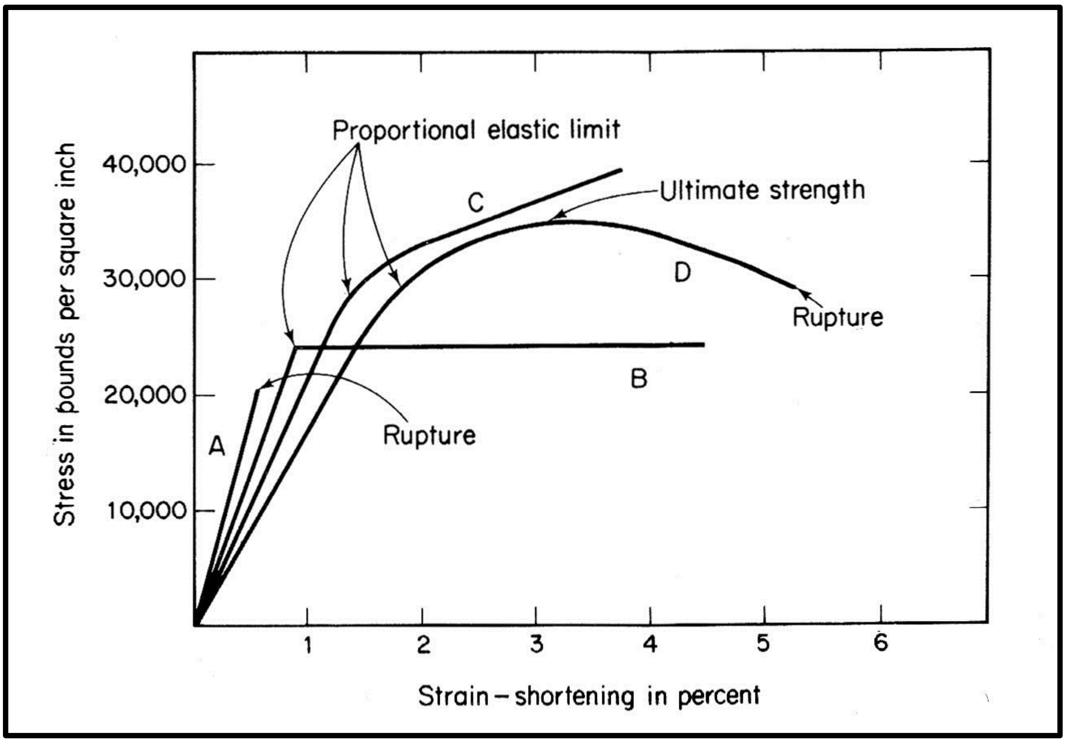
Stress – Strain Diagrams



The relation existing between stress and strain is commonly expressed in graphs known as stress-strain diagrams. The stress is plotted on the ordinate (vertical axis), whereas the strain is plotted on the abscissa (horizontal axis). In the above Fig. the material is under compression and the compressive stress parallel to the axis of the cylinder is in pounds per square inch. With increasing stress the specimen becomes shorter and the strain is plotted in terms of the percentage of the shortening of the specimen.

Curve A *is the stress-strain diagram of a brittle substance. It deforms elastically up to a stress of 20,000 lb/in.^{2} and has shortened one-half of one percent; it then fails by rupture.*

***Curve* B** *is an ideal plastic substance. It behaves elastically at first. At a stress of about 24,000 lb/in.^{2} it reaches the* proportional elastic limit, *which is the point at which the curve departs from the straight line.*

The shortening is slightly less than one percent. Thereafter the specimen deforms continuously without any added stress.

