \frac{b^2 h^2}{4I}$$

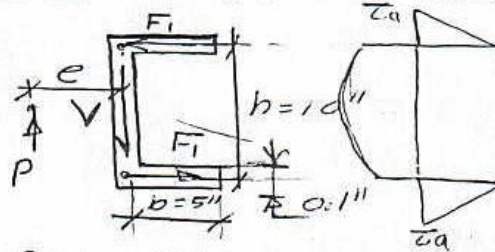
← Note that:
① e is independent of magnitude of applied force as well as of its location along the beam.
i.e., e is a property of a section



Def: Shear Center

The Shear Center for any cross section is a point define as the point in the Plane of cross section through which the resultant of transverse shearing stresses due to flexure (no torsion) will pass for any transverse loads.

Ex1 Find the shear center.



$$P \cdot e = F_1 h$$

$$F_1 = \frac{Z_a}{2} \times (5 \times 0.1)$$

$$Z_a = \frac{V \times (5 \times 0.1 \times 0.1 \times 5)}{I \times 0.1}$$

$$I = \frac{10^3 \times 5 \times 0.1^3}{12} + 2 \left[\frac{0^3 + 5 \times 0.1 \times 5^3}{12} \right]$$

$$I = \frac{10^3 \times 0.1}{12} + 2 \left[0 + 5 \times 0.1 \times 5^3 \right]$$

$$= 33.3 \text{ in}^4$$

$$Z_a = 0.75 V$$

$$F_1 = \frac{0.75 V}{2} \times 5 \times 0.1 = 0.187 V$$

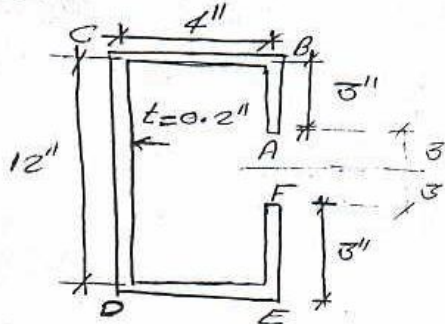
$$P = V$$

$$V \cdot e = 0.187 V \times 10$$

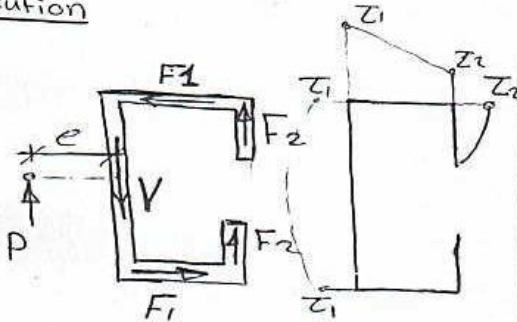
$$e = 1.87 \text{ in}$$



Ex 2 Find the shear center



solution



$$P \times e = F_1 \times 12 + 2F_2 \times 4$$

$$= \frac{12^3 \times 0.2}{12} + 2 \left[0 + 0.2 \times 4 \times 6^2 \right] + 2 \left[\frac{0.2 \times 3^3}{12} + 0.2 \times 3 \times 4.5^2 \right]$$

$$I = 111.6 \text{ in}^4$$

$$Z_2 = \frac{V \times (4.5 \times 3 \times 0.2)}{111.6 \times 0.2} = 121V$$

$$Z_2 = 0.121V$$

$$Z_1 = \frac{V (6 \times 4 \times 0.2 + 4.5 \times 3 \times 0.2)}{111.6 \times 0.2}$$

$$Z_1 = 0.336V$$

$$Z_2 = \frac{2}{3} Z_2 \times 3 \times 0.2$$

$$= \left(\frac{2}{3} \times 0.121 \times 3 \times 0.2 \right) V$$

$$F_2 = 0.0484V$$

$$0.0484V$$

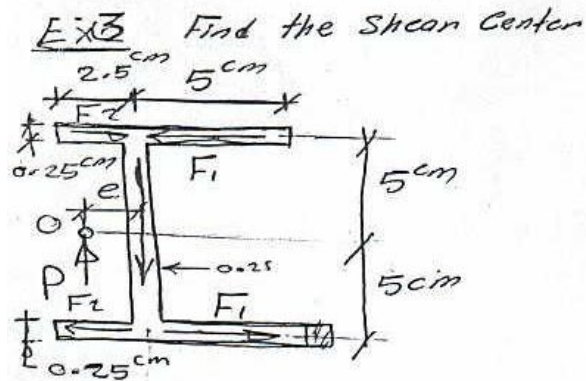
$$F_1 = \frac{0.336V + 0.121V}{2} \times 4 \times 0.2$$

$$F_1 = 0.183V$$

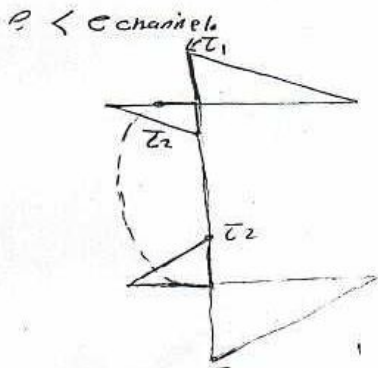
$$P = V$$

$$V \times e = 0.183V \times 12 + 2 \times 0.0484V \times 4$$

$$e = 2.58 \text{ in}$$



$$Pxe = (F_1 - F_2)h$$



$$F_1 = \frac{V (5 \times 0.25)}{I \times 0.25}$$

$$\tau = \frac{25V}{I} \quad I = \frac{10^3 \times 0.25}{12} + 2 \left[0 + 7.5 \times 0.25 \times 25 \right] = 114.5 \text{ in}^3$$

$$F_1 = 0.218 V$$

$$F_1 = \frac{F_1}{2} \times 5 \times 0.25$$

$$F_1 = 0.136 V$$

$$F_2 = \frac{V (2.5 \times 0.25)}{114.5 \times 0.25}$$

$$F_2 = 0.109 V$$

$$F_2 = \frac{F_2}{2} \times 2.5 \times 0.25$$

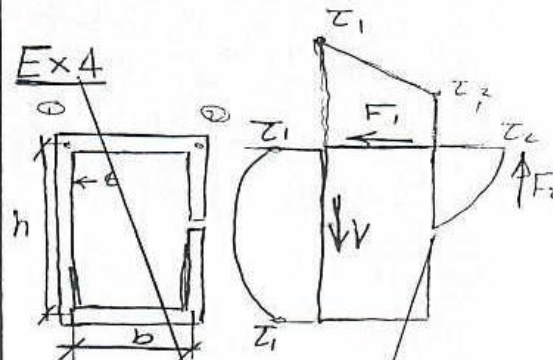
$$F_2 = \left(\frac{0.109}{2} \times 2.5 \times 0.25 \right) V$$

$$F_2 = 0.0341 V$$

$$P = V$$

$$Pxe = (0.136 V - 0.0341 V) \times 10$$

$$e = 1.002$$



$$Pxe = F_1 h + 2 F_2 \times b$$

$$F_2 = \frac{V \times (h/4 \times h/2 \times t)}{I \times t}$$

$$F_2 = \frac{V h^2}{8 I}$$

$$F_1 = V \left[\frac{h/4 \times h/2 \times t}{I \times t} + \frac{h/2 \times b \times t}{I \times t} \right]$$

$$F_1 = V \left[\frac{h^2}{8} + \frac{bh}{2} \right] / I$$

$$F_2 = \frac{2}{3} F_1 \times \frac{h}{2} \times t$$

$$= \frac{2}{3} \times \frac{V h^2}{8 I} \times \frac{h}{2} \times t$$

$$F_2 = \frac{V h^3 t}{24 I}$$

$$F_1 = \frac{F_1 + F_2}{2} \times b \times t$$

$$= \frac{V \left[\frac{h^2}{8} + \frac{bh}{2} + \frac{h^2}{8} \right]}{2 I} \times b \times t$$

$$F_1 = V \left[\frac{h^2}{8} + \frac{bh}{4} \right] \frac{b \times t}{I}$$



Problem 7.19

- Computing the internal forces in the Specified Section.

- compute the reaction
- compute the internal forces.

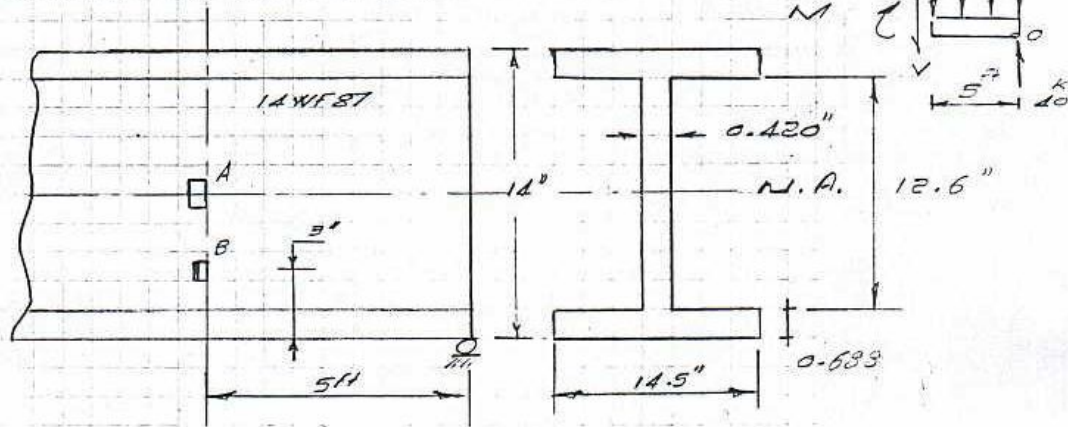
$$\sum F_y = 0 \downarrow \Rightarrow V = 20 \text{ KIPS} \quad R_x = 0$$

$$\sum M_A = 0$$

$$M_A - 20 \times 5 = 4 \times 5 \times 2.5 = 0$$

$$\therefore M_A = 150 \text{ K.ft} = 1800 \text{ K.in.}$$

- Compute the stresses.



From table A $I = 966.9 \text{ in}^4$

Element A

$$\sigma_A = \frac{M \cdot y_A}{I} = 0$$

$$\tau = 3.68 \text{ KSI}$$

As the element A is on the N.A. of beam, $y = 0$.

$$\tau = \frac{V \cdot Q}{I \cdot t}, \quad Q = 0.688 \times 14.5 \times (7 - 0.688/2) + 6.3 \times 0.420 \times 6.3/2 = 74.7 \text{ in}^3$$

$$\therefore \tau = \frac{20 \times 74.7}{966.9 \times 0.420} = 3.68 \text{ KSI}$$

Element B

$$\sigma_B = \frac{M \cdot y_B}{I} = 1800 \times 4 / 966.9 = 7.45 \text{ KSI}$$

$$Q = 0.688 \times (14.5 - 0.420) \times (7 - 0.688/2) + 3 \times 0.420 \times (4 + 1.5) = 64.5 + 6.93 = 71.4 \text{ in}^3$$

$$\therefore \tau = \frac{20 \times 71.4}{966.9 \times 0.420} = 3.52 \text{ KSI}$$

$$\sigma = 7.45 \text{ KSI}$$

$$\tau = 3.52 \text{ KSI}$$



Application of Shear Formula for the Analysis and Design of Monolithic Sections.

Problem 7.10.

- Analysis for Flexure

$$\sigma = \frac{M \cdot c}{I} \Rightarrow \sigma_{all} = \sigma_{all} \times \frac{I}{c} \quad \therefore \sigma_{all} \text{ Tension} < \sigma_{all} \text{ (Cast Iron) Comp.}$$

$\therefore \sigma_{all} \text{ Tension} < \sigma_{all} \text{ Comp.} \quad \therefore \sigma_{all} \text{ will be computed based on the tension stress.}$

$$I = \frac{8^3 \times 3}{12} - 2 \left[\frac{\pi \times 2^4}{64} + \frac{\pi \times 2^2}{4} \times 1.5^2 \right] = 112 \text{ in}^4$$

$$\therefore \sigma_{all} = 7 \text{ ksi} \times \frac{112 \text{ in}^4}{4} = 196 \text{ K.in Ans.}$$

- Analysis for Shear.

$$\tau = \frac{VQ}{It} \Rightarrow \frac{V}{\sigma_{all}} = \frac{I}{t} \cdot \frac{t}{Q}$$

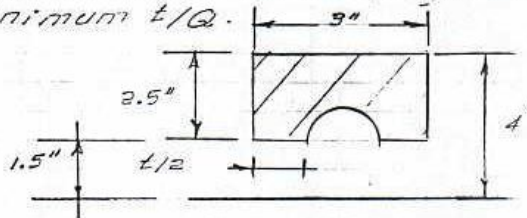
As the τ_{all} and I are constants for whole cross section, then the V_{all} will be computed based on the longitudinal section that has the minimum t/Q .

t/Q @ Sec. a-a.

$$Q = 3 \times 2.5 \times (1.25 + 1.5) - \frac{\pi \times 2^2}{2 \times 4} \times \left(\frac{3 \times 1}{4\pi} + 1.5 \right)$$

$$Q = 20.6 - 2.73 = 17.9 \text{ in}^3$$

$$t = 3 - 2 = 1" \Rightarrow t/Q = 0.0559 \text{ in}^{-2} \text{ @ Sec. a-a.}$$



t/Q @ N.A.

$$Q = (3 \times 4) \times 2 - (\pi \times 2^2/4) \times 1.5 = 19.3 \text{ in}^3$$

$$t = 3" \Rightarrow t/Q = 0.156 \text{ in}^{-2} \text{ N.A.}$$

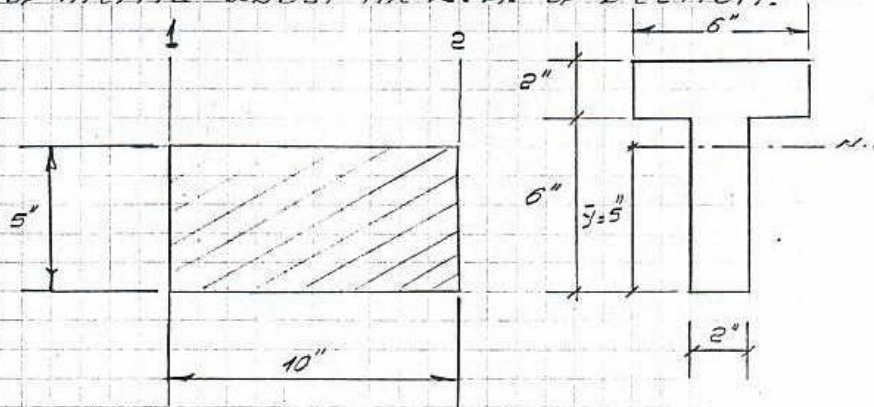
$\therefore$ the critical longitudinal section is @ a-a.

$$\therefore V_{all} = 8 \times 112 \times 0.0559 = 50.1 \text{ Kips Ans.}$$



7.20

To use the flexure and Shears formulas, we must compute the moment of inertia about the N.A. of section.



Computing In.A.

$$\bar{y} \times A = \sum A_i y_i \Rightarrow \bar{y} = \frac{(2 \times 6) \times 7 + 6 \times 2 \times 3}{2 \times 6 + 6 \times 2} = 5 \text{ in}$$

$$I_{N.A.} = \frac{5^3 \times 2}{3} + \frac{1^3 \times 2}{3} + \left[\frac{2^3 \times 6}{12} + 2 \times 6 \times 2^2 \right] = 136 \text{ in}^4$$

For Section 1-1

$$M = -1360 \times 50 = 68000 \text{ lb.in}$$

$$V = +1360 \text{ lb}$$

$$\sigma_1 = \frac{M \cdot y}{I} = 2500 \text{ psi}$$

Bottom I

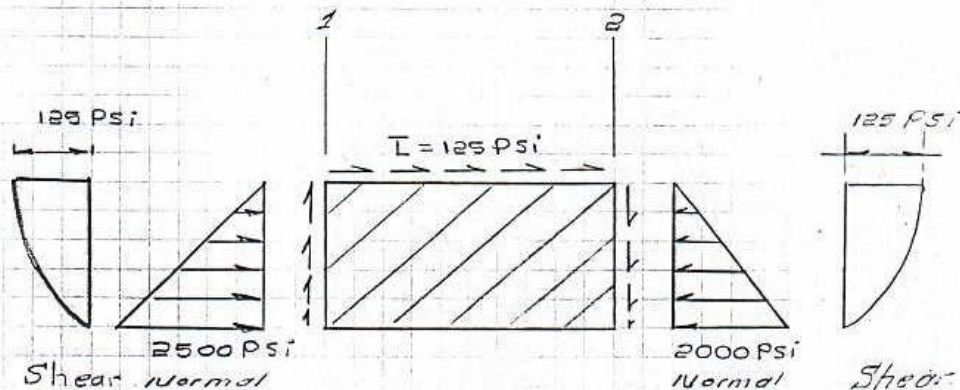
$$Q_1 = 5 \times 2 \times 2.5 = 25 \text{ in}^3 \Rightarrow \tau_1 = \frac{VQ}{I \cdot t} = \frac{1360 \times 25}{136 \times 2} = 125 \text{ psi}$$

For Section 2-2

$$M = -1360 \times 40 = 54400 \text{ lb.in} \Rightarrow \sigma_2 = 2000 \text{ psi}$$

$$\tau = 125 \text{ psi, As the shear is constant through the beam length.}$$

N.A.





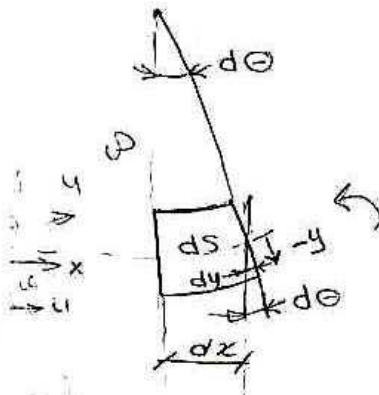
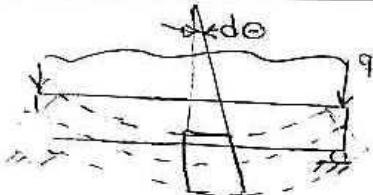
Chapter Eleven Deflection of Beams

Deflection of beam

بما أن كل حسابات الانحرافات في الـ
beams تكون في 2D فنعلم بأن الانحراف
الانحراف في تلك الانحرافات لا يتغير
أما في 3D فيكون الانحراف في 3D

Part A

The governing differential equation
for the deflection of elastic beams is:-



$$ds = ds \cos \theta \rightarrow ds \cos \theta = \frac{ds}{\cos \theta}$$

$$\text{but } ds \cos \theta = \frac{du}{dx} \quad \text{or } du = -y d\theta$$

$$\therefore \frac{du}{y} = - \frac{d\theta}{\cos \theta} \quad \text{or } d\theta$$

$$\frac{du}{ds} = - \frac{y}{\cos \theta} \quad \text{or}$$

$$\text{but } \frac{du}{ds} = E \quad \text{or}$$

$$E = -y/\cos \theta$$

$$E = \frac{d\theta}{ds}$$

$$\frac{d\theta}{ds} = \frac{1}{\cos \theta}$$

$$\text{but } \sigma = \frac{M \cdot y}{I}$$

$$\therefore \frac{-M \cdot y}{IE} = \frac{-y}{\cos \theta}$$

$$\therefore \frac{1}{\cos \theta} = k = \frac{M}{EI}$$

curvature

but from calculus we know
that $\frac{d^2u}{dx^2} = k$

$$k = \frac{1}{\left[1 + \left(\frac{du}{dx}\right)^2\right]^{3/2}}$$

where

x = locate the point on the elastic curve.

u = gives the def.

for small deflection
 $\frac{du}{dx}$ small

$$\left(\frac{du}{dx}\right)^2 \approx 0 \quad \text{or } ds \approx dx$$

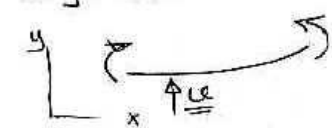
$$\therefore k \approx \frac{d^2u}{dx^2} = \theta''$$

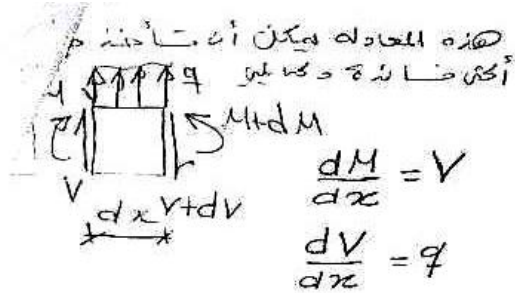
$$\therefore \frac{d^2u}{dx^2} = \frac{M}{EI}$$

← differential equation for the deflection of an elastic beam, was

formulated by James Bernoulli, 1694.

