# *Stress and Strain Ellipsoids*

*Stress Ellipsoid: Is the graphical representation of stress, is the three dimensional figure with three principal axes representing the principal stress axes (the greatest, the intermediate and the least). All the three principal stresses can be compressive or tensile or combination.* 



*Strain ellipsoid (deformation ellipsoid): Are the three dimensional figure enclosing three perpendicular strain axes which are:*

- *ϵ 1: the greatest strain axes (largest deformation axes).*
- *ϵ 2: the intermediate strain axes (intermediate deformation axes).*
- *ϵ 3: the least strain axes (shortest deformation axes).*



## *Relation of rupture planes to strain ellipsoids*

*A convenient way of visualizing deformation is to imagine the change in shape of an imaginary sphere in the rocks. The most general solid resulting from the deformation of a sphere is an ellipsoid. This imaginary figure may be called the strain ellipsoid or the deformation ellipsoid.* 

*1- Tension fractures: are parallel to the plane that contains the least and intermediate strain axes and the fracture plane is perpendicular to the greatest strain axes.* 

*2- If all of the strain ellipsoids are known the position of tension fractures can be predicted and vise versa, if the tension fracture is known the strain ellipsoid orientation might be known.* 

*3- If two sets of shear fractures are present and are the product of the same deformation, the line formed by their intersection is parallel to the intermediate axes of the strain ellipsoid. Moreover, the least strain axis bisects the acute angle between the shear fractures.* 



*The strain ellipsoid doesn't give us a direct evidence of the external forces that caused the deformation! Because an ellipsoid may be formed from a sphere by:* 

- *Simple compression (A)*
- *Tension (B)*
- *Couple forces (C)*

*This can be illustrated in two dimensions figure where the intermediate axis is perpendicular to the page.* 



# *Homogeneous and heterogeneous deformation*

*Where the deformation applied to a rock volume is identical throughout that volume, the deformation is homogeneous. Rigid rotation and translation by definition are homogenous, so it is always strain and volume or area change that can be heterogeneous (the amount of strain in all parts of body is equal).*

*Thus homogeneous deformation and homogeneous strain are equivalent expressions. For homogeneous deformation, originally straight and parallel lines will be straight and parallel also after the deformation.* 

*Further, the strain and volume/area change will be constant throughout the volume of rock under consideration.*

#### *Homogeneous deformation:*

- *Straight lines remain straight.*
- *Parallel lines remain parallel.*
- *Circles (spheres in 3D) become ellipses (ellipsoids in 3D)*



#### *Inhomogeneous strain (Heterogeneous deformation)*

*The amount of strain in different parts of the body is unequal.* 

- *Straight lines become curved.*
- *Parallel lines become non parallel.*



### *Pure shear and simple shear*

*Pure shear (non-rotational) is a perfect coaxial deformation. This means that a marker that is parallel to one of the principal axes has not rotated away from its initial position.*

*Uniaxial strain, where the rock shortens or extends in one direction, is another example of coaxial deformation.*

*Coaxial deformation implies that lines along the principal strain axes have the same orientation as they had in the undeformed state (the orientation of the principle strain axes X and Y have not changed during the deformation with no volume change).*



#### *Simple shear (rotational)*

*Simple shear is a special type of constant volume plane strain deformation. There is no stretching or shortening of lines or movement of particles in the third direction. Unlike pure shear, it is a non-coaxial deformation, meaning that lines parallel to the principal strain axes have rotated away from their initial positions.*



*For non-coaxial deformations, the orientations of the principal strain axes are different for different amounts of strain, while for coaxial deformations they always point in the same directions (same orientation, different lengths).*

## *Progressive deformation*

*The finite strain is achieved by adding successive strain increments (additions) to the initial unstrained shape.* 

strain path **Final** state Initial state (undeformed) (deformed)

### *Strain markers*

*Are objects or features in rocks that indicate the finite strain?*



*We place a marble within a cube of clay, and as we load the block it deforms into a rectangular box. After sectioning, the marble remains undistorted! So, using the marble as the basis for strain analysis we conclude there is zero percent strain (or X/Y = 1), yet looking at the clay you clearly see that strain has accumulated. In fact, we determine a strain ratio, X/Y, of 1.8 for the clay.*

*Now we carry out a second experiment, in which we replace the space occupied by the marble with a fluid. The result is quite different from that of the previous run. In the second case both materials show finite strain, but the strain measured from the bubble, which now has the shape of an ellipsoid, is higher than that measured from the shape of the deformed clay block.*

*There is zero strain for the marble, and finite strain for the fluid bubble is greater than that for the clay block. These different results simply reflect the response of materials with different strengths. The clay is weaker than the marble, but the fluid bubble is weaker than the clay. We therefore identify strain markers of two types: passive (inactive) and active markers.*



*Passive strain markers: are elements in the body that have no mechanical contrast or differences; they deform in a manner indistinguishable (unclear) to that of the whole body.*

*Such markings are rare in nature, but inclusions of the same composition as the matrix are close to this condition; for example, quartz grains in a quartzite in a carbonate. In the case of passive markers, we say that our body behaves as a homogeneous system for strain.*

*Active strain markers: are elements in the body that have mechanical contrast with their matrix and may behave quite differently.*

*The marble and fluid inclusions in the above experiments are both examples of active markers.*

*Conglomerate clasts in a shale matrix or garnets in mica schist's are natural examples of active strain markers, which represent a heterogeneous system for strain.*

### *Strain distribution in the hinge zone of a folded layer*

*When we bend a layer of clay or a metal bar we obtain a fold geometry that seems identical to one produced by flexural folding, but with a distinctly different strain pattern. This is illustrated by tracking the distortion of circles drawn on the sides of the undeformed layer.* 

*On all three surfaces we find that circles have become ellipses, including the folded top and bottom surface. On the top folded surface, the long axis of each ellipse is perpendicular to the hinge line, but on the bottom the long axis is parallel to the hinge line.* 

*In the profile plane the long axis is parallel or perpendicular to the top and bottom surfaces of the folded layer, depending on where we are in that plane.* 

*There must, therefore, be a surface in the fold where there is no strain. This zerostrain surface gives the model its name, neutral-surface fold.* 

*Because strain accumulates in the folded surfaces during neutral surface folding, the* 

*orientation of any features on these surfaces changes with position in the fold. In the outer arc an initial angle with the hinge line increases, while in the inner arc this angle decreases. Only in the* 

*neutral surface is the angle unchanged. Note that the position of the neutral surface is not restricted to the middle of the fold, nor does it necessarily occur at the same relative position across the fold.*