Curve C *represents a more normal type of plastic behavior. At a stress of about 28,000 lb/in.^{2} and a strain of somewhat over 1 percent, the specimen reaches the proportional elastic limit and thereafter deforms plastically. But for every increment of strain an increase in stress is necessary. This is the result of what is called* work hardening; *that is, the specimen becomes progressively more difficult to deform.*

Curve D *represents a very common type of plastic deformation. The specimen deforms elastically up to a stress of about 28,000 lb/in .^{2} and a shortening of somewhat less than 2 percent. At first an increase in stress is necessary for continued deformation. But when the shortening is somewhat over 3 percent, progressively less stress is necessary to continue the deformation.*

This high point on the curve is the **ultimate strength**. *However, the ultimate strength of a rock is a function of many variables, such as confining pressure and temperature.*

*The term "**strength**" is a rather meaningless term unless all the environmental conditions are specified. The value of the breaking strength—normally applied to brittle materials under temperature and pressures at the surface of the earth.*

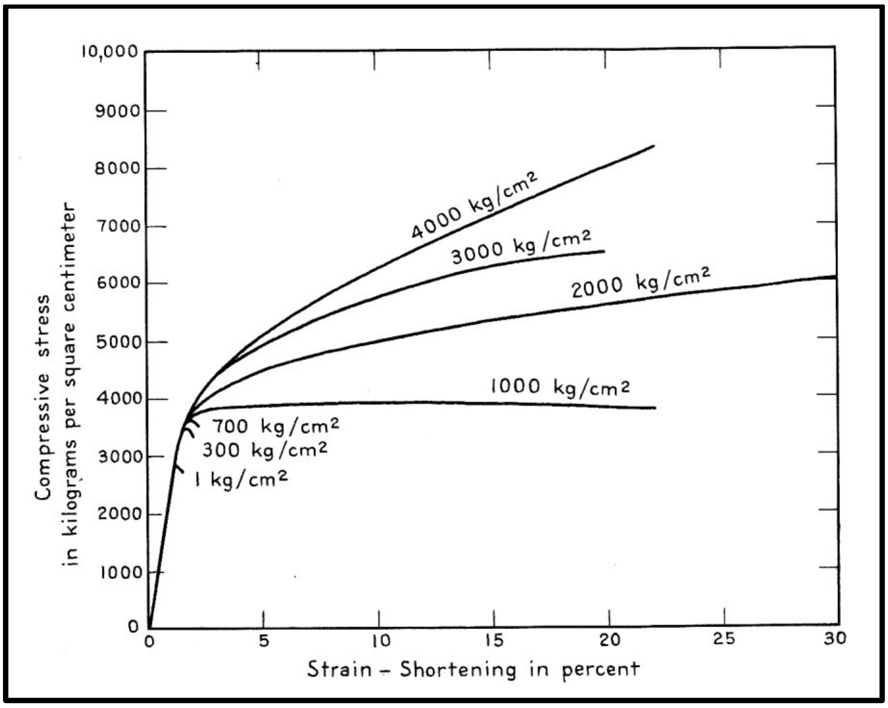
Factors controlling behavior of materials

The mechanical behavior of rocks is controlled not only by their inherent properties—mineralogy, grain size, porosity, fractures, etc.—but also by factors that are of little or no concern in planning man-made structures at the surface of the earth. These factors are confining pressure, temperature, time, and solutions.

***Inherent properties**: are properties required during the formation of the rock like mineralogy, grain size (texture), porosity, fractures, etc.*

1- Confining Pressure:

Cylinders of rock are prepared for the experiments; usually the length is several times the diameter. In some experiments the cylinders are small, the length not exceeding one or a few inches. By using small cylinders it is possible to employ both high confining pressures and high temperatures.



The above figure illustrates the behavior of Solenhofen Limestone under such conditions. The compressive stress on the ends of the cylinder is given on the ordinate in kilograms per square centimeter. The percentage of shortening of the cylinder is given on the abscissa.