Sign convention.





$$\frac{d^3 \varphi}{dx^3} = \frac{V}{EI}$$

$$\frac{d^4 u}{dx^4} = \frac{q}{EI}$$

لذلك المحادثة المستقيمة أخلقة كسائر أخلاقه
فلهذا الحكيم هو في الدنيا -

1] Direct integration method
or Double integration
method

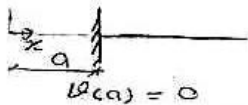
[2] Singularity function method

③ The moment area method.

Boundary conditions

كل حائل حسابي الى deflection
لحاجم لعماد التقاطع بحسب لدرجته الى
و كذا :-

7] Fixed or clamped support



$$W(a) = 0$$

$$\psi'(a) = \Theta(a) = 0$$

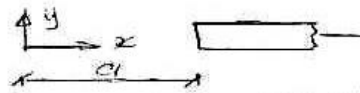


Simply supports

$$\psi(a) = 0$$

$$M(x) = EI \psi''(x) = 0$$

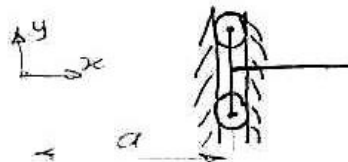
☒ Free support



$$M(a) = EI u''(a) = 0$$

$$V(a) = EI \psi'''(a) = 0$$

d) Guided support



$$U(a) = \Theta(a) = C$$

$$V = EI \psi'''(a) = 0$$

basically two different type of boundary condition.

static boundary condition

Kinematic boundary conditions.



The direct integration method

$$EI v^{(4)} = q(x)$$

$$E[I(x)] = \int_0^{25} f(x) dx + C_1$$

$$w'' = \int_0^x \int_0^x f(x) dx + C_1 x + C_2$$

$$= I(\omega') = \int_{-1}^x dx \int_0^x dx \int_0^x f(x) dx$$

$$T.E = \int_0^x dx \int_0^x dx \int_0^x dx \int_0^x 4 \cos x dx$$

$$+ \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

General Solution

- Homogeneous solution

$$\frac{C_1 x^3}{6} + \frac{C_2 x^2}{4} + C_3 x + C_4$$

 C_1, C_2, C_3, C_4

initial specimen	initial moment	initial rotation	initial deflection
---------------------	-------------------	---------------------	-----------------------

- Particular Solution.)

$$\int_0^x dx \int_0^x dx \int_0^x dx \int_0^x 7x dx$$

- depends only on the loading condition of beam

• تكون طريقة direct integration

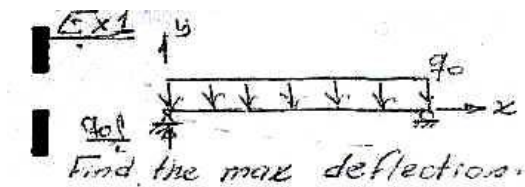
خوفیه : حسه کون ال - head

EI $\oint$ moment $\rightarrow$ Shear

دوال حضرت و جلالہ والہ

سنگینگی: محتاج لفظی ال

- Functions



1st method

$$EI \theta'' = M$$

$$M = \frac{q_0 l}{2} x - \frac{q_0 x^2}{2}$$

$$EI \theta'' = \frac{q_0 l}{2} x - \frac{q_0 x^2}{2} \quad \text{--- (1)}$$

$$EI \theta' = \frac{q_0 l}{4} x^2 - \frac{q_0 x^3}{6} + C_1$$

$$EI \theta = \frac{q_0 l}{12} x^3 - \frac{q_0 x^4}{24} + C_1 x + C_2$$

boundary condition

at $x = 0$ $\theta = 0$
 $M = 0$

$$EI \theta(0) = C_2 = 0$$

at $x = l$ $\theta = 0$

$$EI \theta(l) = \frac{q_0 l^4}{12} - \frac{q_0 l^4}{24} + C_1 l = 0$$

$$C_1 = \frac{-q_0 l^3}{24}$$

$$EI \theta = \frac{q_0 l}{12} x^3 - \frac{q_0 x^4}{24} - \frac{q_0 l^3}{24} x$$

$$EI \theta' = 0 \leftarrow \text{For max deflection}$$

$$EI \theta = 0$$

but from symmetry

$$EI \theta = 0 \text{ at } l/2$$

Max. def. occurs at $l/2$

2nd

$$EI \theta = \frac{q_0 l^4}{8 \times 12} - \frac{q_0 l^4}{16 \times 24} - \frac{q_0 l^4}{48}$$

$$EI \theta_{\text{max}} = \frac{(4 - 1 - 2) q_0 l^4}{384}$$

$$\Delta_{\text{max}} = \frac{-5 q_0 l^4}{384 EI}$$

بما أن الحمل هو ثابت في كل نقطة من طول العنصر Δx في العنصر Δx $\therefore EI$

or

2nd method

$$EI \theta'''' = -q_0$$

$$EI \theta''' = -q_0 x + C_1$$

$$EI \theta'' = -\frac{q_0 x^2}{2} + C_1 x + C_2$$

at $x = 0$ $M = EI \theta'' = 0$

$$C_2 = 0$$

at $x = l$

$$M = EI \theta'' = 0$$

$$0 = -\frac{q_0 l^2}{2} + C_1 l$$

$$C_1 = \frac{q_0 l}{2}$$

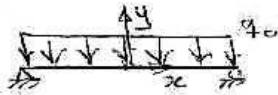
$$EI \theta'' = -\frac{q_0}{2} x^2 + \frac{q_0 l}{2} x$$

بما أن الحمل هو ثابت في كل نقطة من طول العنصر Δx في العنصر Δx $\therefore EI$

بما أن الحمل هو ثابت في كل نقطة من طول العنصر Δx في العنصر Δx $\therefore EI$



3rd method



$$EI w'''' = q_0$$

$$w'''' = -q_0 x + c_1$$

$$EI w''' = -\frac{q_0 x^2}{2} + c_1 x + c_2$$

$$EI w'' = -\frac{q_0 x^3}{6} + \frac{c_1 x^2}{2} + c_2 x + c_3$$

$$EI w = -\frac{q_0 x^4}{24} + \frac{c_1 x^3}{6} + \frac{c_2 x^2}{2} + c_3 x + c_4$$

$$EI w = -\frac{q_0 x^4}{24} + \frac{c_2 x^2}{2} + c_4$$

At $x = l/2$ $w = 0$ --- (1)

$$EI w'' = M = 0 \text{ --- (2)}$$

from (2)

$$c_2 = \frac{q_0 l^2}{16}$$

$$EI w = -\frac{q_0 x^4}{24} + \frac{q_0 l^2}{16} x^2 + c_4$$

from (1)

$$0 = -\frac{q_0 l^4}{16 \times 24} + \frac{q_0 l^4}{16 \times 4} + c_4$$

$$c_4 = \frac{+1 - 6}{384} q_0 l^4$$

$$c_4 = \frac{-5 q_0 l^4}{384}$$

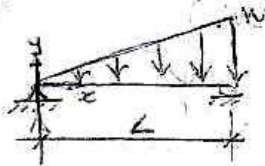
$$w = -\frac{q_0 x^4}{24} + \frac{q_0 l^2}{16} x^2 - \frac{5 q_0 l^4}{384}$$

$$w_{max} = \frac{-5 q_0 l^4}{384 EI}$$

at
mid span



Ex 2



$$EI y^{(4)} = q(x)$$

$$q(x) = \frac{W}{L} x$$

$$EI y^{(4)} = \frac{W}{L} x$$

$$EI y''' = \frac{W}{2L} x^2 + C_1 x$$

$$EI y'' = \frac{W}{6L} x^3 + \frac{C_1 x^2}{2} + C_2$$

$$\text{at } x=0, \theta=0 \Rightarrow C_2=0$$

$$x=L, \theta=0$$

$$\frac{W}{6L} x^3 + \frac{C_1 x^2}{2} = 0$$

$$C_1 = \frac{W}{9}$$

$$EI y'' = \frac{W}{6L} x^3 + \frac{W x^2}{9}$$

$$EI y' = \frac{W}{24L} x^4 + \frac{W x^3}{9} + C_3$$

$$EI y' = \frac{W}{24L} x^4 + \frac{W x^3}{9} + C_3$$

moment equations

$$\text{at } x=0, M=0 \Rightarrow C_3=0$$

$$\text{at } x=L, M=0$$

$$0 = \frac{W}{24L} x^4 + \frac{W x^3}{9}$$

$$C_3 = -\frac{WL}{9}$$

$$EI y' = \frac{W}{24L} x^4 - \frac{WL}{9}$$

$$EI y = \frac{W}{24L} x^5 - \frac{WL}{9} x + C_4$$

$$EI y = \frac{W}{120L} x^5 - \frac{WL}{9} x + C_4$$

$$\text{at } x=0, y=0$$

$$C_4=0$$

$$\text{at } x=L, y=0$$

$$\frac{W}{120L} x^5 - \frac{WL}{9} x + C_4 = 0$$

$$C_3 = \frac{(-3+10)WL^3}{360}$$

$$C_3 = \frac{7WL^3}{360}$$

$$EI y = \frac{W}{120L} x^5 - \frac{WL}{36} x^3 + \frac{7WL^3}{360} x$$

$$EI y = \frac{WL}{360L} \left[3x^5 - 10L^2 x^3 - 7L^3 x^4 \right]$$



EX3 Find the deflection



$$EI \omega'''' = q = q_0 \sin \frac{\pi x}{l}$$

$$EI \omega''' = q_0 \times \frac{l}{\pi} \cos \frac{\pi x}{l} + C_1$$

$$EI \omega'' = + \frac{q_0 l^2}{\pi^2} \sin \frac{\pi x}{l} + C_1 x + C_2$$

at $x=0$ $\omega = 0 \rightarrow C_2 = 0$ eq.

at $x=l$ $\omega = 0 \rightarrow C_1 = 0$

$$EI \omega'' = + \frac{q_0 l^2}{\pi^2} \sin \frac{\pi x}{l}$$

$$EI \omega' = - \frac{q_0 l^3}{\pi^2} \cos \frac{\pi x}{l} + C_3$$

$$EI \omega = - \frac{q_0 l^4}{\pi^4} \sin \frac{\pi x}{l} + C_3 x + C_4$$

at $x=0$ $\omega = 0 \rightarrow C_4 = 0$

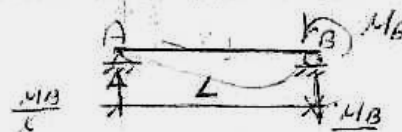
at $x=l$ $\omega = 0 \rightarrow C_3 = 0$

$$\omega = - \frac{q_0 l^4}{\pi^4 EI} \sin \frac{\pi x}{l}$$

ω_{max} occurs at mid-span

$$\omega_{max} = - \frac{q_0 l^4}{\pi^4 EI}$$

EX4 Find the def.



$$EI \omega'''' = M = \frac{M_B}{l} x$$

$$EI \omega''' = \frac{M_B}{2l} x^2 + C_1$$

$$EI \omega'' = \frac{M_B}{6l} x^3 + C_1 x + C_2$$

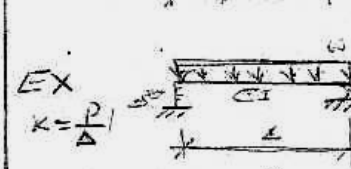
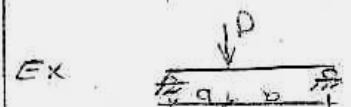
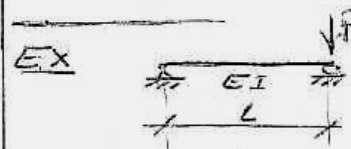
at $x=0$ $\omega = 0 \rightarrow C_2 = 0$

at $x=l$ $\omega = 0$

$$\rightarrow \frac{M_B}{6l} x^3 + C_1 x = 0$$

$$C_1 = - \frac{M_B l}{6}$$

$$EI \omega = \frac{M_B}{6l} x^3 - \frac{M_B l}{6} x$$



$$A = \frac{R}{K}$$

$$= \frac{w_0 l}{2K}$$

$$EI \omega'''' = \frac{w_0 l}{2K}$$

$$\omega'' = \frac{w_0 l}{2K} (1 - x/l)$$

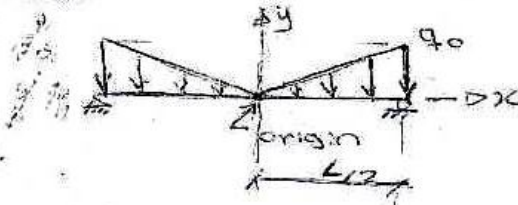
$$\omega = \frac{w_0 l^3}{12} x + \frac{w_0 l^3}{24} - \frac{w_0 l^3}{24} x$$

$$= \frac{w_0 l^3}{24} (1 - x/l)$$



هذا المثال يمثل القوس الكروي. يكون الاستطالة
الاستطالة صغيرة جداً بحيث يمكن تجاهلها
في الحساب التالى.

Ex 3



$$q(x) = \frac{q_0}{L} x = EI \theta^{(4)}$$

$$EI \theta^{(4)} = \frac{-q_0}{L} x$$

$$EI \theta^{(3)} = V = -\frac{q_0}{L} x^2 + C_1$$

$$EI \theta^{(2)} = M = \frac{-q_0}{3L} x^3 + C_1 x + C_2$$

$$EI \theta^{(1)} = \theta = \frac{-q_0}{12L} x^4 + \frac{C_1}{2} x^2 + C_2 x + C_3$$

$$EI \theta = \frac{-q_0}{60L} x^5 + \frac{C_1}{6} x^3 + \frac{C_2}{2} x^2 + C_3 x + C_4$$

1st method
From Symmetry

$$C_1 = 0 \quad \text{eq. to } V_{x=0} = 0$$

$$C_2 = 0 \quad \text{eq. to } \theta_{x=0} = 0$$

$$EI \theta = \frac{-q_0}{60L} x^5 + \frac{C_2}{2} x^2 + C_4$$

$$\text{at } x = L/2 \quad \theta = 0$$

$$C_2 = \frac{q_0 L^3}{24}$$

$$EI \theta = \frac{-q_0}{60L} x^5 + \frac{q_0 L^3}{48} x^2 + C_4$$

$$\text{at } x = L/2 \quad \theta = 0$$

$$0 = \frac{-q_0 L^4}{1920} + \frac{q_0 L^4}{192} + C_4$$

$$C_4 = \frac{-9 q_0 L^4}{1920}$$

$$EI \theta = \frac{-q_0}{60L} x^5 + \frac{q_0 L^3}{48} x^2 - \frac{9 q_0 L^4}{1920}$$

know occur at $x=0$

$$EI \theta_{max} = \frac{-9 q_0 L^4}{1920 EI}$$

في هذه الحالة يكون الزخم
الزخم في الوسط

$$V = 0 \quad \& \quad \theta = 0$$

→ also, $\theta = 0$ at $x = L/2$



$$\theta = 0 \quad V = 0$$

Guided Support

Ex 4

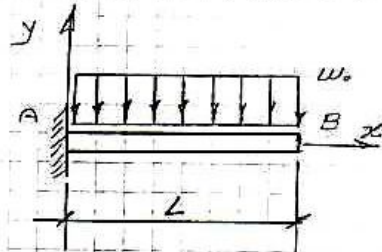


Applications of Direct Integration Method to Cantilever Beams.

Example 1. Compute the maximum deflection for the beam shown.

Solution

Start from the 4th order differential equation to avoid the derivation of moment equation.



$$\therefore EI v'''' = f(x) \rightarrow EI v'''' = -w_0$$

$$\therefore EI v''' = -w_0 x + C_1 \quad \text{initial shear}$$

$$EI v''' = V = 0 \text{ @ } x = L \quad \text{"1st boundary Condition"}$$

$$\therefore 0 = -w_0 L + C_1 \Rightarrow C_1 = w_0 L \quad \text{is equal to initial shear} \quad \text{OK.}$$

$$EI v'' = -w_0 \frac{x^2}{2} + w_0 L x + C_2 \quad \text{initial moment}$$

$$\text{@ } x = L, EI v'' = 0 \quad \text{"Free end"}$$

$$\therefore 0 = -w_0 \frac{L^2}{2} + w_0 L^2 + C_2 \Rightarrow C_2 = -w_0 L^2/2 \quad \text{is equal to initial moment} \quad \text{OK.}$$

$$EI v' = -w_0 \frac{x^3}{6} + w_0 L \frac{x^2}{2} - w_0 \frac{L^2}{2} x + C_3 \quad \text{initial rotation} \quad \times EI.$$

$$\therefore \theta = 0 \text{ @ } x = 0 \Rightarrow C_3 = 0$$

$$EI v = -w_0 \frac{x^4}{24} + w_0 L \frac{x^3}{6} - w_0 \frac{L^2}{4} x^2 + C_4 \quad \text{initial deflection} \quad \times EI$$

$$\therefore v = 0 \text{ @ } x = 0 \Rightarrow C_4 = 0$$

$$v = \frac{1}{EI} \left[-\frac{w_0 x^4}{24} + w_0 L \frac{x^3}{6} - w_0 \frac{L^2}{4} x^2 \right]$$

$v_{\max}$ occurs at $x = L$ "Can you prove that by using calculus method!!"

$$v_{\max} = \frac{-w_0 L^4}{8EI}$$



Example 2. Calculate the maximum deflection for the beam shown.

