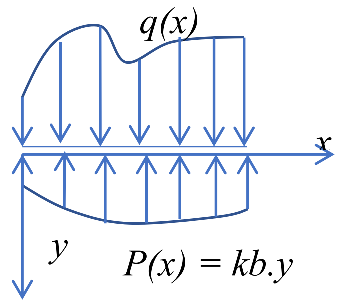
Beams on Elastic Foundation:



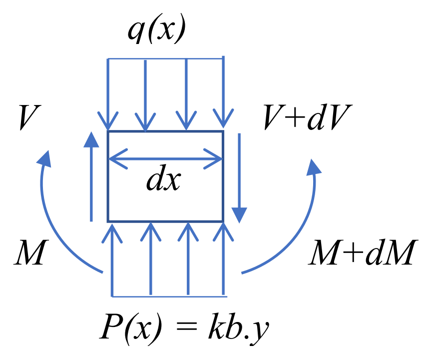
$$\uparrow + \sum F_y = 0 \rightarrow [\, V - (V + dV) + P(x)dx - q(x)dx = 0] \div dx$$

$$p(x) - q(x) - - - - - - (1)$$

$\frac{dV}{dx} =$

$$\sum M_{center} = 0 \rightarrow \left[M - (M + dM) + V\frac{dx}{2} + (V + dV)\frac{dx}{2} \right] \div dx$$

$$\frac{dM}{dx} = V \rightarrow \frac{d2M}{dx^2} = \frac{dV}{dx} - - - - - - (2)$$



Sub. eq.(1) in eq.(2)

$$\frac{d2M}{dx^2} = p(x) - q(x) - - - - (3)$$

But $\quad EI\frac{d2y}{dx^2} = -M \rightarrow EI\frac{d4y}{dx^4} = -\frac{d2M}{dx^2} - - - - (4)$

Sub. eq.(3) in eq.(4)

$$\left[EI\frac{d4y}{dx^4} = -\big(p(x) - q(x)\big) \right] \div EI$$

$$\frac{d4y}{dx^4} = \frac{q(x) - p(x)}{EI} \rightarrow \frac{d4y}{dx^4} + \frac{p(x)}{EI} = \frac{q(x)}{EI}$$

$$\therefore \; \frac{d4y}{dx^4} + \frac{kb}{EI}y = \frac{q(x)}{EI}$$

Where: k is the foundation modulus (unit : $N/m^2/m$)

b is the Foundation width (m)

p(x) is the pressure (reaction of soil) (N/m^2)

y is the deflection (m)

E is the modulus of Elasticity (N/m^2)

I is the moment of Inertia (m^4)

Solution of the equation:

The equation for a uniform beam on Winkler foundation:

$$\therefore \ \frac{d4y}{dx^4} + \frac{kb}{EI} y = \frac{q(x)}{EI}$$

Let $4\beta^4 = \frac{kb}{EI}$

1) To find homogeneous solution Put $\frac{q(x)}{EI} = 0$

$$D^4 + 4\beta^4 = 0$$

$$D^4 + 4\beta^2 D^2 + 4\beta^4 - 4\beta^2 D^2 = 0$$

$$(D^2 + 2\beta^2)^2 - (2\beta D)^2 = 0$$

$$(D^2 - 2\beta D + 2\beta^2)(D^2 + 2\beta D + 2\beta^2) = 0$$

$$D_{1,2} = \frac{2\beta \pm \sqrt{(-2\beta)^2 - 4(2\beta^2)}}{2} \rightarrow D_{1,2} = +\beta \pm \beta i \quad , i = \sqrt{-1}$$

$$D_{3,4} = \frac{-2\beta \pm \sqrt{(-2\beta)^2 - 4(2\beta^2)}}{2} \rightarrow D_{1,2} = -\beta \pm \beta i$$

$$y_h = \ e^{-\beta x}(c_1 \cos(\beta x) + c_2 \sin(\beta x) + e^{\beta x}(c_3 \cos(\beta x) + c_4 \sin(\beta x)$$

$$\frac{d4y}{dx^4} + 4\beta^4 y = \frac{q(x)}{EI} \qquad \beta = \sqrt[4]{\frac{kb}{4EI}}$$

$$y_h = e^{-\beta x}\left[c_1 \cos(\beta x) + c_2 \sin(\beta x)\right] + e^{\beta x}\left[c_3 \cos(\beta x) + c_4 \sin(\beta x)\right]$$

$$y_h' = \beta e^{-\beta x}\left[(c_2 - c_1)\cos(\beta x) - (c_1 + c_2)\sin(\beta x)\right] + \beta e^{\beta x}\left[(c_4 - c_3)\cos(\beta x) + (c_3 + c_4)\sin(\beta x)\right]$$

$$y_h'' = 2\beta^2 e^{-\beta x}\left[-c_2 \cos(\beta x) + c_1 \sin(\beta x)\right] + 2\beta^2 e^{\beta x}\left[c_4 \cos(\beta x) - c_3 \sin(\beta x)\right]$$