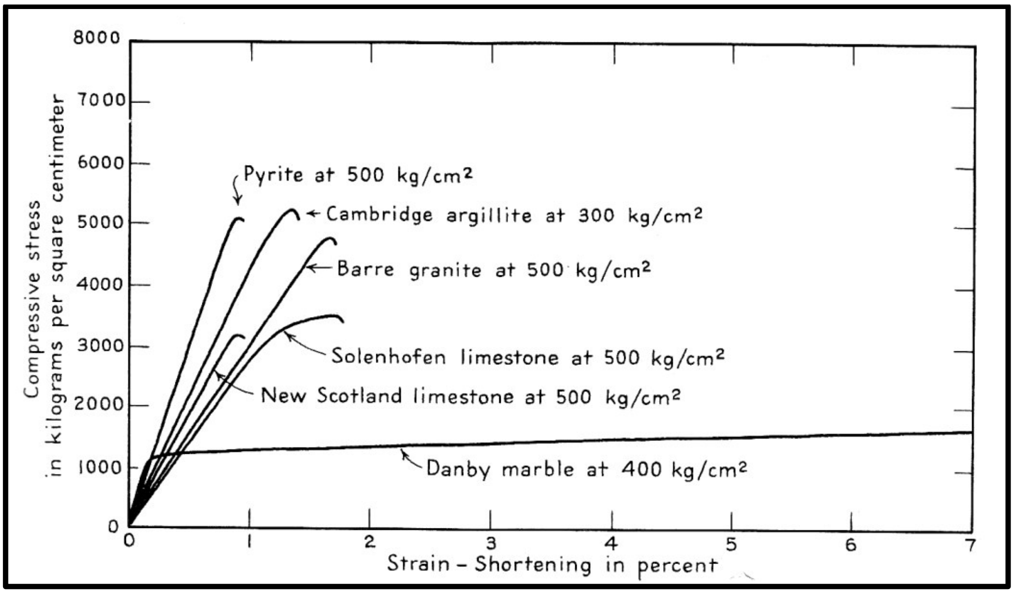
- *Seven separate experiments are shown at confining pressures of **1**, **300**, **700**, **1000**, **2000**, **3000**, and **4000** kilograms per square centimeter. Separate curves are given for the behavior at each of these confining pressures.*

- *Below a compressive stress of **3700** kg/cm2 the curves run together and appear as one.*
- *One experiment was run in air so that the confining pressure was equal to **1** kg/cm^2, that is, 14.7 lb/in.^{2}, or **1** atmosphere.*
- *This specimen behaved elastically up to a compressive stress of 2800 kg/cm^2, when it failed by rupture.*
- *The specimens tested under confining pressures of 300 and 700 kg/cm2 deformed elastically, went through a short stage of plastic deformation—that portion of the lines that is bending—and then failed by rupture.*
- *The specimens tested under confining pressures of 1000 or more kg/cm^2 began to deform plastically at a compressive stress of about 4000 kg/cm^2 and continued to deform plastically.*
- *The specimen tested under a confining pressure of 2000 kg/cm^2 had shortened **30** percent.*
- *When the test was terminated. The curves representing the tests at a confining pressure of 1000, 2000, 3000, and 4000 kg/cm^2 end, **not because of failure** by rupture, but because the tests were not carried any further.*

"Such experiments indicate that rocks exhibiting very little plastic defor-mation near the surface of the earth may be very plastic under high confining pressure".

It is also readily apparent that the strength increases with the confining pressure.