Solution:

To avoid the derivation of the triangular load from $w(x)$, we start with the 4th order d.e.

$$E \cdot v'''' = q(x)$$

$$\frac{q(x)}{L-x} = \frac{-w_0}{L} \Rightarrow q(x) = -w_0 + w_0 \frac{x}{L}$$

$$\therefore EI v'''' = -w_0 + w_0 \frac{x}{L}$$

$$EI v''' = -w_0 x + w_0 \frac{x^2}{2L} + C_1 \quad \text{Initial shear}$$

$$\therefore V = 0 \text{ at } x = L \Rightarrow C_1 = w_0 L \quad \text{Initial shear is OK.}$$

$$EI v'' = -w_0 \frac{x^2}{2} + w_0 \frac{x^3}{6L} + w_0 \frac{x}{2} + C_2 \quad \text{Initial moment}$$

$$\therefore M = 0 \text{ at } x = 0 \Rightarrow C_2 = 0 \quad \text{Initial moment is OK.}$$

$$EI v' = -w_0 \frac{x^3}{6} + w_0 \frac{x^4}{24L} + w_0 \frac{Lx^2}{4} - w_0 \frac{x^2}{2} + C_3 \quad \text{Initial rotation}$$

$$\therefore \theta = 0 \text{ at } x = 0 \Rightarrow C_3 = 0$$

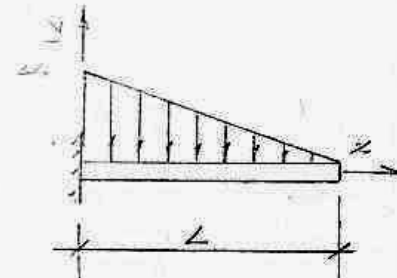
$$EI v = -w_0 \frac{x^4}{24} + w_0 \frac{x^5}{120L} + w_0 \frac{Lx^3}{12} - w_0 \frac{L^2x}{12} + C_4 \quad \text{Initial deflection}$$

$$\therefore v = 0 \text{ at } x = 0 \Rightarrow C_4 = 0$$

$$\therefore v = \frac{-w_0 L^4}{120 EI} \left[+5Lx^2 - x^3 - 10L^2x - 10L^3 \right]$$

$$v_{\max} \text{ occurs at } x = L \quad \text{"Can you prove this by using Calculus method!!"}$$

$$v_{\max} = - \frac{w_0 L^4}{30 EI}$$



Please use, check the correctness @ $x=0$ & $x=L$.



2nd Method

$EI v'' = N(x)$, then we must derive the equation of $N(x)$.

$$\sum M_0 = 0$$

$$N(x) = -PL + Px.$$

$$EI v'' = -PL + Px$$

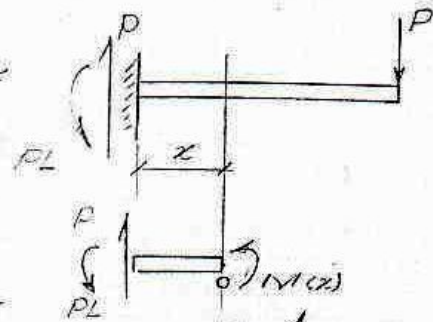
$$EI v' = -PLx + Px^2/2 + C_3 + \text{initial rotation} \times EI$$

$$\therefore \theta = 0 \text{ @ } x=0 \Rightarrow C_3 = 0$$

$$EI v = -PL \frac{x^2}{2} + \frac{Px^3}{6} + C_4 + \text{initial deflection} \times EI$$

$$v = -\frac{PL}{6EI} (3L - x)$$

$$v_{\max} \text{ occurs @ } x=L, \therefore v_{\max} = -\frac{PL^3}{3EI}$$



Must be assumed in the positive direction of sign convention

Example 3: Compute $v_{\max}$ for the beam shown.

Solution:

Start with 4th order d.e.

$$EI v'''' = q(x), \text{ but } q(x) = 0 \text{ for our case.}$$

$$\therefore EI v'''' = 0$$

$$EI v''' = C_1 + \text{initial shear}$$

$$V = 0 \text{ @ } x=L \text{ "Free end"}$$

$$\therefore 0 = C_1 \Rightarrow C_1 = 0 + \text{equal to initial shear} \therefore \text{O.K.}$$

$$EI v'' = C_2 + \text{initial moment}$$

$$M = -M_0 \text{ @ } x=L \Rightarrow -M_0 = C_2 = \text{initial moment} \therefore \text{O.K.}$$

$$\therefore EI v' = -M_0 x + C_3 + \text{initial rotation} \times EI$$

$$\therefore \theta = 0 \text{ @ } x=0 \Rightarrow C_3 = 0$$

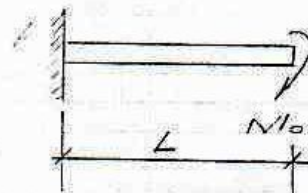
$$EI v = -M_0 \frac{x^2}{2} + C_4 + \text{initial deflection} \times EI$$

$$\therefore v = 0 \text{ @ } x=0 \Rightarrow C_4 = 0$$

$$\therefore v = -\frac{M_0 x^2}{2EI}$$

$$\therefore v_{\max} \text{ occurs @ } x=L \text{ "Can You Use Calculus to Prove that"}$$

$$\therefore v_{\max} = -\frac{M_0 L^2}{2EI}$$



H.w. Using the 2nd order d.e. to solve the same problem.



Example 6.

Compute v_{max} for the beam shown.

Solution

Start with 4th order d. e.

$$EI v'''' = q(x) = 0$$

$$\therefore EI v''' = C_1$$

$$\therefore V = 0 \text{ @ } x = L \Rightarrow C_1 = 0$$

$$EI v'' = C_2$$

$$\therefore M = -M_0 \text{ @ } x = L \Rightarrow C_2 = -M_0$$

$$EI v' = -M_0 x + C_3 \quad \text{initial rotation} \times EI$$

$$\therefore \theta = -\frac{M_0}{K} \text{ @ } x = 0 \Rightarrow C_3 = -\frac{M_0}{K} \times EI$$

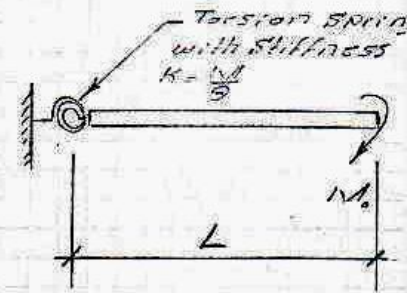
$$\therefore EI v = -\frac{M_0}{2} x^2 - \frac{M_0 EI}{K} x + C_4 \quad \text{initial deflection} \times EI$$

$$\therefore v = 0 \text{ @ } x = 0 \Rightarrow C_4 = 0$$

$$v = -\frac{M_0 x^2}{2EI} - \frac{M_0 x}{K}$$

v_{max} occurs @ $x = L$

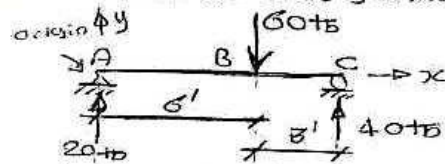
$$\therefore v_{max} = -\frac{M_0 L^2}{2EI} - \frac{M_0 L}{K}$$





2 Singularity Functions 8 -

Ex1 Find the θ and θ_{max} .



هذا المثال يتبع الطريقة السابقة
والتي استخدمنا فيها دالة
ممكن. وعليه يجب تبديل
معادلاتنا على x وأيضاً
هنا نرى نقطة الانعطاف.

نلاحظ هنا أن لدينا
نقطة انعطاف واحدة على طول
الbeam.

Portion AB

$$I = EI\theta'' = 20x$$

Portion BC

$$M = 20x - 60(x-6)$$

نلاحظ هنا أن لدينا
نقطة انعطاف واحدة على طول
الbeam.

$$M = 20\langle x \rangle - 60\langle x-6 \rangle$$

where

$$\langle x-a \rangle = x-a \text{ if } x-a > 0 \\ = 0 \text{ if } x-a < 0$$

أولاً نلاحظ أن لدينا
نقطة انعطاف واحدة على طول
الbeam.

$$EI\theta'' = 20\langle x \rangle - 60\langle x-6 \rangle$$

$$EI\theta' = 10\langle x \rangle^2 - 30\langle x-6 \rangle^2 + C_3$$

$$EI\theta = \frac{10\langle x \rangle^3}{3} - 10\langle x-6 \rangle^3 + C_3\langle x \rangle + C_4$$

B.C.

$$\text{at } x=0 \quad \theta=0 \quad C_4=0$$

$$\text{at } x=9 \quad \theta=0$$

$$0 = \frac{10 \times 9^3}{3} - 10 \times 3^3 + C_3$$

$$C_3 = -240$$

$$EI\theta = \frac{10}{3}\langle x \rangle^3 - 10\langle x-6 \rangle^3 - 240\langle x \rangle$$

Portion AB

$$EI\theta = \frac{10}{3}x^3 - 240x$$

Portion BC

$$EI\theta = \frac{10}{3}x^3 - 10(x-6)^3 - 240x$$

assume that θ_{max} occurs
in the portion AB.

$$EI\theta' = 0 = 10x^2 - 240$$

from θ_{max}

$$EI\theta' = 0 = 10x^2 - 240$$

$$x^2 = 24$$

$$x = 4.9 \text{ ft} < 6' \approx \frac{9k}{5}$$



Symmetry & Singularity Function

Ex 4

Diagram showing a beam of length 6m, divided into three 2m segments. A uniformly distributed load of 60 kN/m is applied over the middle 2m segment. The beam is supported by a pin at the left end (x=0) and a roller at the right end (x=6). A vertical dashed line at x=2m is labeled "Plane of Symmetry".

Free body diagram of the left half (0 ≤ x ≤ 2):

- Reaction at x=0: $240 \times 6 - 240 \times 4 = 480$ (upward)
- Reaction at x=2: 240 (upward)
- Load: 60 kN/m (downward)

Equations for the left half:

$$M(x) = 480x - \frac{60}{2} \langle x-2 \rangle^2$$

$$EI \theta' = 480x - \frac{60}{6} \langle x-2 \rangle^2 + C_3$$

at $x=0$ $\theta=0$ $C_3=0$

$$EI \theta = 240x^2 - \frac{10}{4} \langle x-2 \rangle^4 + C_4$$

at $x=0$ $\theta=0$ $C_4=0$

Final equation for the left half:

$$EI \theta = 240x^2 - \frac{10}{4} \langle x-2 \rangle^4$$



1 solution sec. ① - ①

$$M(x) = \frac{M_0}{L} x - M_0 \langle x-a \rangle^0$$

$$EI\theta' = \frac{M_0}{2L} x^2 - M_0 \langle x-a \rangle^1 + C_3$$

$$EI\theta = \frac{M_0}{6L} x^3 - \frac{M_0}{2} \langle x-a \rangle^2 + C_3 x + C_4$$

$$\text{at } x=0 \quad \theta=0 \quad C_4=0$$

$$\text{at } x=0 \quad \theta=0$$

$$0 = \frac{M_0}{6L} L^3 - \frac{M_0}{2} \langle L-a \rangle^2 + C_3 L$$

$$0 = \frac{M_0}{6L} L^3 - \frac{M_0}{2} L^2 + M_0 a - \frac{M_0 a^2}{2} + C_3 L$$

$$C_3 = -\frac{M_0}{6} L + \frac{M_0}{2} L - M_0 a + \frac{M_0}{2L} a^2$$

$$C_3 = \left(\frac{M_0 L}{3} - M_0 a + \frac{M_0 a^2}{2L} \right)$$

$$EI\theta = \frac{M_0}{6L} x^3 - \frac{M_0}{2} \langle x-a \rangle^2 + \left(\frac{M_0 L}{3} - M_0 a + \frac{M_0 a^2}{2L} \right) x$$

$$EI\theta(a) = \frac{M_0}{6L} a^3 + \frac{M_0 L}{3} a - M_0 a^2 + \frac{M_0 a^3}{2L}$$

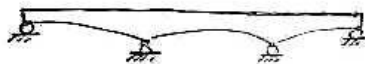
$$EI\theta(a) = \frac{M_0 a}{3L} \left[\frac{a^2}{2} + L^2 - 3La + \frac{3a^2}{2} \right]$$

$$EI\theta(a) = \frac{M_0 a}{3L} \left[2a^2 + L^2 - 3La \right]$$



3 Introduction to the moment area method :-

باعتبارنا الجسور كجسور عتبات
bridges



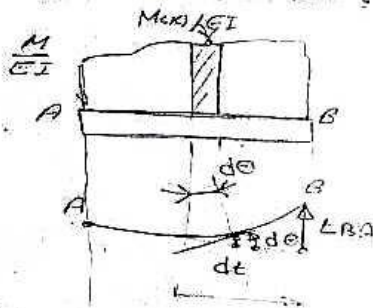
Haunched or tapered beam.

deflection

في الجسور عتباتنا الجسور كجسور عتبات
على شكل عتبات عتباتنا الجسور كجسور عتبات

تكون العتبات عتباتنا الجسور كجسور عتبات
ذلك فكل عتباتنا الجسور كجسور عتبات

deflection
أما في الجسور عتباتنا الجسور كجسور عتبات



$$EI \theta'' = M(x)$$

$$\theta'' = \frac{d^2\theta}{dx^2} = \frac{M(x)}{EI}$$

$$\text{but } \frac{d^2\theta}{dx^2} = \frac{d\theta}{dx}$$

$$\therefore \frac{d\theta}{dx} = \frac{M(x)}{EI}$$

$$d\theta = \frac{M(x)}{EI} dx$$

d

The area under the $\frac{M}{EI}$ diagram is equal to the change between two adjoining tang.

$$d\theta = x d\theta = \frac{M}{EI} x dx$$

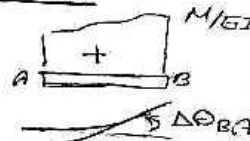
From equation ① we can obtain

$$\int_A^B d\theta = \theta_B - \theta_A = \Delta\theta_{BA} = \int_A^B \frac{M}{EI} dx$$

1st theorem

يقضي أن الفجوة في العتبات
بين العتبات عتباتنا الجسور كجسور عتبات

Note

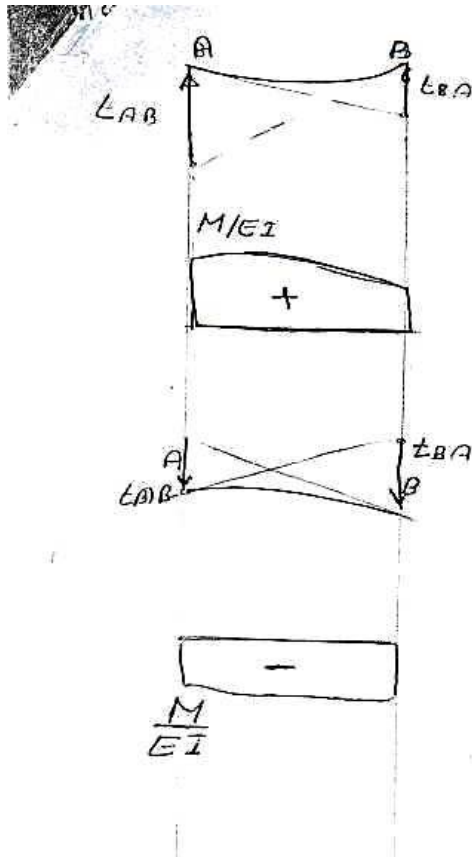


$$\Delta\theta_{BA} = \int_A^B d\theta = \int_A^B \frac{M}{EI} dx$$

2nd Theorem

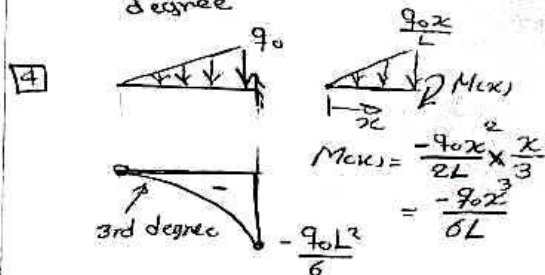
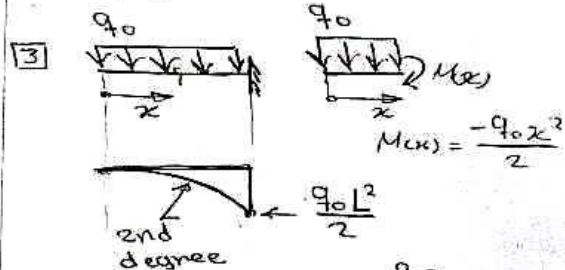
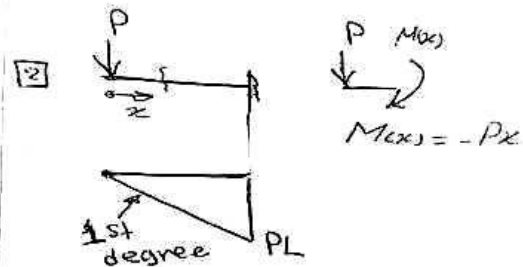
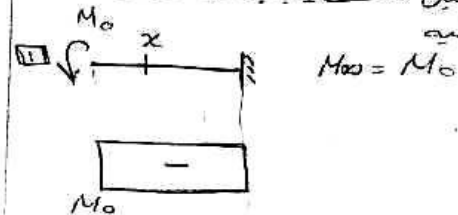
tangential deviation of a point B from Point A.