$$y_h''' = 2\beta^3 e^{-\beta x}\left[(c_1 + c_2)\cos(\beta x) + (c_2 - c_1)\sin(\beta x)\right] + 2\beta^3 e^{\beta x}\left[(c_4 - c_3)\cos(\beta x) - (c_3 + c_4)\sin(\beta x)\right]$$

or

$$y_h = A_1 \cosh(\beta x)\cos(\beta x) + A_2 \cosh(\beta x)\sin(\beta x) + A_3 \sinh(\beta x)\cos(\beta x) + A_4 \sinh(\beta x)\sin(\beta x)$$

$$y_h' = \beta[(A_2 + A_3)\cosh(\beta x)\cos(\beta x) + (A_4 - A_1)\cosh(\beta x)\sin(\beta x) + (A_1 + A_4)\sinh(\beta x)\cos(\beta x) + (A_2 - A_3$$

$$y_h'' = 2\beta^2[A_4 \cosh(\beta x)\cos(\beta x) - A_3 \cosh(\beta x)\sin(\beta x) + A_2 \sinh(\beta x)\cos(\beta x) - A_1 \sinh(\beta x)\sin(\beta x)]$$

$$y_h''' = 2\beta^3[(A_2 - A_3)\cosh(\beta x)\cos(\beta x) - (A_4 + A_1)\cosh(\beta x)\sin(\beta x) + (A_4 - A_1)\sinh(\beta x)\cos(\beta x) - (A_2 +$$

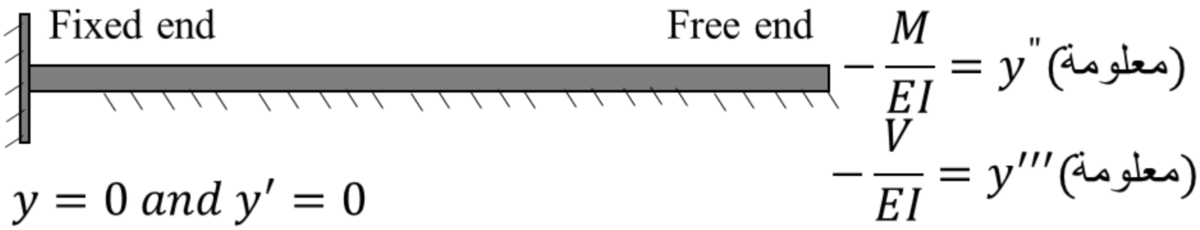
$$y = y_h + y_p$$

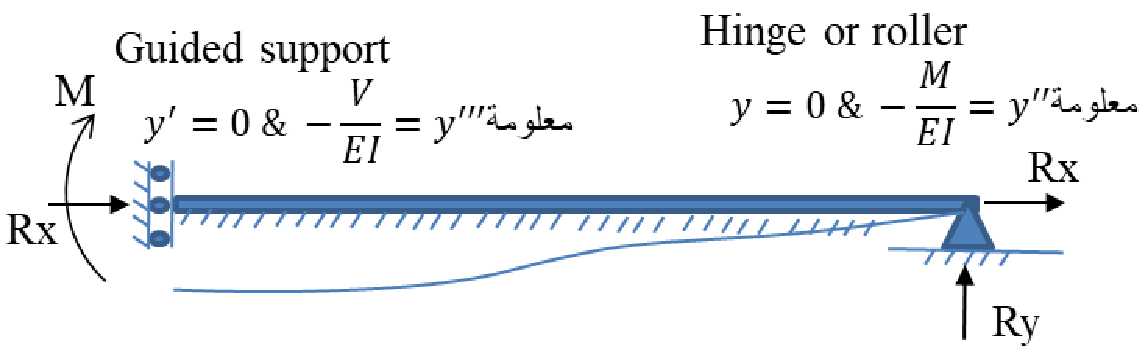
2) the particular solution (y_p) depends on the load function p(x)

3) the deflection equation $y = y_h + y_p$

تحوي معادلة الهطول على اربعة ثوابت فنحتاج على الاقل اربعة شروط معلومة لايجاد هذة الثوابت