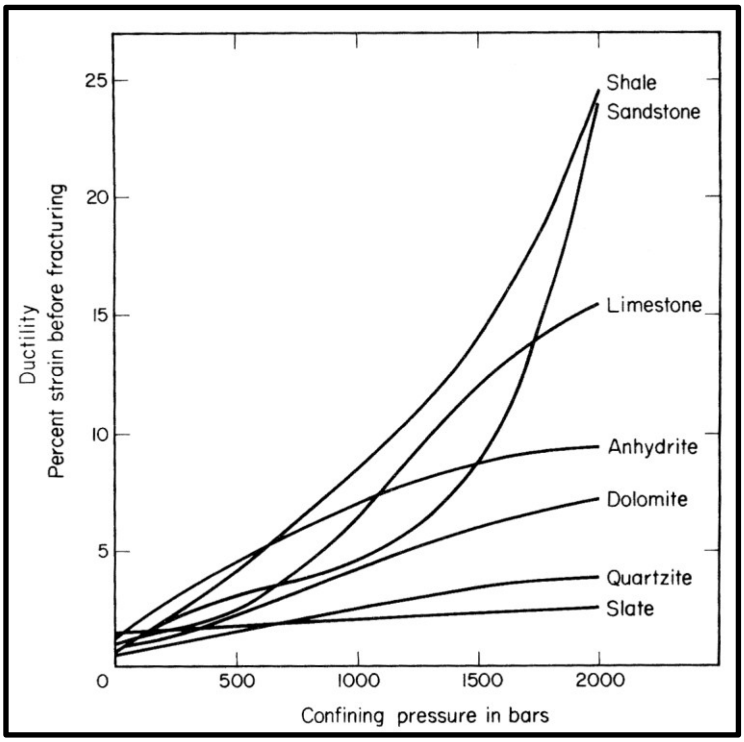
***Different rocks, of course, behave differently**. The following figure shows the stress-strain diagram for several rocks and one mineral. The results are not strictly comparable because, as the figure shows, the confining pressure was not the same in all experiments, ranging from 300 to 500 kg/cm^2. Pyrite, Cambridge Argillite, and Barre Granite are relatively* *brittle rocks, which behave elastically up to a compressive stress of over 4500 kg/cm^2. Above the elastic limit there is a small zone of plastic deformation, and then rupture takes place. New Scotland limestone was elastic up to a compressive stress of about 3000 kg/cm^2, deformed plastically for a short interval, and then ruptured at 3200 kg/cm2. Solenhofen Limestone shows a still larger range of plastic deformation. Danby Marble is much weaker. It deforms elastically up to a compressive stress of 1000 kg/cm^2 and then deforms plastically. Although the curve scale ends at 7 percent, the original data show that the specimen shortened 14 percent before the test was ended.*



The figure below illustrates the effect of confining pressure on the breaking strength of several different rocks.

- *At atmospheric pressure—confining pressure of one bar—the rocks deform only a few percent before fracturing.*
- *Under a confining pressure of 1000 bars the sandstone and shale deform more than 5 percent before rupturing.*

- *Under a confining pressure of 2000 bars the limestone deforms nearly 15 percent and shale and sandstone over 20 percent before rupturing.*



***<u>Conclusion</u>**: by increasing confining pressure the rocks pass into larger plastic deformation before fracturing. Therefore ductility (plastic deformation – before fracturing) increases with increasing confining pressure.*

2- *Temperature*

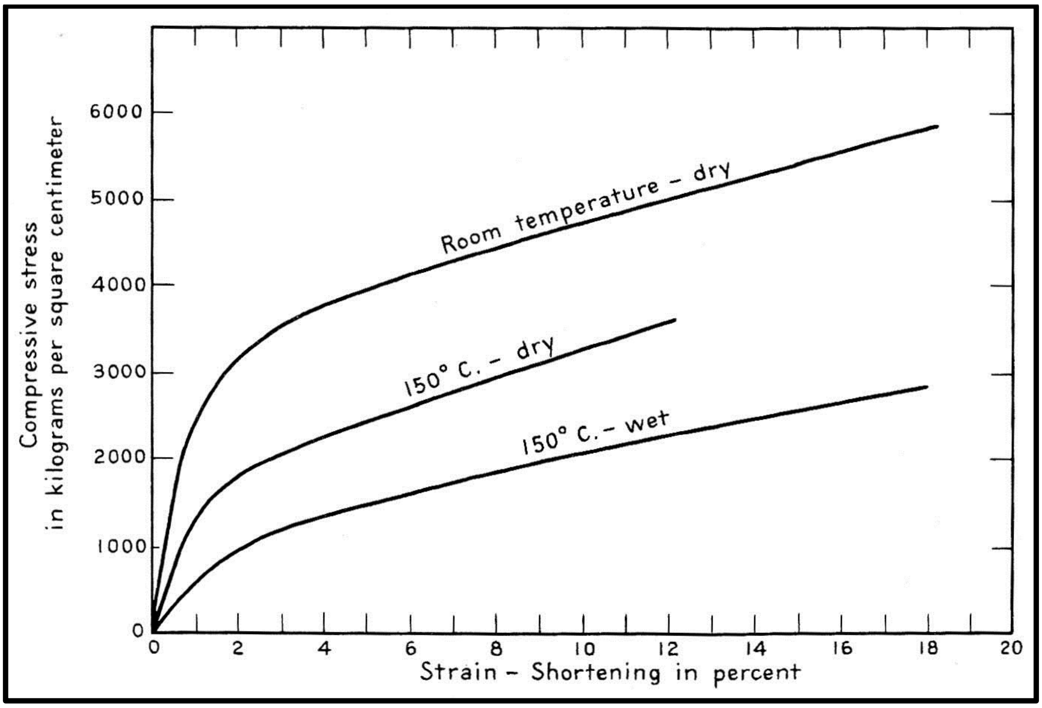
Changes in temperature modify the strength of rocks. Hot steel, for example, undergoes plastic deformation much more readily than does cold steel. The following figure shows two tests run on Yule Marble. Conditions were identical except for temperature; the axes of the cylinders were perpendicular to the foliation, the confining pressure was 10,000 atmospheres, and the deformation was produced by compressive stress.