يقضي أن الفجوة في العتبات
بين العتبات عتباتنا الجسور كجسور عتبات



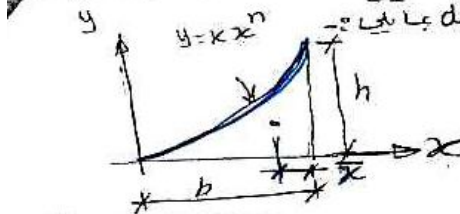
Moment Diagrams By Parts 2-

method
- مرسوم منحنى العزوم M على طول المساحة M/EI في اتجاه واحد
- مرسوم منحنى العزوم M على طول المساحة M/EI في اتجاه واحد
- مرسوم منحنى العزوم M على طول المساحة M/EI في اتجاه واحد
- مرسوم منحنى العزوم M على طول المساحة M/EI في اتجاه واحد





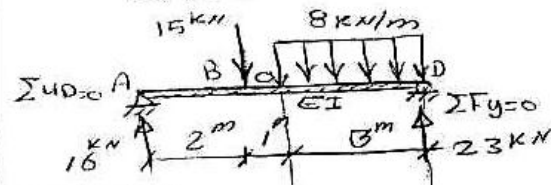
في كل حالة السالبة ، فالتحيز يكون
نحو المحاور القياسية ، والتي هي



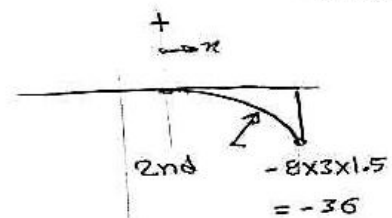
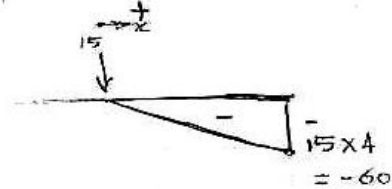
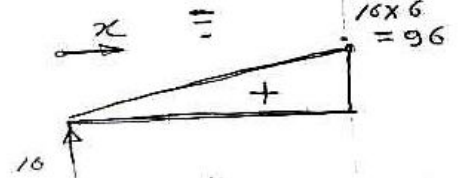
$$Area = \frac{b \times h}{n+1}$$

$$\bar{x} = \frac{b}{n+2}$$

Ex 1 Draw the B.M.D. by
Parts 2-



method ①

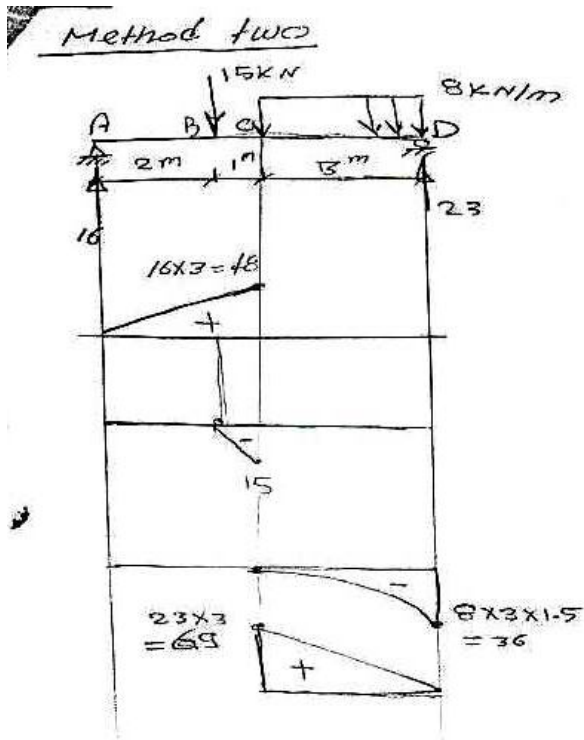


Total moment

$$M_D = 96 - 60 - 36 = 0 \text{ ok}$$

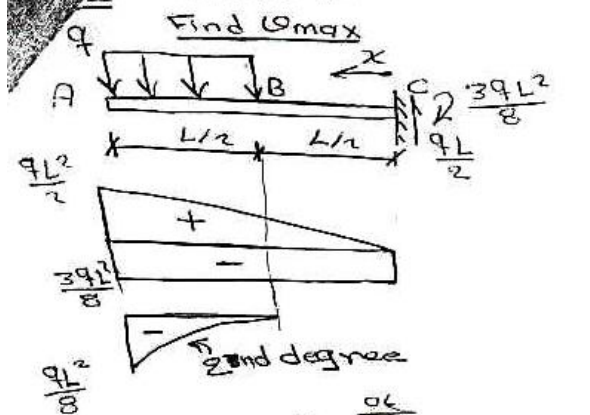
$$M_A = 0 + 0 + 0 = 0 \text{ ok}$$

$$M_C = \frac{96}{2} - \frac{60}{4} + 0 = 33$$





Applications of the moment area method to cantilever beams :-



Check $M_A = \frac{qL^2}{2} - \frac{3qL^2}{8} - \frac{qL^2}{2} = 0$



$\Delta_{max} = \Delta_{AC}$

$$= \left[\left(\frac{qL^2}{2} \times \frac{L}{2} \right) \times \frac{1}{3} - \left(\frac{3qL^2}{8} \times L \right) \times \frac{1}{2} - \left(\frac{qL^2}{8} \times \frac{L}{2} \right) \times \frac{1}{3} \times \frac{L/2}{4} \right] \frac{1}{EI}$$

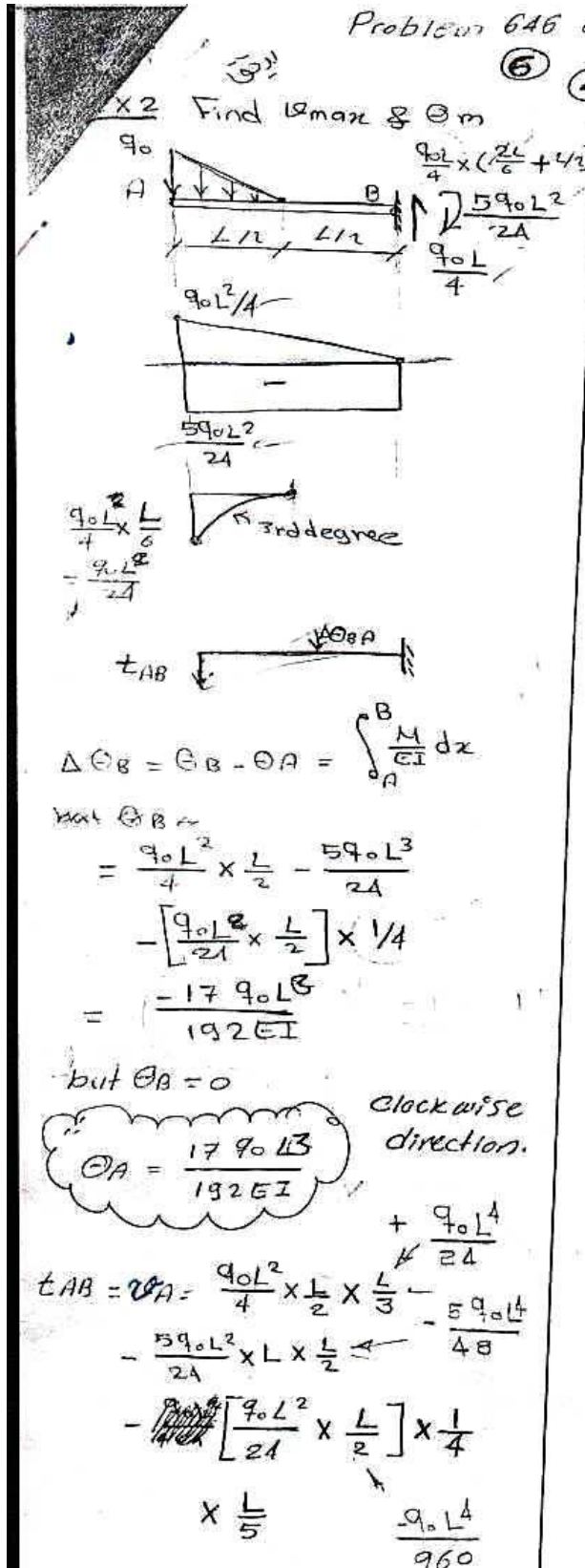
$$= \left[\frac{1}{12} - \frac{3}{16} - \frac{1}{384} \right] \frac{qL^4}{EI}$$

$\Delta_{max} = \frac{qL^4}{384 EI}$ ← From
Singer
2nd Edition
Prob.
643.

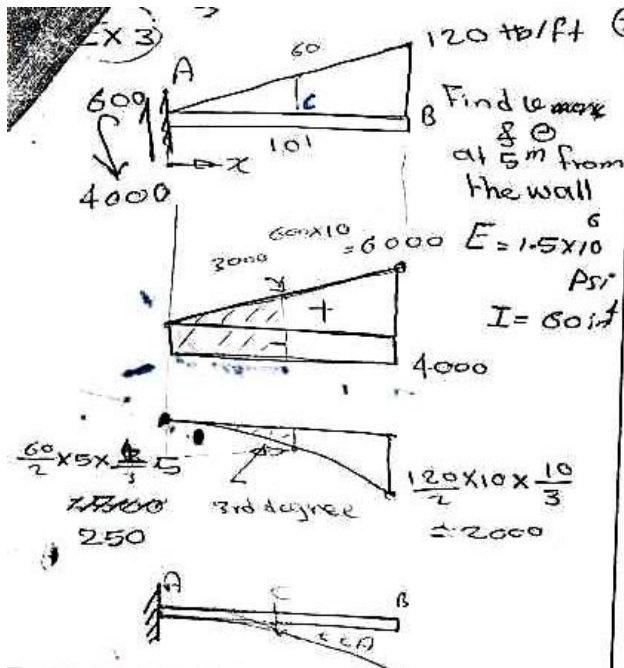
الاشارة
السابقة
تنسجم مع النتيجة
من المثال.



Problem 646 of Singer 2nd Edition



$$C_A = \frac{-61}{960} \frac{q_0 L^4}{EI} ?$$



$$\Delta_c = \frac{t_c A}{EI}$$

$$= \frac{1}{EI} \left[\frac{3000}{2} \times 5 \times (0.333 \times 5) - 4000 \times 5 \times 2.5 - \frac{250 \times 5}{4} \times \frac{5}{5} \right]$$

$$= \frac{-37,825 \text{ lb}\cdot\text{ft}^3}{EI \text{ lb}\cdot\text{ft}^2}$$

$$= \frac{-37,825 \times 12^3 \text{ lb}\cdot\text{in}^3}{1.5 \times 10^6 \times 60}$$

$$= -0.726 \text{ in}$$

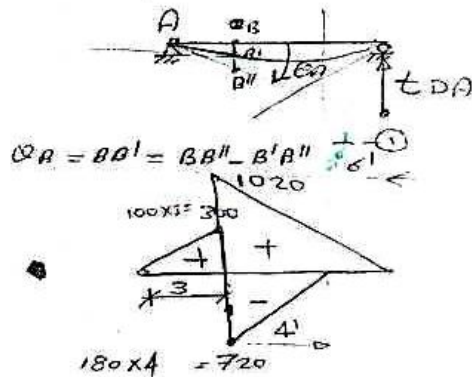
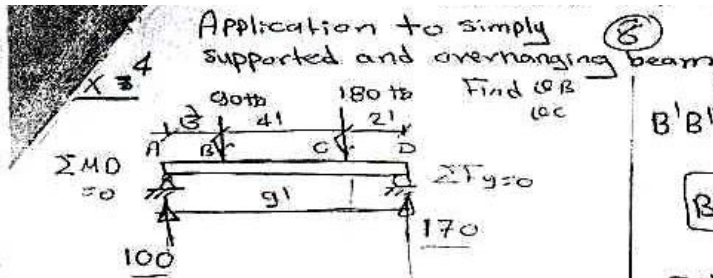
$$\Delta_c = 0.726 \text{ in} \downarrow \text{down}$$

$$\Delta_c = \frac{t_c A}{EI} = \frac{1}{EI} \left[\frac{3000}{2} \times 5 - 4000 \times 5 - \frac{250 \times 5}{4} \right] = \frac{-12,812.5 \text{ lb}\cdot\text{ft}^2}{EI \text{ lb}\cdot\text{ft}^2}$$

$$c = \frac{-12,812.5 \times 12}{1.5 \times 10^6 \times 60} = -0.0205 \text{ Rad}$$

$$\theta_c = -1.175^\circ$$

$$\therefore \theta_c = 1.175^\circ \downarrow$$



$\theta_A = \theta_D = \theta_B = \theta_C$

$\theta_A = \theta_D = \theta_B = \theta_C$

$\theta_A = \theta_D = \theta_B = \theta_C$

moment of Area under $\frac{M}{EI}$ about Point D

$$\frac{1}{EI} \left[\frac{300}{2} \times 3 \times (1+6) + \frac{1020}{2} \times 6 \times 4 - \frac{720}{2} \times 4 \times \left(\frac{8}{3}+4\right) \right]$$

$$= + \frac{8670}{EI}$$

$$\theta_A = \frac{\theta_D}{L} = \frac{8670}{9EI}$$

$$\theta_A = \theta_D = \theta_B = \theta_C = \frac{963.3}{EI}$$

$$BB' = \theta_A \times 3$$

$$BB' = 2890/EI$$

$$B'B'' = \theta_B A$$

$\theta_B A =$ moment of area under $\frac{M}{EI}$ between A & B about B.

$$B'B'' = \left[\frac{300}{2} \times 3 \times 1 \right] = \frac{450}{EI}$$

$$B'B'' = \frac{450}{EI}$$

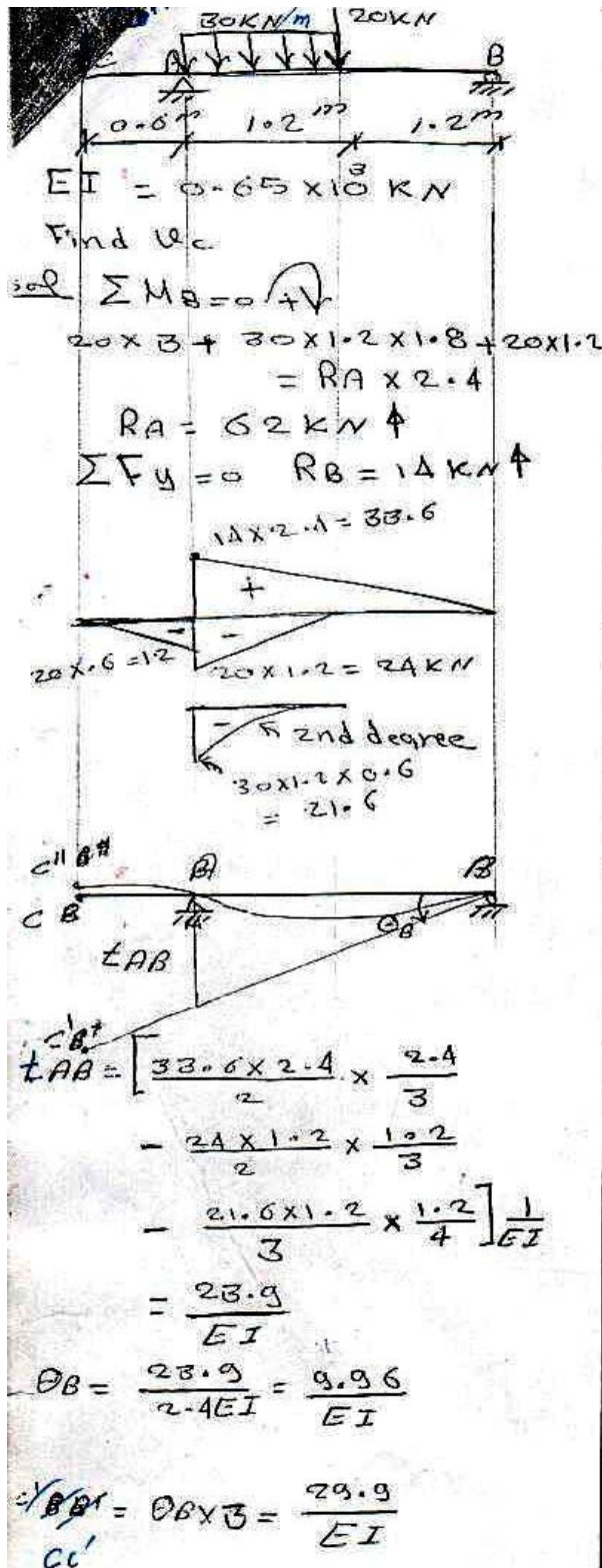
Sub. into eq. (1)

$$\theta_B = \frac{2440}{EI} \text{ down ward}$$

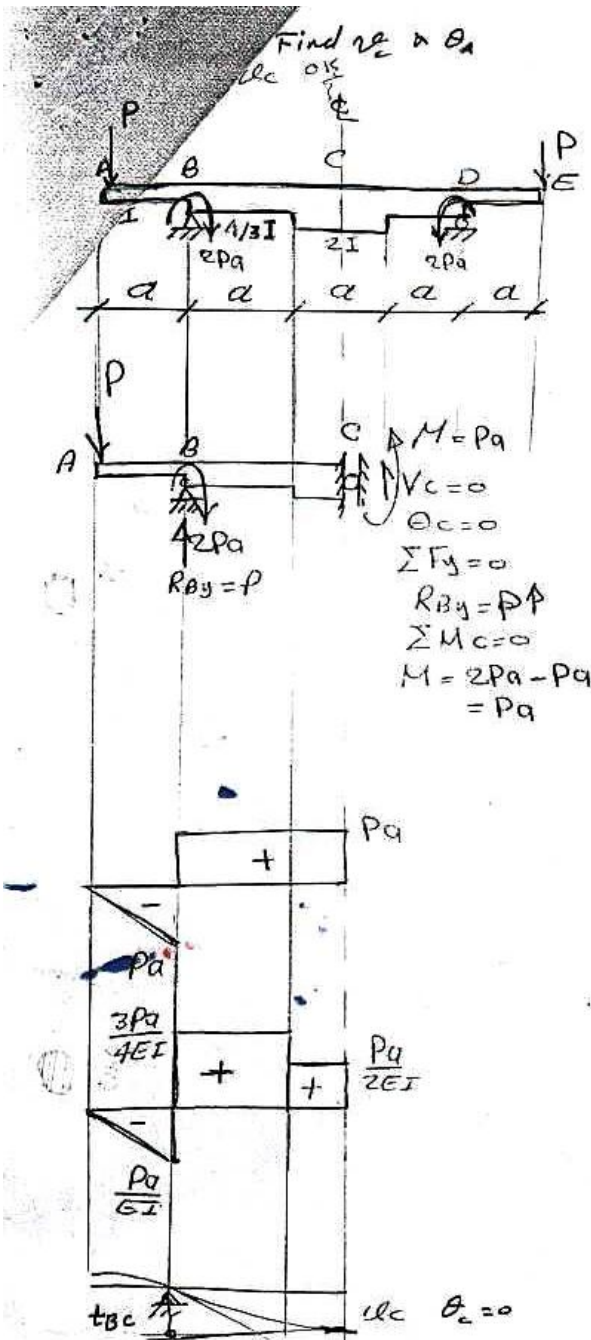
θ_C home work

Ans.

$$\theta_C = \frac{1990}{EI} \text{ down ward}$$



$$\begin{aligned}
 C''C' &= \angle AB \\
 &= \left[\frac{-12}{2} \times 0.6 \times 0.4 \right. \\
 &\quad + \frac{33.6}{2} \times 2.4 \times \left(\frac{2.4}{3} + 0.6 \right) \\
 &\quad - \frac{24}{2} \times 1.2 \times \left(\frac{1.2}{3} + 0.6 \right) \\
 &\quad \left. - \frac{21.6 \times 1.2}{3} \left(\frac{1.2}{4} + 0.6 \right) \right] \times \frac{1}{EI} \\
 &= \frac{32.83}{EI} \\
 \theta_C &= C''C' - C'C \\
 &= \frac{1}{EI} [32.83 - 29.9] \\
 \theta_C &= \frac{2.93}{EI} \\
 &= 4.5 \times 10^{-3} \text{ m} \\
 \theta_C &= 4.5 \text{ mm}
 \end{aligned}$$



$$\theta_C - \theta_A = \frac{-Pa}{2EI} \cdot a + \frac{3Pa}{4EI} \cdot a$$

$$= \frac{Pa^2}{EI} \left[-\frac{1}{2} + \frac{3}{4} + \frac{1}{4} \right]$$

$$\theta_C - \theta_A = \frac{Pa^2}{EI}$$

but from symmetry $\theta_C = 0$

$$\therefore \theta_A = -\frac{Pa^2}{2EI}$$

$$\Delta_c = t_{BC} = \frac{3Pa}{4EI} \cdot a \cdot \frac{a}{2}$$

$$+ \frac{Pa}{2EI} \cdot \frac{a}{2} \cdot \frac{5a}{4}$$

$$= \frac{Pa^3}{EI} \left[\frac{3}{8} + \frac{5}{16} \right]$$

$$= \frac{11Pa^3}{16EI}$$



Example 6.

Compute v_{max} for the beam shown.

Solution

Start with 4th order d.e.

$$EI v^{(4)} = f(x) = 0$$

$$\therefore EI v''' = C_1$$

$$\therefore V = 0 \text{ @ } x = L \Rightarrow C_1 = 0$$

$$EI v'' = C_2$$

$$\therefore M = -M_0 \text{ @ } x = L \Rightarrow C_2 = -M_0$$

$$EI v' = -M_0 x + C_3 = \text{initial rotation} \times EI$$

$$\therefore \theta = -\frac{M_0}{K} \text{ @ } x = 0 \Rightarrow C_3 = -\frac{M_0}{K} \times EI$$

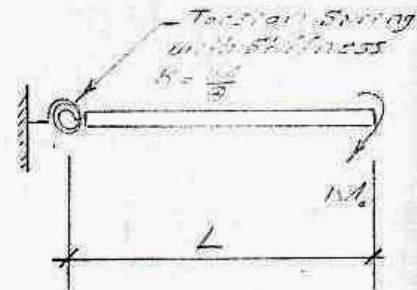
$$\therefore EI v = -\frac{M_0}{2} x^2 - \frac{M_0 EI}{K} x + C_4 = \text{initial deflection} \times EI$$

$$\therefore v = 0 \text{ @ } x = 0 \Rightarrow C_4 = 0$$

$$v = -\frac{M_0 x^2}{2EI} - \frac{M_0 L}{K}$$

v_{max} occurs @ $x = L$

$$\therefore v_{max} = -\frac{M_0 L^2}{2EI} - \frac{M_0 L}{K}$$





Applications of Moment Area Method for the Overhanging Beams.

Example 1 For the beam shown, determine v_c .
Solut. op.

Draw B.M.D. by Parts

Reaction: $\sum \uparrow \downarrow = 0 \Rightarrow R_A = 160 \uparrow$

$v_c = C'C' - C''C$

but $C'C' = \frac{t_{CA}}{EI}$

$$= \frac{1}{EI} \left[-\frac{960 \times 6}{2} \times (2+4) - \frac{960 \times 4}{2} \times \frac{3}{4} \times 4 \right] \text{ E.M.D.}$$

$$= \frac{1}{EI} [-17280 - 3840]$$

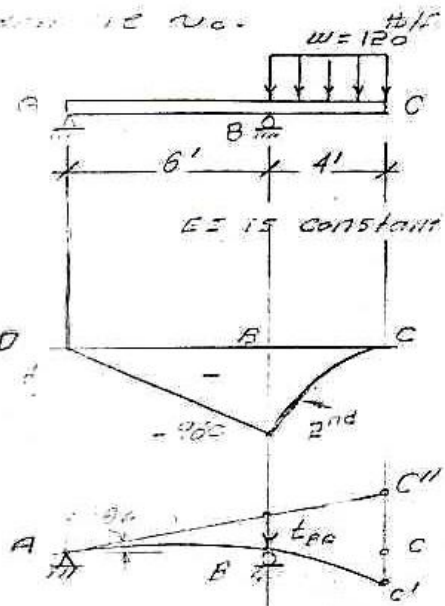
$C'C' = -\frac{21120}{EI}$

$C''C = \theta_A \times 10$, but $\theta_A = \frac{t_{BA}}{6}$

$\frac{1}{EI} = \frac{1}{EI} \left[-\frac{960 \times 6}{2} \times 2 \right] = -\frac{5760}{EI}$

$\therefore C''C = -\frac{9600}{EI}$

$\therefore v_c = -\frac{11520}{EI}$



Example 2 Compute v_c for the beam shown.
Solution

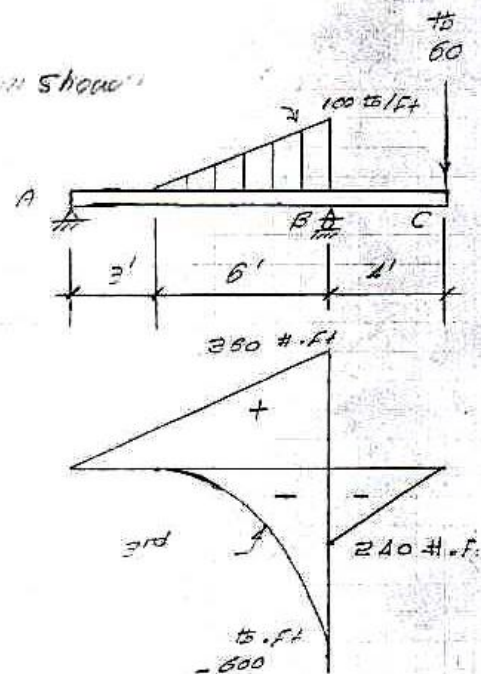
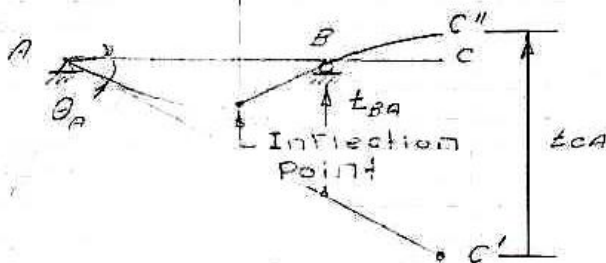
Draw B.M.D. by Parts

Compute R_A , $\sum \uparrow \downarrow = 0$

$R_A = \frac{1}{9} [50 \times 4 - 100 \times \frac{6}{9} \times 2]$

$\therefore R_A = -40 \therefore R_A = 40 \uparrow$

Assume that C'' is above C





$$\text{but } \delta_c = C''C' - CC'$$

$$= \frac{1}{EI} \left[\frac{360 \times 9}{2} \times (3+1) - \frac{600 \times 6}{4} \times \left(\frac{6}{5} + 4 \right) - 240 \times \frac{4}{2} \times \frac{5}{3} \times 4 \right]$$

$$\delta_{CA} = \frac{1}{EI} [+11340 - 4680 - 1280]$$

$$\delta_{CA} = \frac{5380}{EI} = C''C'$$

$$CC' = 18 \times \theta_A = 18 \times \frac{180}{EI}$$

$$\delta_{BA} = \frac{1}{EI} \left[\frac{360 \times 9}{2} \times 3 - \frac{600 \times 6}{4} \times \frac{5}{5} \right]$$

$$\delta_{BA} = \frac{1}{EI} [4860 - 1080] = \frac{3780}{EI}$$

$$CC' = \frac{5460}{EI} > C''C' \therefore \text{the assumption that } C'' \text{ is above } C \text{ is not correct.}$$

$$\therefore \delta_c = CC' - C''C' = \frac{804}{EI}$$

Apply the result for section

$$I = 864 \text{ in}^4$$

$$\delta_c = 5.33 \times 10^{-5} \text{ in}$$



Section E = 3 x 10 PSI



Example 3. For the beam shown, compute the Value force P that will cause the tangent to the deflected Curve at Point B to be horizontal. Then compute v_c .

Solut. on
Draw the S.M.D.
by parts.

$$\theta_B - \theta_A = \left[\frac{600 - 3P}{2} \times 10 - 600 \times \frac{6}{3} \right] \frac{1}{EI}$$

$$\therefore EI \theta_B = 3000 + 15P + 1800$$

$$\therefore EI \theta_B = 15P + 1800 \quad \text{--- (1)}$$

but $\theta_B = -\frac{t_{BA}}{L_{BA}}$

$$t_{BA} = - \left[\frac{10EI}{2} \times \frac{5}{10} \times \frac{10}{3} - \frac{600 \times 6}{2} \times \frac{6}{3} \right]$$

$$t_{BA} = [10000 - 50P - 3600]$$

$$\frac{t}{BA} = -6400 + 50P$$

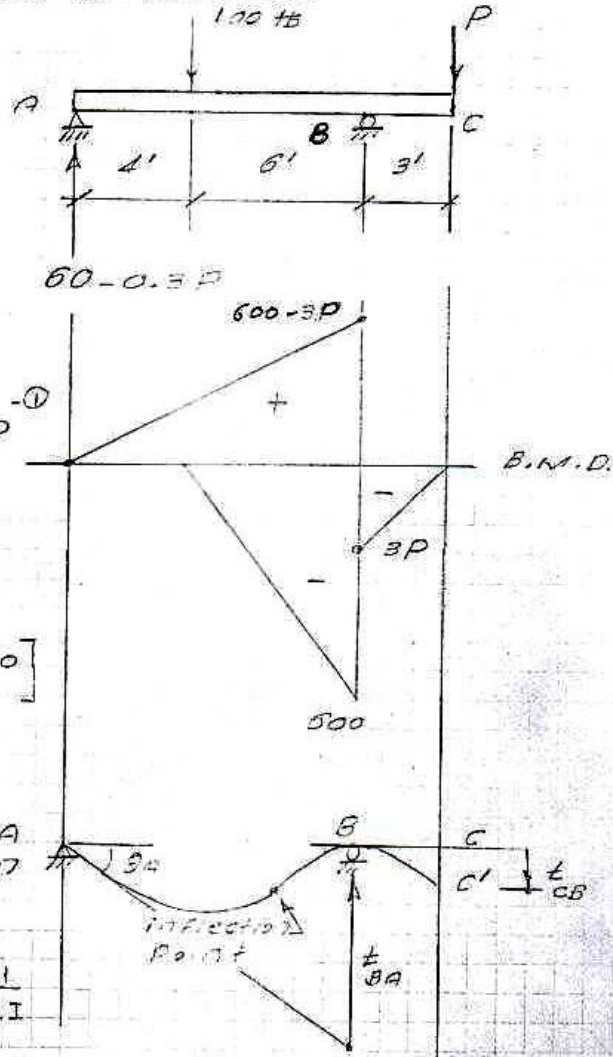
$$EI \theta_B = 6400 + 50P \quad \text{--- (2)}$$

Solve (1) and (2) to obtain P

$$P = 56 \text{ k}$$

$$v_c = t_{CB} = \left[\frac{(8 \times 56) \times 3 \times 2}{2} \right] \frac{1}{EI}$$

$$v_c = - \frac{504}{EI}$$





Chapter Eight Compound Stresses

Introduction:

The following table contains a summary of stresses and deformations that have been studied in the previous chapters:

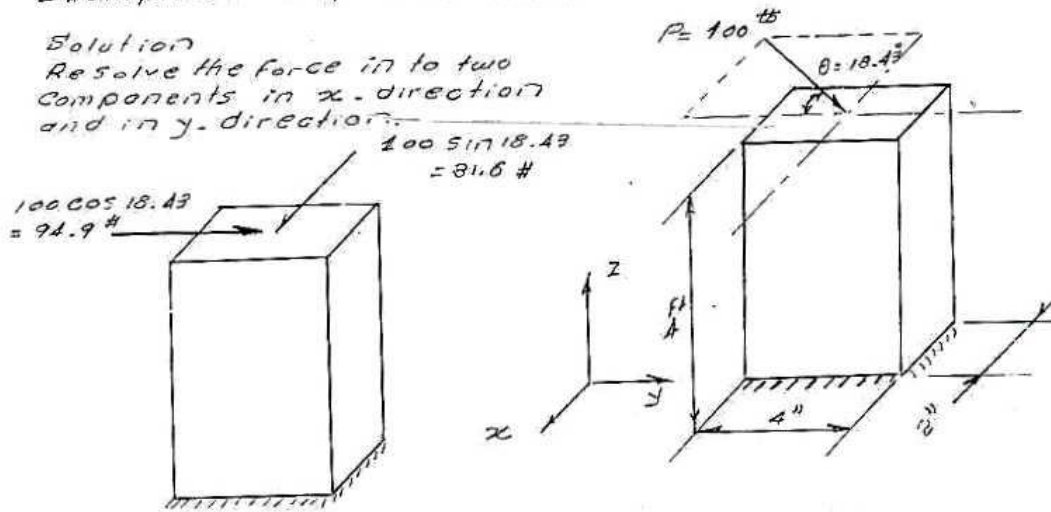
loading case	section type	elastic stresses	deformation elastic
Axial	any	P/A	$\epsilon_x = \frac{P}{AE}$
Torsion	Circular	$\tau = \frac{T \cdot y}{J}$	$\frac{d\phi}{dx} = \frac{T}{\pi R^4 G}$
	Rectangular	$\tau_{max} = \frac{T}{\alpha b c^2}$	$\frac{d\phi}{dx} = \frac{T}{\beta b c^3 G}$
	Thin closed Thin walled Tubular	$\tau = \frac{T}{2 A t}$	$\frac{d\phi}{dx} = \frac{T}{4 A^2 G} \int \frac{ds}{t}$
Bending	Any	$\sigma = \frac{M \cdot y}{I}$	$E I \phi'' = M$
Principal axes must be used			
Shearing in beams	Any	$\tau = \frac{V Q}{I t}$	Generally very small and can be neglected



Skew Bending

Example 1. Compute the maximum normal stress.

Solution
Resolve the force in to two components in x-direction and in y-direction.



As the bending moments about x and y axes are maximum at base, then maximum normal stresses occur at base too.

$$M_z = 94.9 \text{ lb} \times 4 \text{ ft} = 380 \text{ lb} \cdot \text{ft} = 4555 \text{ lb} \cdot \text{in}$$

$$M_y = 31.6 \text{ lb} \times 4 \text{ ft} = 126 \text{ lb} \cdot \text{ft} = 1517 \text{ lb} \cdot \text{in}$$

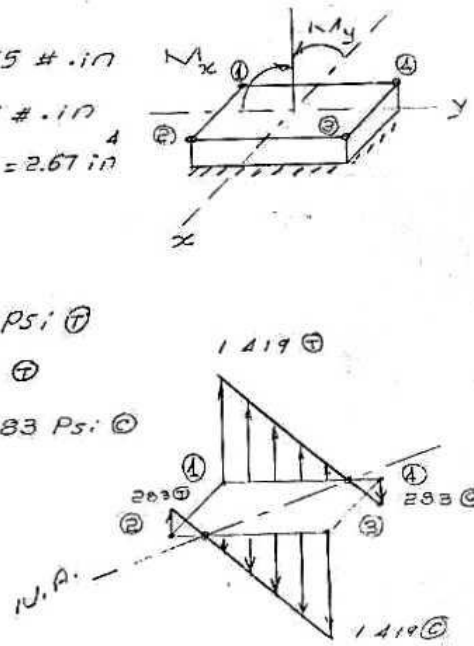
$$I_x = \frac{4^3 \times 2}{12} = 10.7 \text{ in}^4, \quad I_y = \frac{2^3 \times 4}{12} = 2.67 \text{ in}^4$$

$$\sigma_1 = \frac{4555 \times 2}{10.7} + \frac{1517 \times 1}{2.67} =$$

$$= 851.0 + 568.0 = 1419 \text{ psi} \text{ (T)}$$

$$\sigma_2 = 851.0 - 568.0 = 283 \text{ psi} \text{ (T)}$$

$$\sigma_3 = -1419 \text{ psi} \text{ (C)} \quad \sigma_4 = -283 \text{ psi} \text{ (C)}$$



Maximum compressive stress occurs at point ③:

$$\sigma_1 = - \frac{(7.5 \times 12) \times 2}{\frac{4^3 \times 6}{12}} - \frac{(26 \times 12) \times 3}{\frac{6^3 \times 4}{12}} =$$

$$= -5.63 - 13 = -18.6 \text{ psi}$$

Maximum tension stress occurs at point ②:

$$\sigma_2 = +18.6 \text{ psi}$$

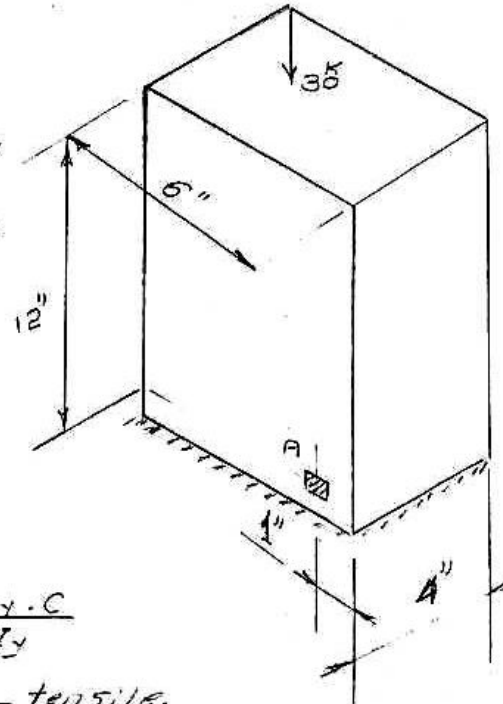
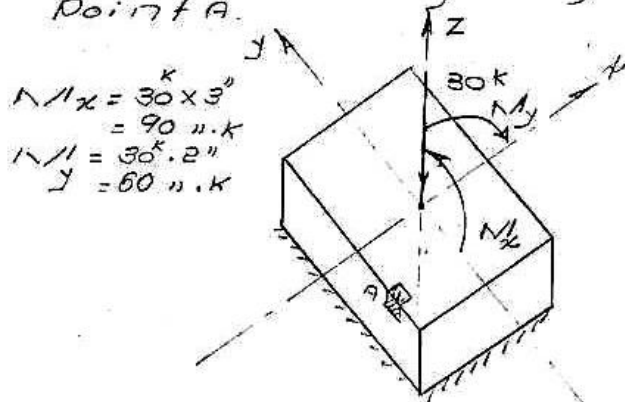


Simultaneous Action of Axial Force and Bending Moment "Eccentrically Loaded Members":

Example 1. Find σ_A .

Solution

- Compute internal forces and moments on cross section passing through Point A.



$$\sigma_A = -\frac{P}{A} + \frac{M/A_x \cdot y}{I_x} + \frac{M/A_y \cdot x}{I_y}$$

Compressive tensile.

$$\sigma_A = -\frac{30}{(6 \times 4)} + \frac{90 \times 2}{\frac{6^3 \times 4}{12}} + \frac{60 \times 2}{\frac{4^3 \times 6}{12}}$$

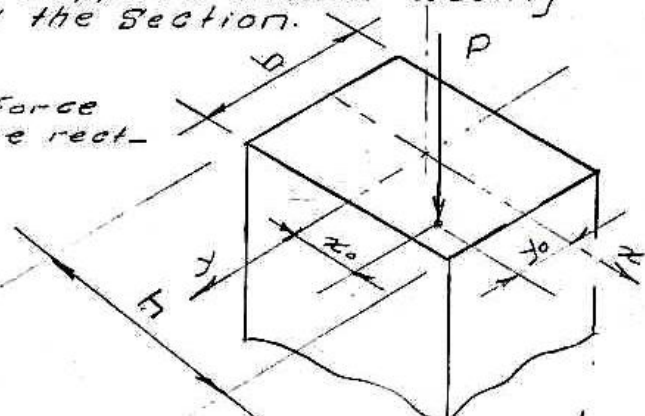
$$\sigma_A = -1.25 \text{ ksi} + 2.5 \text{ ksi} + 3.75 \text{ ksi}$$

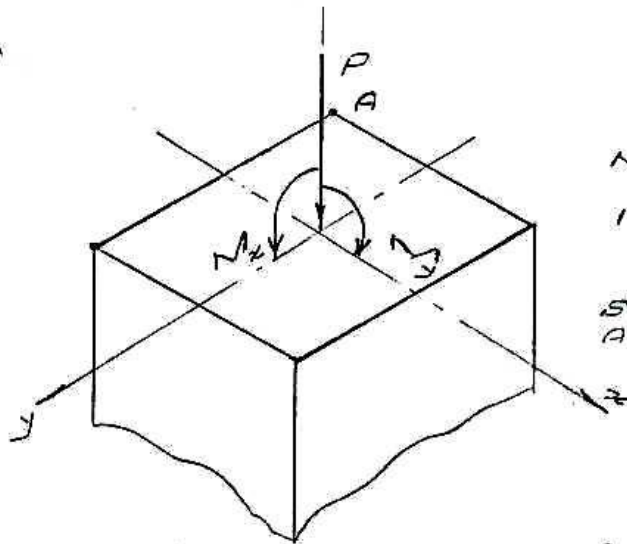
$$\sigma_A = 5 \text{ ksi} \text{ (T)}$$

Example 2. Find the zone over which the vertical downward force P may be applied without causing any tensile stresses at the section.