*The **uppermost** curve is that obtained at **room temperatures**, whereas the **intermediate** curve is that obtained at a temperature of **150°C**.*

- *At room temperature the elastic limit is at a compressive stress of about 2000 kg/cm^2,*
- *At 150°C the elastic limit is at about 1000 kg/cm^2. Moreover, to produce a given strain far less stress is necessary when the specimen is hot than when it is cold.*

For example, to produce a strain of 10 percent at 150°C the compressive stress is 3000 kg/cm^2, but at room temperature the stress necessary to produce a similar deformation is 4500 kg/cm^2.

It is apparent that plastic deformation is far less common near the surface of the earth, where the confining pressure and the temperature are low, than it is at greater depths, where higher temperatures and greater confining pressure increase the possibility of plastic deformation.



***Conclusion**: an increase in temperature leads to increase in ductility of rocks or plastic deformation.*

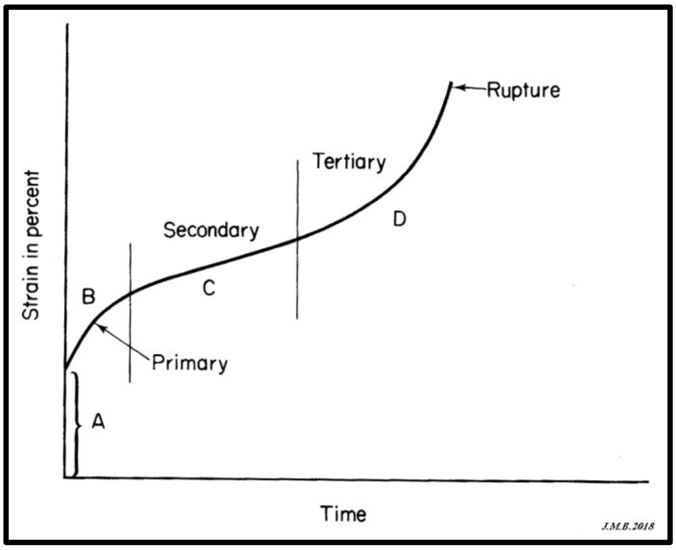
3- Time

*Geological processes have **great lengths of time** in which to operate. Although geologic time is impossible to duplicate experimentally, it is possible from experiments to make some conclusions concerning the influence of time. An analysis of the effects of time is concerned with such subjects as **creep**, **strain-rate**, and **viscosity**.*

- ***Creep** (sometimes called **cold flow**):*

Refers to the slow continuous deformation with the passage of time. The stresses may be above or below the elastic limit, but we are especially interested in creep caused by stresses below the elastic limit.