Solution

Transform the applied force P to the centroid of the rectangular section.





$$M_x = P \cdot y_0$$

$$M_y = P \cdot x_0$$

The maximum tensile stress occurs at point A.

$$\sigma_A = 0 = -\frac{P}{A} + \frac{M_x \cdot C_x}{I_x} + \frac{M_y \cdot C_y}{I_y}$$

compression tension tension

$$0 = -\frac{P}{h \cdot b} + \frac{(P \cdot y_0) \times b/2}{b^3 h / 12} + \frac{(P \cdot x_0) \times h/2}{h^3 b / 12}$$

$$-\frac{P}{h \cdot b} + \frac{P \cdot y_0}{b^2 h / 6} + \frac{P \cdot x_0}{h^2 b / 6} = 0$$

$$\therefore \frac{y_0}{b/6} + \frac{x_0}{h/6} = 1$$

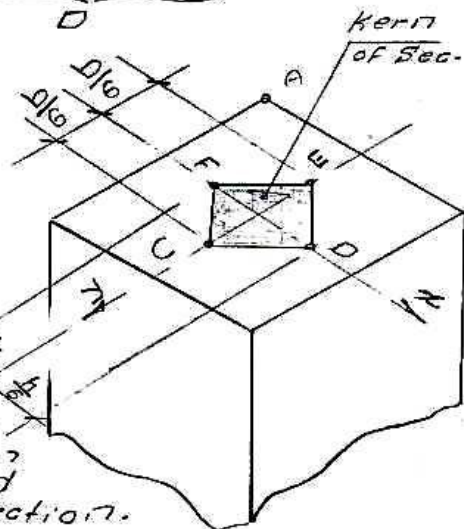
@ $x_0 = 0, y_0 = b/6$ and @ $y_0 = 0, x_0 = h/6$

Point C D

In the same approach one can locate point E and F.

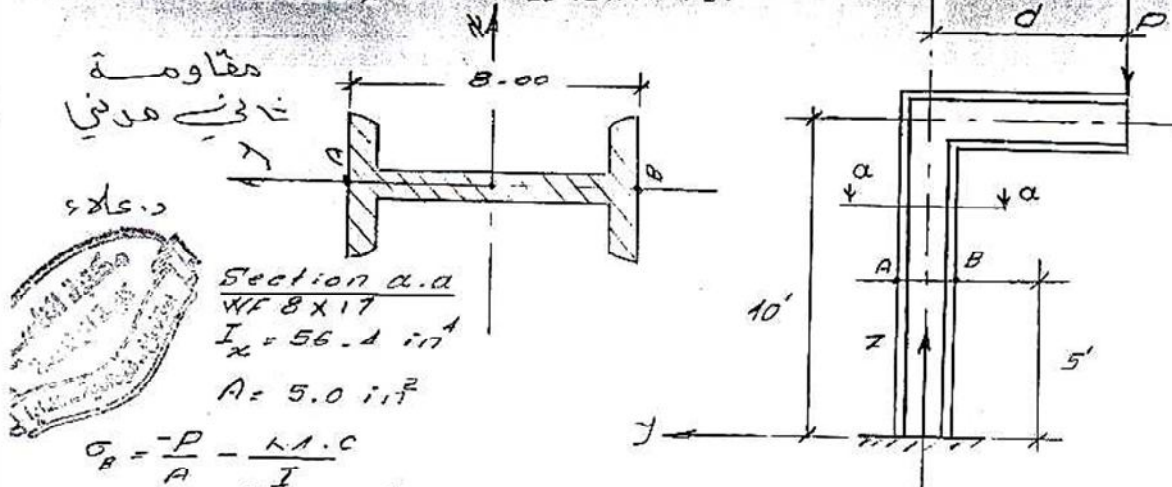
The vertical force P can be applied anywhere within the shaded area without causing tensile stress at the whole section. The shaded area is known as the Kern of Section.

Kern Concept is very useful in the design of unreinforced masonry wall and Column, where the dimensions of wall or column are computed to make the applied force P acting in the Kern of Section.





Example 3. For the frame shown below, compute the applied force P and the distance d such that the strains at $A = 600 \times 10^{-6}$ and at $B = -200 \times 10^{-6}$. $E = 30 \times 10^6 \text{ PSI}$



$$\sigma_B = -\frac{P}{A} - \frac{M \cdot C}{I}$$

$$(-200 \times 10^{-6} \times 30 \times 10^6) = -\frac{P}{A} - \frac{M \cdot C}{I}$$

$$\therefore -6000 = -\frac{P}{A} - \frac{M \cdot C}{I} \quad \text{--- ①}$$

$$\sigma_A = -\frac{P}{A} + \frac{M \cdot C}{I} \Rightarrow +600 \times 10^{-6} \times 30 \times 10^6 = -\frac{P}{A} + \frac{M \cdot C}{I}$$

$$\therefore 18000 \text{ PSI} = -\frac{P}{A} + \frac{M \cdot C}{I} \quad \text{--- ②}$$

$$\text{①} - \text{②} \Rightarrow -24000 \text{ PSI} = -2 \frac{M \cdot d}{56.4 \text{ in}^4} \Rightarrow M = 169200 \text{ in} \cdot \text{lb}$$

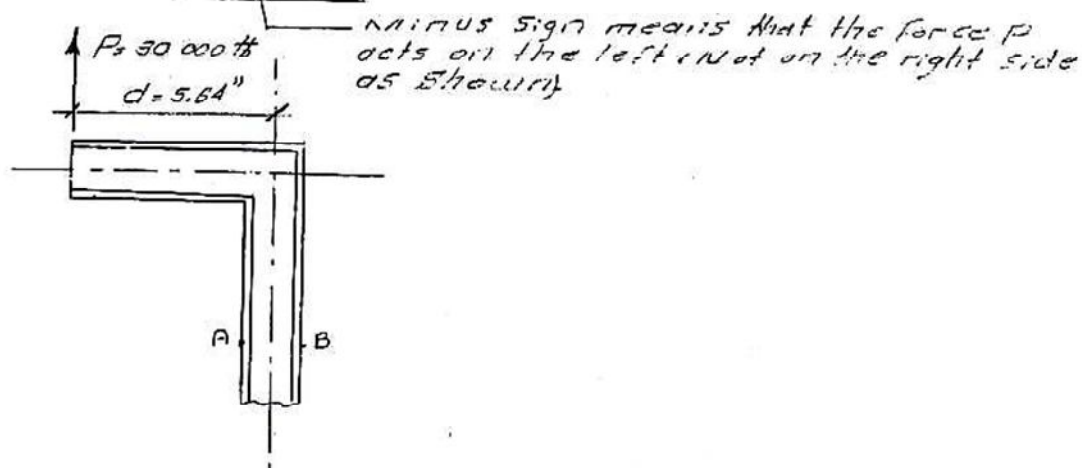
$$\text{①} + \text{②} \Rightarrow 12000 \text{ PSI} = -2 \frac{P}{5.0 \text{ in}^2} \Rightarrow P = -30000 \text{ lb}$$

But $M = P \times d$

$$\therefore 169200 = -30000 \times d$$

$$\therefore d = -5.64$$

Minus Sign means that the force acting in upward direction.

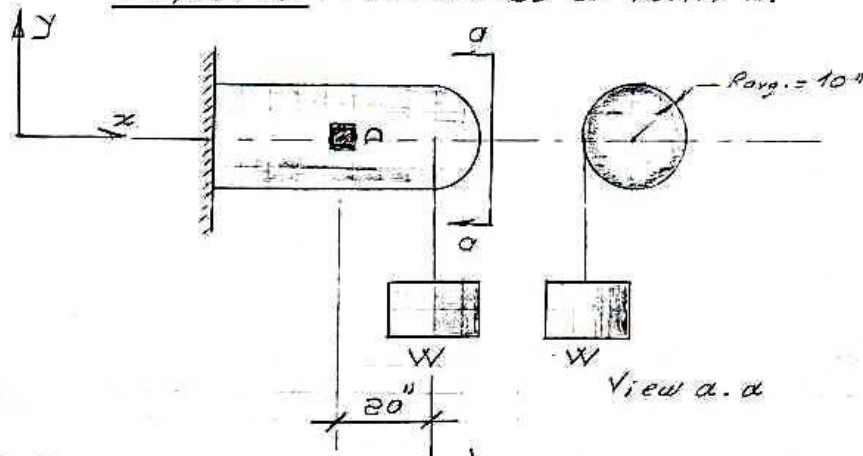




Pressure Vessel under Compound Stresses

Example 1. Given:- For the Pressure Vessel shown:-
Diameter = 20 in, Wall thickness = 0.25 in,
Internal Pressure $P = 250 \text{ psi}$ and
Applied weight $W = 31.4 \times 10^3 \text{ lb}$

Required :- Stresses at Point A.



Solution
Normal Stresses
From diagram of
element A:-

$$\sigma_x = \sigma_L + \sigma_b \leftarrow \text{Normal stress due to bending} = \frac{M \cdot y}{I}$$

Longitudinal stress due to internal pressure = $\frac{Pr}{2t}$

$$\therefore \sigma_x = \frac{250 \times (20/2)}{(2 \times 0.25)} + \frac{M \cdot 0}{I} \quad \text{As Element A is on the neutral Axis, } y \text{ is taken as equal to zero.}$$

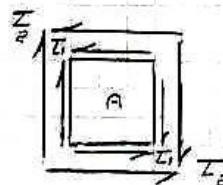
$$\sigma_x = 5000 \text{ Psi } \oplus$$

$\sigma_y = \sigma_t \leftarrow$ Tangential (or hoop) stress due to internal pressure.

$$\sigma_y = \frac{P \cdot r}{t} = \frac{250 \times (20/2)}{0.25}, \quad \sigma_y = 10000 \text{ Psi } \oplus$$

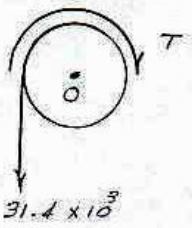
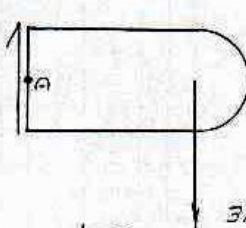
Shear Stresses

$J = J_1 - J_2$
Shear stress due to torque, T .
Shear stress due to shear force, V .





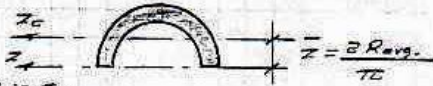
$$I = \frac{VQ}{I_t} + \frac{T \cdot r}{I}$$

$\Sigma M_o = 0$
 $T = (31.4 \times 10^3) \times 10^{in} = 31.4 \times 10^4 \text{ in. lb}$
 $\Sigma F_y = 0$
 $V = 31.4 \times 10^3 \text{ lb}$

For this thin-walled pressure vessel, the following approximate relations can be used for the computing of "I" and "Q".

$I = \pi R_{avg}^3 \cdot t = \pi \times 10^3 \times 0.25 \Rightarrow I = 785 \text{ in}^4$
 $\therefore J = 2 \times I = 1570 \text{ in}^4$
 $Q = A \times \bar{Z}_c = (R_{avg} \times \pi \times t) \times \frac{2 R_{avg}}{\pi}$
 $Q = 50 \text{ in}^3$

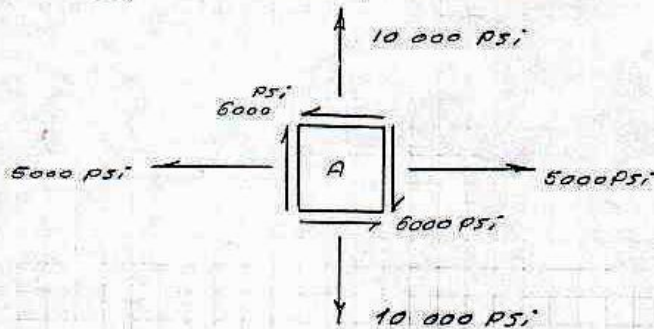


$$\therefore \tau = \frac{(31.4 \times 10^3 \text{ lb}) \times 50 \text{ in}^3}{785 \text{ in}^4 \times (2 \times 0.25) \text{ in}} + \frac{(31.4 \times 10^3 \text{ lb}) \times 10 \text{ in}}{1570 \text{ in}^4}$$

$$\tau = 4000 \text{ PSI} + 2000 \text{ PSI}$$

$$\therefore \tau = 6000 \text{ PSI}$$

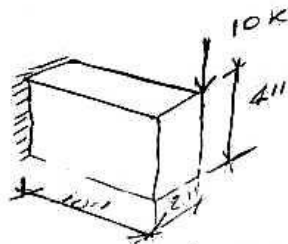
Summary of Stresses on Element A :-



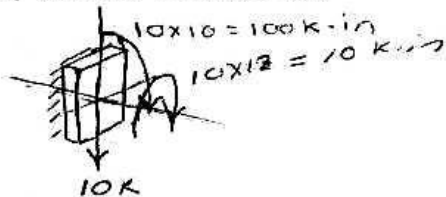


Suppose the stresses of Shear stress -

Ex 4.5



Find the max. stress shear stress at the fixed end of the above cantilever.



shearing stress =

$$\tau_{max} = \frac{VQ}{Ib}$$

$Q = \frac{1}{2} \times \frac{b}{2} \times \frac{b}{2} \times \frac{b}{2} = \frac{b^3}{8}$
 $I = \frac{1}{12} b c^3$
 $\tau_{max} = \frac{V \times \frac{b^3}{8}}{\frac{1}{12} b c^3 \times b} = \frac{3V}{4c}$

$$b = 2 \text{ in } c = 2 \text{ in}$$

$$\therefore b/c = 1$$

$$\therefore \alpha = 0.246 \text{ From table Page 167}$$

$$\tau_B = \frac{10 \times 2 \times 2 \times 1}{\frac{4^3 \times 2}{12} \times 2} + \frac{10}{0.246 \times 4 \times 2}$$

$$\tau_B = 1.875 + 2.54$$

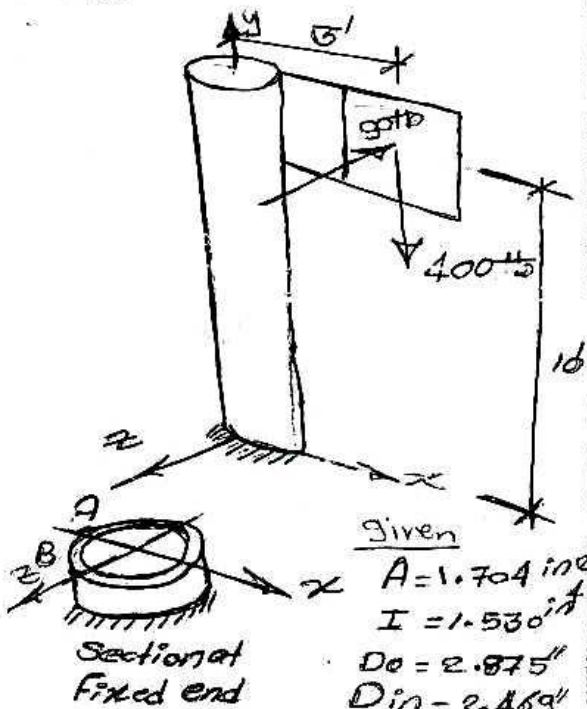
$$\tau_B = 4.4156 \text{ ksi}$$

$$\tau_A = 1.875 - 2.54$$

$$\tau_A = -1.352 \text{ ksi}$$

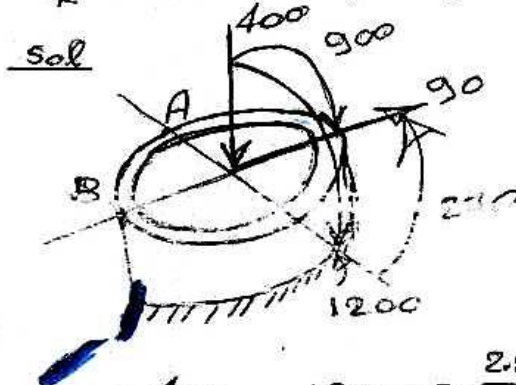


Ex 6



Req'd Find $\sigma_A, \sigma_B, \tau_A$ & τ_B

sol



$$\sigma_A = \frac{-400}{1.704} + \frac{1200 \times 12 \times \frac{2.875}{2}}{1.530}$$

$$+ 0 = -234.7 + 13529$$

$$= 13,295 \text{ PSI}$$

$$\sigma_B = \frac{-400}{1.704} + 0 + \frac{900 \times 12 \times \frac{2.875}{2}}{1.530}$$

$$= -234.7 + 0 + 10,147$$

$$= 9,912 \text{ PSI}$$

$$\tau_A = \frac{VQ}{Ib} + \frac{T \times C}{J}$$

$$Q = \frac{2.875^2 \pi}{4 \times 2} \times \frac{4 \times \frac{2.875}{2}}{3\pi} - \frac{2.469^2 \pi}{4 \times 2} \times \frac{4 \times \frac{2.469}{2}}{3\pi}$$