*Solenhofen Limestone under atmospheric pressure and at room temperature has strength of 2560 kg/cm^2. In a long-time experiment, Solenhofen Limestone subjected to a compressive stress of **1400 kg/cm^2** half the value of the strength deforms **rapidly** at first, then more **slowly.** At the end of **one** day, it has been shortened about **0.006** percent; after **10** days about **0.011** percent; after **100** days about **0.016** percent; and after **400** days a little more than **0.019** percent.*



The general form of a creep curve is shown in the adjacent figure. The ordinate is the total strain and the abscissa is time. The intercept A *on the ordinate represents the sudden strain when the load is added. The first part of the curve,* B, *represents primary creep, when the strain decreases with time. The main part of the curve, C, represents secondary or stable-state creep. Finally, in tertiary creep,* D, *the curve sharply rises just before rupture.*

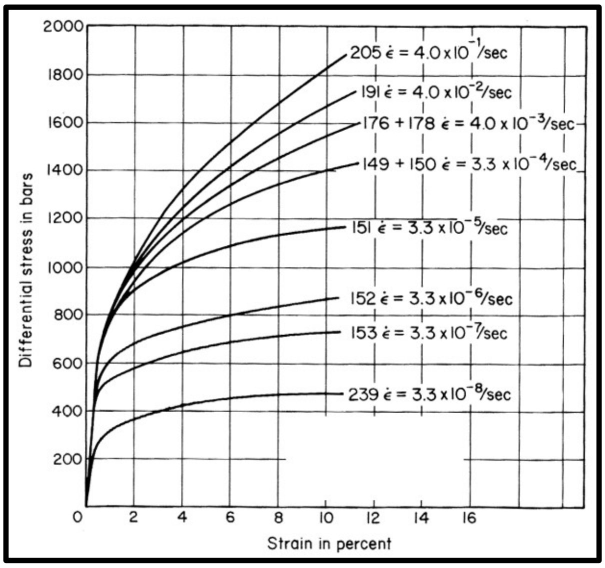
- ***Strain rate:***

Strain rate is the amount of strain divided by the time.

The strain rate at some point within the material <u>measures the rate at which the distances of adjacent parcels of the material change with time in the neighborhood of that point.</u> It comprises both the rate at which the material is expanding or shrinking (expansion rate), and also the rate at which it is being deformed by progressive shearing without changing its volume (shear rate). It is zero if these distances do not change, as happens when all particles in some region are moving with the same velocity (same speed and direction) and/or rotating with the same angular velocity, as if that part of the medium were a rigid body.

$$\epsilon^{-} = \frac{\in}{t}$$

Where ϵ^{-} is strain rate, $\in$ is strain, and t is time.



From the adjacent figure, two important principles may be established.

1- *The slower the strain rate, the less the differential stresses to attain a given strain. A differential stress of 1820 bars is necessary to attain 10 percent lengthening if the strain rate is 4.0 x 10^{-1}/sec, but only 450 bars are necessary if the strain rate is 3.3 x 10^{-8} sec.*

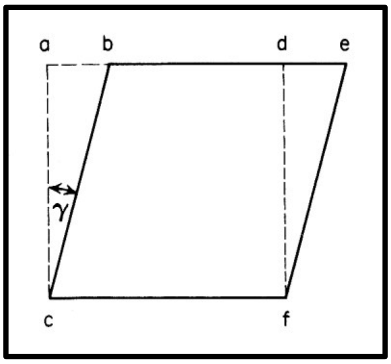
2- *The higher the temperature, the less the required differential stress for a given strain.*

- ***Viscosity:***

Normally we think of ***viscosity*** *in relation to liquids and more specifically the ease with which they flow.* ***Water*** *is much less viscous than* ***syrup (sauce)****, and syrup much less viscous than* ***tar (asphalt)****. But the concept of viscosity, or, more precisely, apparent viscosity, may be applied to solids.*