$$= 1.98 - 1.254$$

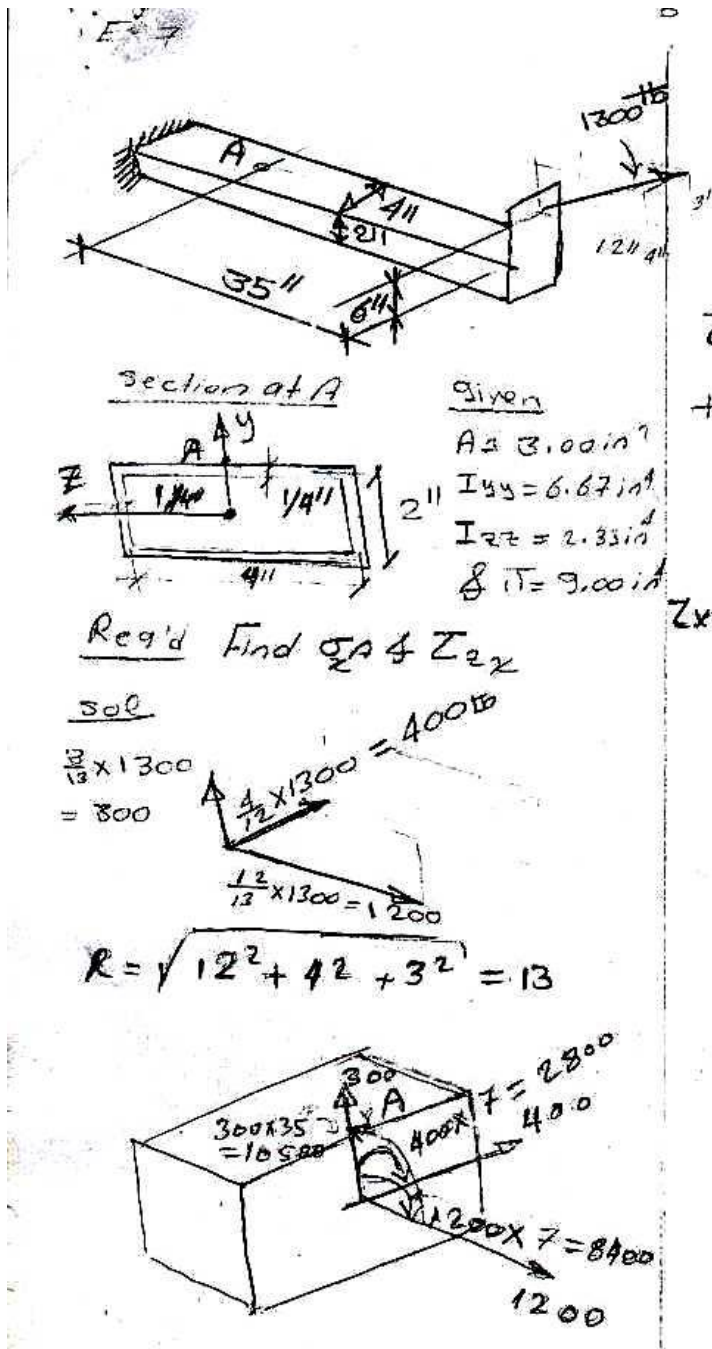
$$Q = 0.725$$

$$J = I_x + I_y = 2 \times 1.53 = 3.06$$

$$\tau_A = \frac{90 \times 0.725}{1.53 \times 2 \times 0.203} - \frac{270 \times 12 \times \frac{2.875}{2}}{3.06}$$

$$\tau_A = 105 - 1522 = -1417 \text{ PSI}$$

$$\tau_B = 0 + 1522 = 1,522 \text{ PSI}$$



$$\sigma_A = \frac{(10,500 - 8400) \times 1}{2.33} + \frac{1200}{3.00} = 501 \text{ Psi}$$

$$Z_{xz} = 0 \leftarrow \text{due to } 300 + 400 \times \left[(2 \times \frac{1}{4} \times 1) \times 2 + 2 \times \frac{1}{4} \times 2 \right] = 6.67 \times 0.5$$

$$+ \frac{2800}{2 \times [2 \times 4] \times \frac{1}{4}}$$

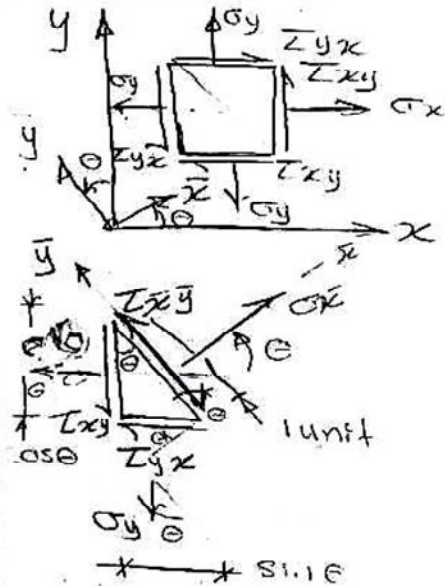
$$Z_{xz} = 239.9 + 700 = 940 \text{ Psi}$$



Chapter Nine Stress and Strain Transformation

Stress and Strain Transformation

Transformation of Stress



$$\sum F_x = 0$$

$$\sigma_{x'} \cdot 1 = (\sigma_x \cos \theta) \cos \theta + (\tau_{xy} \cos \theta) \sin \theta + (\sigma_y \sin \theta) \cos \theta + (\tau_{yx} \sin \theta) \cos \theta$$

$$\sigma_{x'} = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + 2\tau_{xy} \sin \theta \cos \theta$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$2 \sin \theta \cos \theta = \sin 2\theta$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sum F_y = 0$$

$$\tau_{x'y'} = -(\sigma_x \sin \theta) \cos \theta + (\tau_{xy} \cos \theta) \cos \theta + (\sigma_y \sin \theta) \sin \theta - (\tau_{yx} \sin \theta) \sin \theta$$

$$\tau_{x'y'} = -\sigma_x \frac{\sin 2\theta}{2} + \sigma_y \frac{\sin 2\theta}{2} + \tau_{xy} (\cos^2 \theta - \sin^2 \theta)$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$



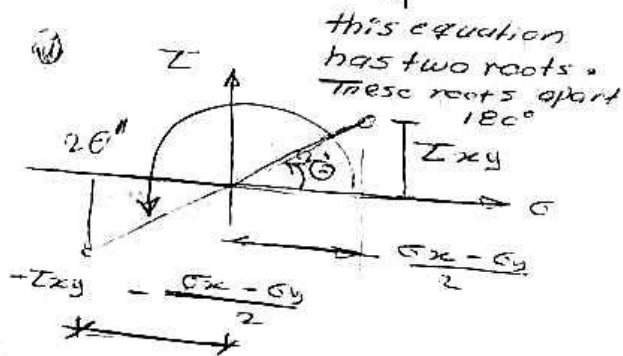
Principal stresses

$$\frac{d\sigma_x}{d\theta} = 0 = \frac{\sigma_x - \sigma_y}{2} \times 2 \sin 2\theta + \tau_{xy} \cos 2\theta = 0$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

$$\tan 2\theta = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} \quad \text{--- (3)}$$

Principal



To locate the plane of zero shear stress

$$0 = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\frac{\tan 2\theta}{\cos 2\theta} = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

$$\tan 2\theta = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} \quad \text{--- (4)}$$

Zero shear plane

on the planes on which max. or min. shear normal stresses occur, there are no shearing stress. these called Principal Planes of stresses.

Sub. the value of θ into eq. (1) to obtain the max. ~~shearing~~ normal stress

$$\sigma_{\max/\min} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{\sigma_x - \sigma_y}{2} \times \frac{(\sigma_x - \sigma_y)/2}{\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}} + \tau_{xy} \frac{\tau_{xy}}{\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}}$$

$$\sigma_{\max/\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{--- (5)}$$

Finally to find the max. shearing stress.

$$\frac{d\tau_{xy}}{d\theta} = -\frac{\sigma_x - \sigma_y}{2} \times 2 \cos 2\theta + \tau_{xy} \times 2 \sin 2\theta = 0$$

$$\tan 2\theta = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

max. shearing

Note that

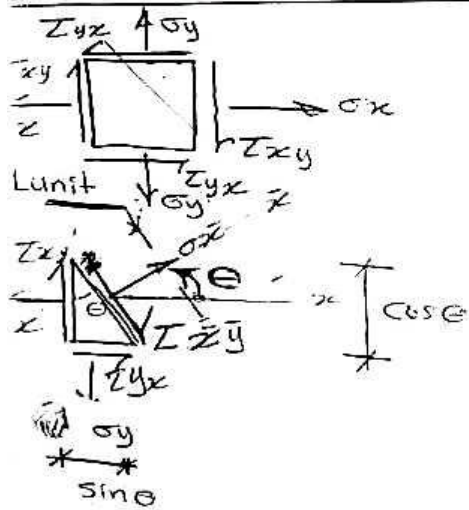
$$\tan 2\theta = \frac{-1}{\text{Principal } \tan 2\theta_{\max. \text{ shearing}}}$$

this means that the angles which locate the plane of maximum ~~normal~~ shearing stress form angles of 45° with plane of principal stresses.

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



Mohr's Circle for Stress transformation:-



$$\bar{\sigma}_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \quad \text{--- (a)}$$

$$\bar{\tau}_{xy} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \quad \text{--- (b)}$$

$$\bar{\sigma}_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\bar{\tau}_{xy} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \quad \text{--- (c)}$$

by squaring both these eqs.
adding the results we obtain

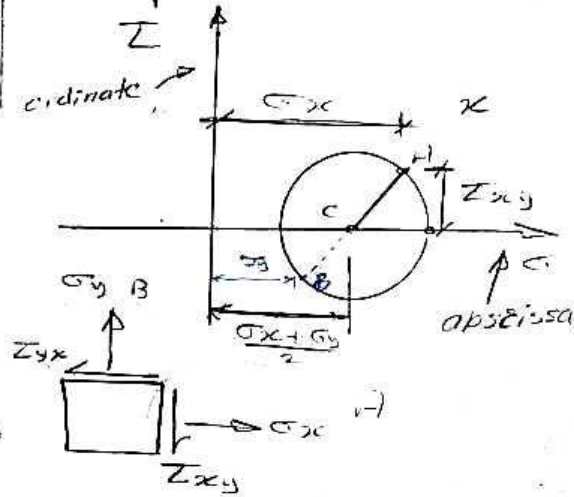
$$\left(\bar{\sigma}_x - \frac{\sigma_x + \sigma_y}{2} \right)^2 + (\bar{\tau}_{xy})^2 = \left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2$$

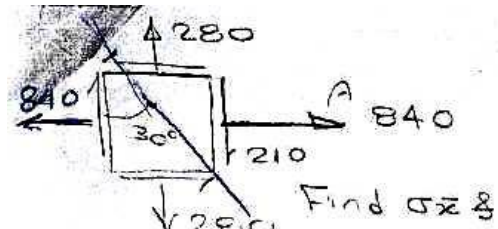
The above is a circle equation with center

$$C \left(\frac{\sigma_x + \sigma_y}{2} : 0 \right)$$

and radius

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$





sol ① Using the equations

$$\sigma_{\bar{x}} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$



$$+ \tau_{xy} \sin 2\theta$$

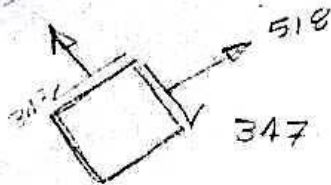
$$\sigma_{\bar{x}} = \frac{840 + 280}{2} + \frac{840 - 280}{2} \cos 60^\circ + (-210) \sin 60^\circ$$

$$\sigma_{\bar{x}} = 518$$

$$\tau_{\bar{x}\bar{y}} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{840 - 280}{2} \sin 60^\circ + (-210) \cos 60^\circ$$

$$\tau_{\bar{x}\bar{y}} = -347$$



$$\sigma_{\bar{y}} + \sigma_{\bar{x}} = \sigma_x + \sigma_y$$

$$\sigma_{\bar{y}} + 518 = 840 + 280 = 602$$

$$\sigma_{\bar{y}} = 602 - 518 = 84$$

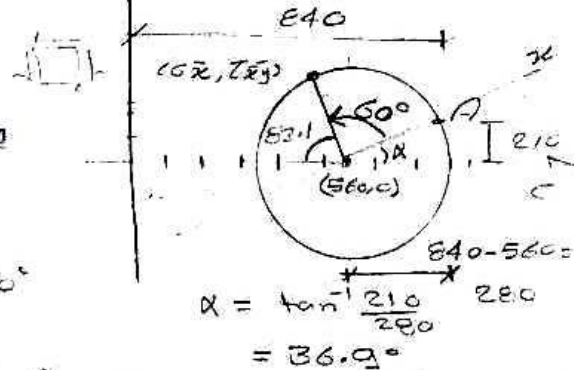
$$\sigma_{\bar{y}} = \frac{840 + 280}{2} - \frac{840 - 280}{2} \cos 60^\circ + (-210) \sin 60^\circ$$

$$= 602 \text{ (tension)}$$

② Using Mohr Circle

$$\text{Center} = \frac{840 + 280}{2}$$

$$= 560$$



$$\theta = 180 - 36.9 - 60 = 83.1$$

$$\sigma_{\bar{x}} = 560 - R \cos 83.1$$

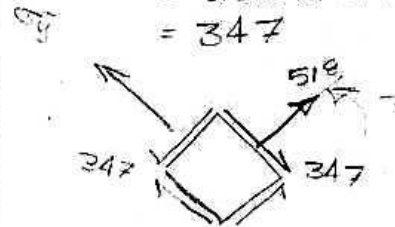
$$R = \sqrt{280^2 + 210^2} = 350$$

$$\sigma_{\bar{x}} = 560 - 350 \cos 83.1 = 518$$

$$\sigma_{\bar{x}} = 518 \text{ (tension)}$$

$$\tau_{\bar{x}\bar{y}} = R \sin 83.1$$

$$= 350 \times \sin 83.1 = 347$$

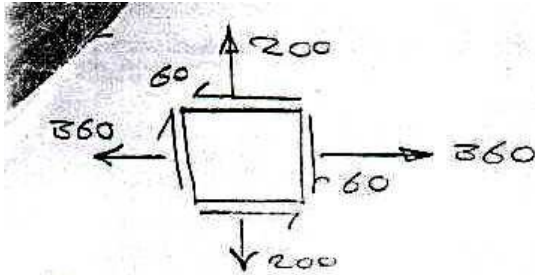


Note that

$$\sigma_{\bar{y}} + \sigma_{\bar{x}} = \sigma_x + \sigma_y = 602$$

$$\sigma_{\bar{y}} + 518 = 840 + 280$$

$$\sigma_{\bar{y}} = 602$$



Find σ_1 & σ_2 & its orientation

solⁿ ① Using the eqn.s

$$\sigma_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$+ \tau_{xy} \sin 2\theta$$

$$\frac{d\sigma_x}{d\theta} = 0 = (\sigma_x - \sigma_y) \sin 2\theta$$

$$+ 2\tau_{xy} \cos 2\theta$$

$$\tan 2\theta = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

$$\tan 2\theta = \frac{-60}{\frac{360 - 200}{2}}$$

$$= -0.75$$

$$2\theta = -36.9^\circ \leftarrow \text{principal stress}$$

$$\text{or } 2\theta = -36.9^\circ + 180^\circ$$

$$= 143^\circ$$

$$\therefore \theta = -18.45^\circ$$

or

$$\theta = 71.5^\circ$$

$$\sigma_1 = \frac{360 + 200}{2} + \frac{360 - 200}{2} \cos 36.9^\circ$$

$$\theta = -18.45^\circ$$

$$- 60 \sin(-36.9^\circ)$$

$$\sigma_1 = 380$$

$$\theta = -18.45^\circ$$

$$\sigma_2 = \frac{360 + 200}{2}$$

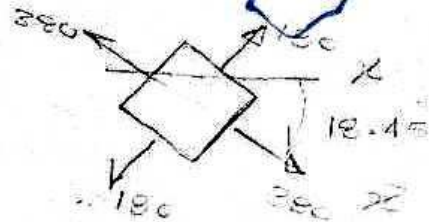
$$- 71.5^\circ$$

$$+ \frac{360 - 200}{2} \cos 143^\circ - 60 \sin 143^\circ \quad \sigma_{min} = 280 - 100 = 180$$

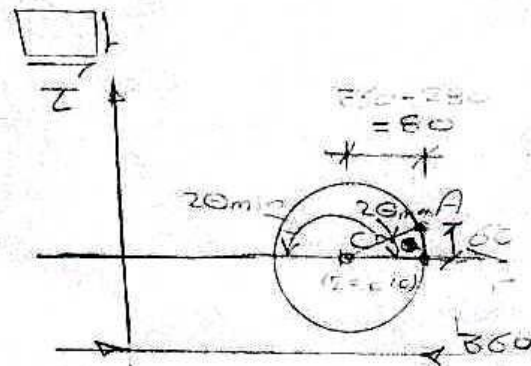
$$\sigma_1 = 180$$

$$\sigma_{max} = 380$$

$$\sigma_{min} = 180$$



② Using the Mohr Circle



$$C = \frac{360 + 200}{2} = 280$$

$$2\theta_{max} = \sin^{-1} \frac{60}{R}$$

$$R = \sqrt{80^2 + 60^2} = 100$$

$$2\theta_{max} = \sin^{-1} 0.6 = 36.9^\circ$$

$$\theta_{max} = -18.43^\circ$$

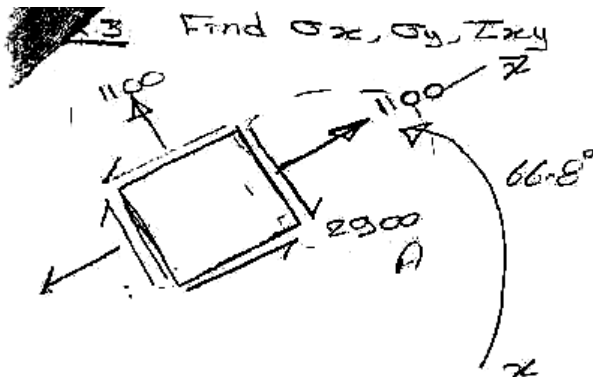
$$2\theta_{min} = 180 - 36.9^\circ = 143.1^\circ$$

$$\theta_{min} = 71.6^\circ$$

$$\sigma_{max} = 280 + R$$

$$= 280 + 100 = 380$$

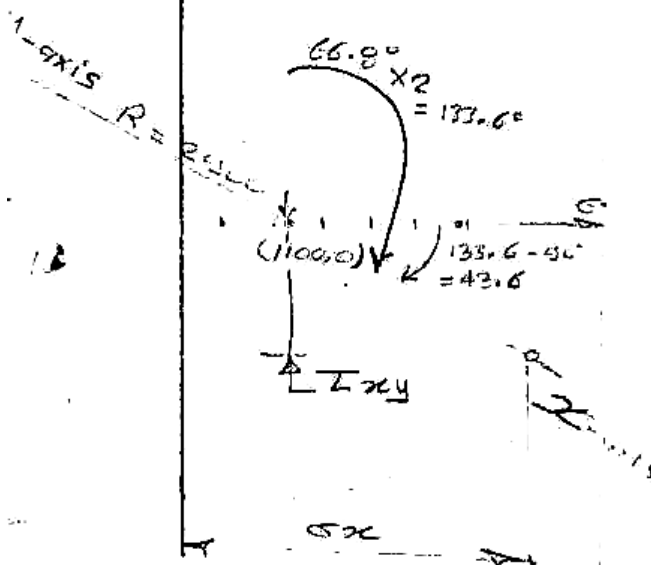
$$\sigma_{min} = 280 - 100 = 180$$



$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_{\bar{x}} + \sigma_{\bar{y}}}{2}$$

$$C = 1100$$

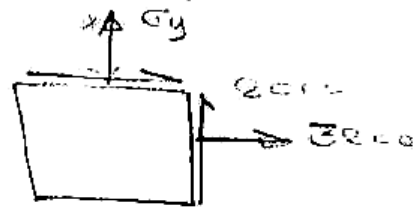
$$R = \sqrt{\frac{(1100 - 1100)^2}{2} + 2900^2} = 2900$$



$$\sigma_x = 2900 \times \cos 43.6^\circ + 1100 =$$

$$\sigma_x = 3200$$

$$\tau_{xy} = -2900 \times \sin 43.6^\circ = -2000$$



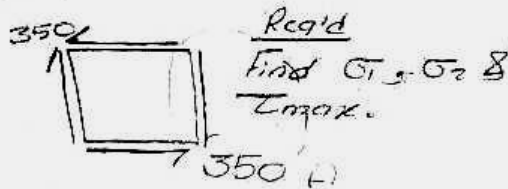
$$\sigma_y + \sigma_x = \sigma_{\bar{x}} + \sigma_{\bar{y}}$$

$$\sigma_y + 3200 = 1100 \times 2$$

$$\sigma_y = -1000$$



Stress Transformation of
2558-
4. Pure Shear

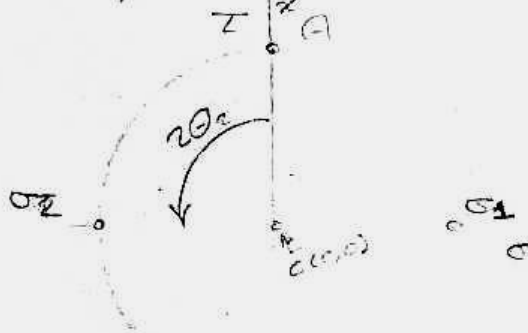


Solution Using Mohr's Circle

$$\text{center} = \frac{\sigma_x + \sigma_y}{2} = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

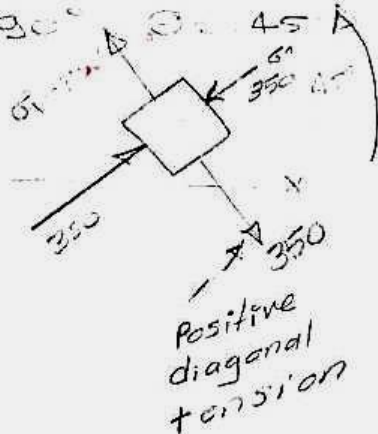
$$R = \sqrt{0 + 350^2} = 350$$



$$\sigma_1 = R = 350 \text{ T}$$

$$\sigma_2 = R = 350 \text{ C}$$

$$2\theta_2 = 90^\circ$$



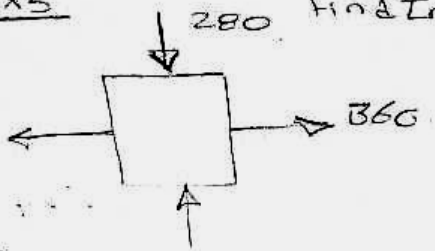
عن هذه الماكينة تقاس
معدل متجه الجهد في عتود
معدل ميل عن الأفق عتود
في. حيث على هذا المخطط
التي إلى diagonal tension
التي درجة 45° ذلك
المستطيل

في التقاطع للخط ل أعلاه تم حساب
شروط إلى max. normal
التي Rankine

Biaxial Stress

Ex 5

280 Find $L_{\max}$



$$\frac{Sc^0}{C_2} = C_1 = 360$$

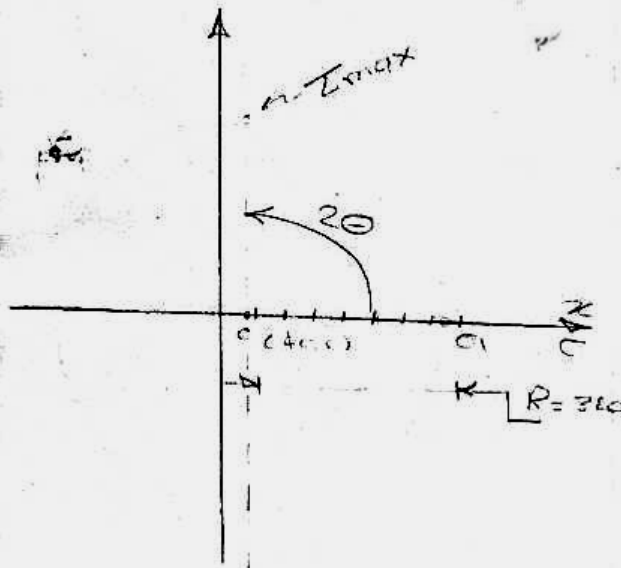
$$T_2 = 40 - 280$$

$$C = \frac{G_1 + G_2}{2} \text{ or } \frac{Gx + Gy}{2}$$

$$C = \frac{360 - 2.80}{2} = 40$$

$$R = \sqrt{\left(\frac{500 - 50}{2}\right)^2 + \left(\frac{100}{0.4}\right)^2}$$

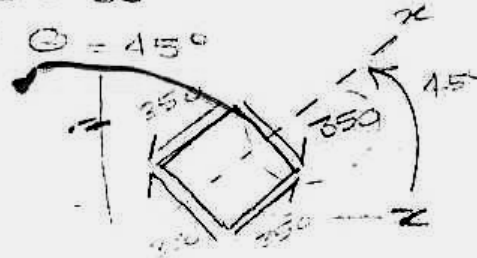
$$= \sqrt{\left(\frac{360 + 250}{2}\right)^2} = 320$$



$$T_{max} = R = 320$$

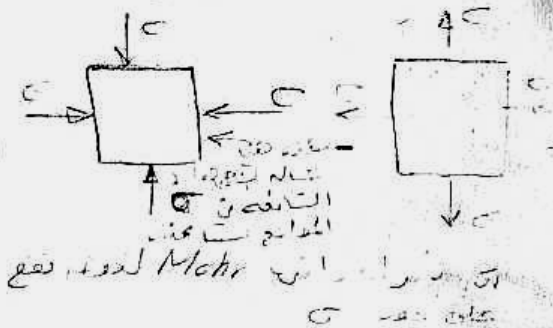
$$2\theta = 90$$

$$\text{Ans } C = 45^\circ$$



Stress -
can be applied to stress
state defined as
ductile material

Shen ١٠٠٠





Chapter Fourteen Buckling of Columns

Columns may be subdivided into groups

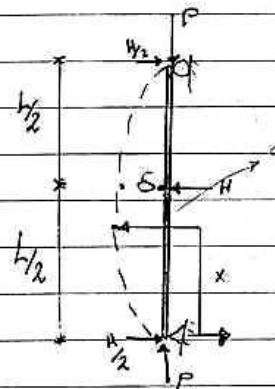
1. Long Columns
 2. Intermediate Columns
- Sombody else add third group 'Short Columns' such as bearing blocks.

Long Columns

Critical load

$$M_{cr} = P \delta$$

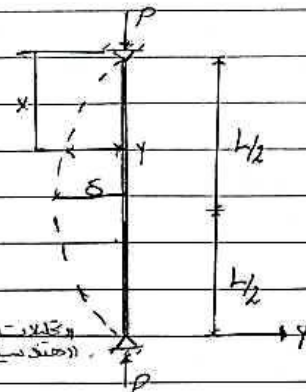
$$EI \frac{d^2 y}{dx^2} = M \quad (\text{from chapter 11})$$



$$EI = \frac{d^2 y}{dx^2} = P(-y) = -Py$$

$$EI = \frac{d^2 y}{dx^2} = -Py$$

This is second order ordinary differential equation



Solution of 2nd order diff equation

$$\text{let } ay'' + by' + cy = 0$$

a second order homogeneous diff equation compare with the above equation

$$EI \frac{d^2 y}{dx^2} = -Py$$

1



$$EI \frac{dy}{dx^2} + py = 0$$

$$ay'' + by' + cy = 0$$

let the solution is $y = e^{mx}$
where m is arbitrary constant sub
the solution into d.e.

هذا هو الحل
ان نجرب الحلات العامة

$$y = e^{mx}$$

$$y' = me^{mx}$$

$$y'' = m^2 e^{mx}$$

$$am^2 e^{mx} + bme^{mx} + ce^{mx} = 0$$

$$e^{mx} (am^2 + bm + c) = 0$$

$$e^{mx} \neq 0$$

$\therefore am^2 + bm + c = 0$ characteristic equation

This al-gebric equation may be solved by:-

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m_1, m_2$$

$$y_1 = e^{m_1 x}, y_2 = e^{m_2 x} \Rightarrow y(x) = C_1 y_1 + C_2 y_2$$

~~if~~

$$y(x) = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$



if $b^2 - 4ac > 0$ (two solutions y_1, y_2)

$b^2 - 4ac = 0$ (one solution $y_1 = y_2$)

$b^2 - 4ac < 0$ (two imaginary solutions)

in our problems

$b = 0$

$a = EI$ (+ve)

$c = P$ (+ve)

$\therefore b^2 - 4ac = 0 - 4ac = -ve$ always.

hence, we have two imaginary roots:-

$$\begin{aligned} m_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0 \pm \sqrt{4ac(-1)}}{2a} \\ &= \pm \frac{2\sqrt{ac}}{2a} \sqrt{-1} \\ &= \pm \frac{2\sqrt{ac}}{2a} i \quad \text{where } i = \sqrt{-1} \\ &= p \pm q i \end{aligned}$$

~~$$y(x) = e^{p_1 x} e^{q_1 i x}$$~~

$$y(x) = C_1 e^{p_1 x} + C_2 e^{-p_1 x}$$

$e^{i\theta} = \cos\theta + i\sin\theta$	rules of complex numbers
$e^{-i\theta} = \cos\theta - i\sin\theta$	



$$\therefore y(x) = C_1 [\cos qx + i \sin qx] + C_2 [\cos qx - i \sin qx]$$

$$y(x) = (C_1 + C_2) \cos qx + (C_1 - C_2) i \sin qx$$

$$EI \frac{d^2 y}{dx^2} + Py = 0$$

$$a = EI, \quad c = P$$

$$\therefore q = \sqrt{\frac{c}{a}} = \sqrt{\frac{P}{EI}}$$

$$\therefore y(x) = A \cos \sqrt{\frac{P}{EI}} x + B \sin \sqrt{\frac{P}{EI}} x$$

Apply the Boundary Conditions:-

$$\text{at } x=0, \quad y=0$$

$$0 = A(1) \Rightarrow A = 0$$

$$\therefore y(x) = B \sin \sqrt{\frac{P}{EI}} x$$

$$\text{at } x=L, \quad y=0$$

$$0 = B \sin \sqrt{\frac{P}{EI}} L$$

$B \neq 0$ (trivial solution)

$$\sin \sqrt{\frac{P}{EI}} L = 0$$

$$\therefore \sqrt{\frac{P}{EI}} L = n\pi \quad (n=1, 2, 3, \dots)$$

For $n=1$

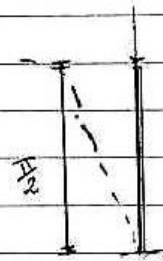
$$\sqrt{\frac{P}{EI}} L = \pi$$

-4-



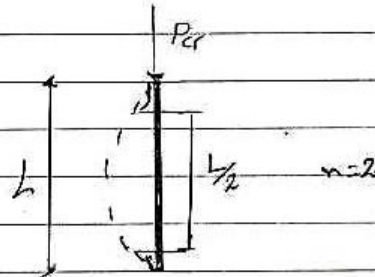
$$\frac{P}{EI} = \frac{\pi^2}{L^2}$$

$\therefore P_{cr} = \frac{\pi^2 EI}{L^2}$ Euler load
Euler equation
is Swiss 1757
mathe.



$$P_{cr} = \frac{EI(\frac{\pi}{2})^2}{L^2}$$

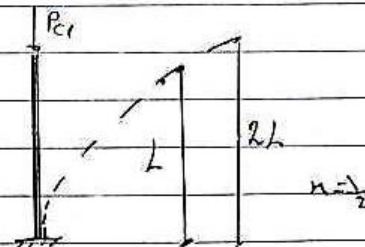
$$= \frac{4EI\pi^2}{L^2}$$



Both ends fixed

$$P_{cr} = \frac{EI(\frac{\pi}{2})^2}{L^2}$$

$$= \frac{EI\pi^2}{4L^2}$$

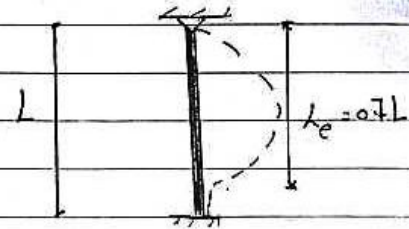


one end free
and other fixed



$$P_{cr} = \frac{EI \pi^2}{L_e^2}$$

$$= \frac{EI \pi^2}{(0.7L)^2} = \frac{2EI \pi^2}{L^2}$$



$$n = \sqrt{2}$$

one end Hinge
other fixed.

End condition	N	L_e = effective Length
Both end Hing	1	L
Fixed ends	4	$\frac{1}{2}L$
one end hinged other fixed	2	$0.7L$
one end free & other fixed.	$\frac{1}{4}$	$2L$

N = number of times strength of hinged columns.

$$P = \frac{EI \pi^2}{L^2}$$

$$\frac{P}{A} = \frac{E \frac{I}{A} \pi^2}{L^2}$$

$$\frac{P}{A} = \frac{E r^2 \pi^2}{L^2}$$

$$\frac{P}{A} = \frac{E \pi^2}{\left(\frac{L}{r}\right)^2}$$

Take a steel columns

allowable stress in steel = 200 mpa



$$\frac{P}{A} = \sigma_{all}$$

$$200 = \frac{200 \times 10^3 (\pi)^2}{\left(\frac{L}{r}\right)^2}$$

$$\left(\frac{L}{r}\right)^2 = 10000$$

$$\frac{L}{r} = 100$$

Ex

Select the ^{Lightest} ~~Lightest~~ shape that can be used as a column 7m long to support an axial load of 450 kN with a factor of (3). assume (a) both ends Hinged and (b) one end fixed and the other Hinged use T_{all} steel
200 MPa, $E_{steel} = 200 \times 10^3$ MPa.



design problem

Ex: select the lightest W shape that can be used as a column 7 m long to support an axial load of 450 kN with a factor of safety of 3. assume (a) both ends hinged and (b) one end fixed and the other hinged use A36 steel = 250 MPa
 $E_{\text{steel}} = 200 \times 10^3 \text{ MPa}$

Solution:

$$L = 7.0 \text{ m}$$

$$P = 450 \text{ kN}$$

$$F.O.S. = 3.0$$

$$\sigma_{PL} = \sigma_{yield} = 250 \text{ MPa}; E = 200 \text{ GPa}$$

(Safety Factor) $S.O.F. = \frac{P_u}{P}$
 $S.O.F. = 3$

$$P_u = P \times S.O.F.$$

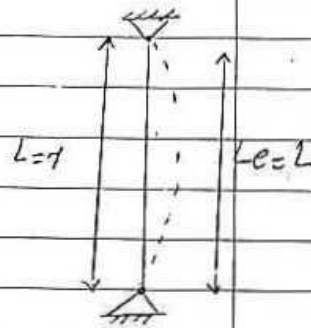
$$= 450 \times 3$$

$$= 1350 \text{ kN}$$

$$P_{cr} = \frac{EI \pi^2}{L^2}$$

$$1350 \times 10^3 = \frac{200 \times 10^3 \times I \times (3.14)^2}{(7 \times 1000)^2}$$

$$I = 33.5 \times 10^6 \text{ mm}^4$$





$$A = \frac{P}{E \pi r^2}$$

$$200 = \frac{200 \times 10^3 \times (3.14)^2}{(4r)^2}$$

$$\frac{L}{r} \approx 100$$

$$r = \frac{L}{100} = \frac{7 \times 1000}{100} = 70 \text{ mm}$$

select a steel w shape section that have

$$I \geq 33.5 \times 10^6 \text{ mm}^4$$

$$r \leq 70 \text{ mm}$$

(r) is the radius of gyration
of the section about the axis of bending

select w 250 x 73

depth 159 mm

this give $I = 38.2 \times 10^6 \text{ mm}^4$
 $r = 64.6 \text{ mm}$

give I about the axis of bending
about y



The section must be checked for compressive stresses $\sigma_{pc} = 200 \text{ MPa}$

$$\sigma_{pc} = \frac{P_u}{A} \Rightarrow A = \frac{P_u}{\sigma_{pc}} = \frac{1350 \times 1000}{200}$$

$$A = 6750 \text{ mm}^2$$

أيضا يجب ان نتحقق من حمل Euler الذي يجب ان يكون أكبر من الحمل المطبق $P = \frac{E \pi^2 I}{(\frac{L}{r})^2}$

$$A \geq \frac{\frac{L}{r} \times P}{E \pi^2}$$

select a steel W shape section that have:

$$A \geq 6750 \text{ mm}^2$$

$$r \geq 76 \text{ mm}$$

This will be satisfied by selection of:

$$W 310 \times 99$$

$$A = 12300 \text{ mm}^2$$

$$r = 76.9 \text{ mm}$$

O.K.

d n l



$$P_{cr} = \frac{E I \pi^2}{(L_e)^2}$$

$$1350 \times 10^3 = \frac{200 \times 10^3 \times I \times (3.14)^2}{(0.7 \times 7 \times 1000)^2} \quad L=7m$$

$$I = 16.4 \times 10^6 \text{ mm}^4$$

$$\frac{L}{r} = 100 \quad (\text{steel}) \quad (\text{as in (a) before})$$

$$r = \frac{0.7 \times 7 \times 1000}{100} = 49.0 \text{ mm}$$

select a W shape have:

$$I > 16.4 \times 10^6 \text{ mm}^4$$

$$r < 49.0 \text{ mm}$$

use W 360 x 64 have:

$$I = 18.8 \times 10^6 \text{ mm}^4$$

$$r = 48.1 \text{ mm}$$

The section must be checked for compression stress $\leq \sigma_{pl} = 200 \text{ MPa}$

$$A = \frac{P_u}{\sigma_{pl}} = \frac{1350 \times 1000}{200} = 6750 \text{ mm}^2$$

select a steel W shape that have:

$$A \geq 6750 \text{ mm}^2$$

$$r > 49 \text{ mm}$$



W 250 * 38 that have $A = 7420 \text{ mm}^2$

$$A = 7420 \text{ mm}^2$$

$$r = 54.4 \text{ mm}$$

Intermediate Columns:

Using Empirical Formulas

1. Extending Euler Formula to cover the intermediate columns

$$\frac{P}{A} = \frac{\bar{E} \pi^2}{(L/r)^2}$$

$\bar{E}$ - is the tangent modulus is less than the secant or initial tangent modulus (E)

$$\bar{E} < E$$

2. Tetmajer and Buss T. H. Johnson, 1886

$$\frac{P}{A} = \sigma = C - \frac{L}{r}$$

C - slope

σ - stress @ $\frac{L}{r} = 0$



3 Tetmajer and Bauschinger

$$\frac{P}{A} = 330 - 1.45 \frac{L_c}{r} \quad (\text{MPa L/100mm})$$

main formula

when using a factor of safety of 3.0

$$\frac{P}{A} = 110 - 0.483 \frac{L_c}{r}$$