Viscosity *is defined as the ratio of shearing stress to the rate of shear.*

Referring to the adjacent figure, if shear stress is applied the rapidity with which the square **adcf** *is deformed into the parallelogram* **becf** is *a function of the viscosity. The rate of shear is measured by the change in angle* **γ** *per unit of time. If the stress is measured in dynes/cm*^{2} *and the rate of shear in seconds, the viscosity is given in dynes-sec/cm*^{2}*; this unit is called the* poise (*to be in balance*).



$$\dot{\gamma} = \frac{\gamma}{t} \qquad\qquad \eta = \frac{\tau}{\dot{\gamma}}$$

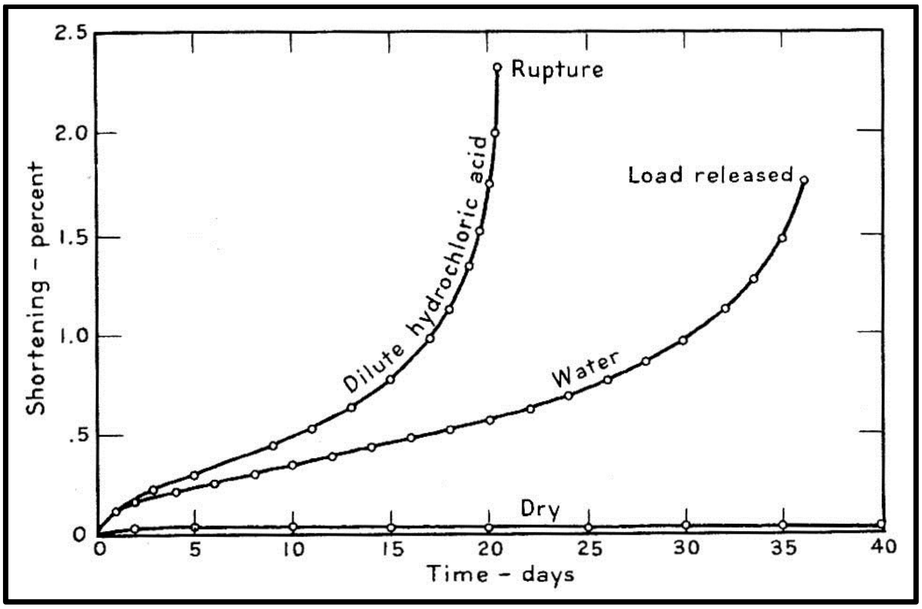
Where $\dot{\gamma}$ *is rate of shear strain,* γ *is shear strain,* ***t*** *is time,* η *is viscosity, and, τ is shearing stress.*

Conclusion*: as viscosity increases the deformation decreases.*

4- solutions:

Much rock deformation takes place while solutions capable of reacting chemically with the rock are present in the pore spaces. This is notably true of metamorphic rocks, in which extensive or complete recrystallization occurs. The solutions dissolve old minerals and precipitate new ones. Under such conditions the mechanical properties of rock are greatly modified.

Creep experiments have been performed on alabaster (a variety of gypsum) with ***solutions present****. In all cases the compressive stress was 205 kg/cm^2 (less than half the normal elastic limit of 480 kg/cm^2), and the temperature 24°C.*



1. *The lowest curve represents the deformation of a* ***dry specimen****. Within a few days the specimen had shortened about 0.03 percent, but there was no further detectable deformation even after 40 days.*
2. *A specimen deformed under such conditions that* ***water had access*** *to the alabaster (intermediate curve) had shortened 1 percent at the end of 30 days and 1.75 percent by the end of 36 days, when the load was released.*

3. *A specimen deformed with access of dilute hydrochloric acid had deformed more than 2 percent before rupturing at the end of 20 days.*

Conclusion*: solutions weaken the rock, so rocks deform more faster in the presence of solutions.*

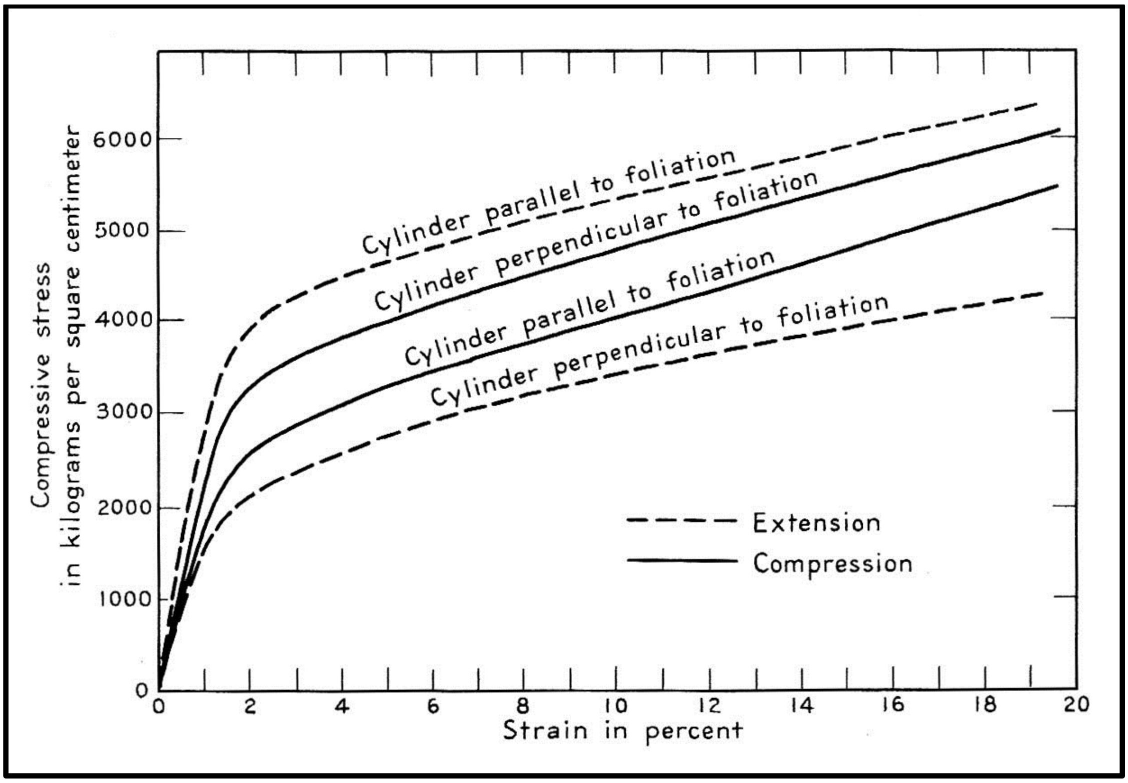
Anisotropy and Inhomogeneity

Most of the tests described in the preceding sections were made on ***isotropic materials****, that is, rocks whose mechanical properties were uniform in all directions.*

Rocks that show bedding, banding, or foliation are ***not isotropic****. The strength of such rocks would depend upon the orientation of the applied forces to the planar structures of the rock.*

This point is well illustrated in the following figure. The rock was Yule Marble, confining pressure was ***10,000 kg/cm***^{2}*, and the tests were run at* ***room temperature****. All the specimens show great plastic deformation.*

- *The* ***solid*** *lines represent experiments under* ***compression****; in this case the stress is compressive and the strain is* ***shortening*** *parallel to the axis of the cylinder.*
- *Under compression the cylinder perpendicular to the foliation is stronger than the cylinder parallel to the foliation.*
- *The broken lines represent tests under* ***tension****; in this case the stress is tensile and the strain is* ***lengthening*** *parallel to the cylinders.*
- *Under tension the cylinder parallel to the foliation is much stronger than the cylinder perpendicular to the foliation.*



***Summary**:*

It is clear that the mechanical properties of rocks are greatly modified by confining pressure, temperature, the time factor, and the presence of reacting solutions.

The combined effect of these factors is so great that it is impossible in the present state of our knowledge to treat rock deformation in a measurable way. Increase in confining pressure increases the elastic limit and the ultimate strength.

- *Increase in the temperature weakens the rocks.*
- *After long continued stress the rocks become much weaker.*
- *Reacting solutions lower the strength of rocks.*