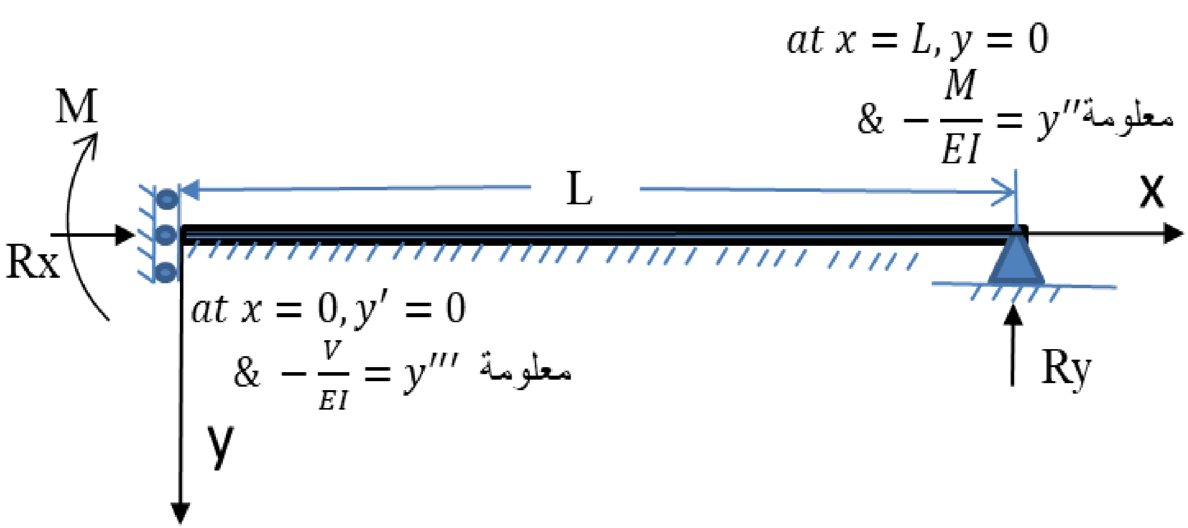
Boundary conditions:



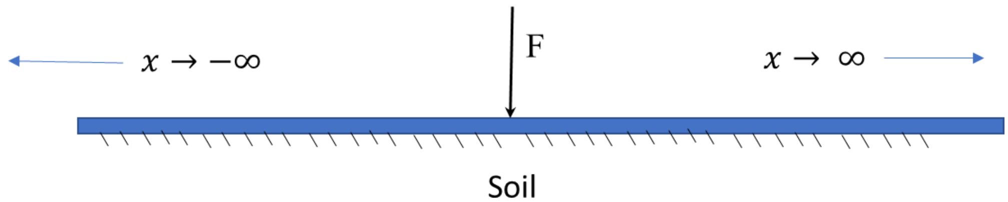


يجب ان نحدد موقع نقطة الاصل (origin) على ال (beam) لان ال (boundary conditions) كاحداثيات (x, y) . حيث x هي البعد الافقي عن نقطة الاصل و قيمة y تمثل الشرط المعلوم و التي ممكن ان تكون (Deflection , rotation , moment , shear)

مثال للتوضيح كيفية تحديد موقع نقطة الاصل و كتابة ال (Boundary conditions):



Example (1): For the Infinite beam with concentrated load shown in figure



Write the deflection equation and draw reaction, shear and bending moment diagram.

Solution:

1- نحدد موقع نقطة الاصل ان لم تحدد بالسؤال و لتكن تحت القوة ليكون الشكل متناظر
2- نجد y_p تفرض حسب نوع الحمل الموجود بالسؤال و بما انه لا يوجد حمل فتكون المعادلة التفاضلية لها حل واحد و هو ال y_h فقط

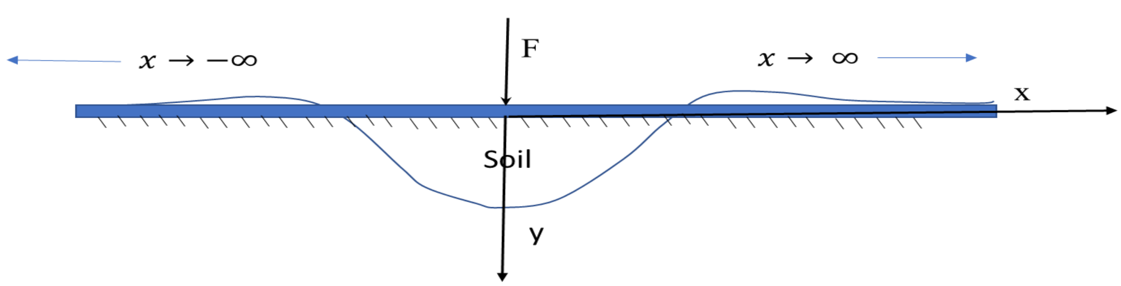
q(x) = zero $\rightarrow \therefore \ \frac{d4y}{dx^4} + \frac{kb}{EI} y = 0$

3- فتكون معادلة الهطول

$y = y_h + y_p = y_h + \text{zero}$

$\therefore \ y = y_h = \ e^{-\beta x}(c_1 \cos(\beta x) + c_2 \sin(\beta x) + e^{\beta x}(c_3 \cos(\beta x) + c_4 \sin(\beta x)$

4- نلاحظ المعادلة تحوي اربعة ثوابت اي نحتاج الى اربعة شروط معلومة(Boundary conditions) لايجاد قيمة هذة الثوابت



1 and 2 At $x \to \infty \ y \ = \ y' = zero \ \to C3 \ and \ C4 = zero$

$y = e^{-\beta x}(c_1 \cos(\beta x) + c_2 \sin(\beta x)$

3 At x = zero $y' = zero \ \to y' = \beta e^{-\beta x}\big((c_2 - c_1)\cos(\beta x) + (c_2 + c_1) \sin(\beta x)\big)$

$c_2 = c_1 = c$

$y = ce^{-\beta x}(\cos(\beta x) + \sin(\beta x)$

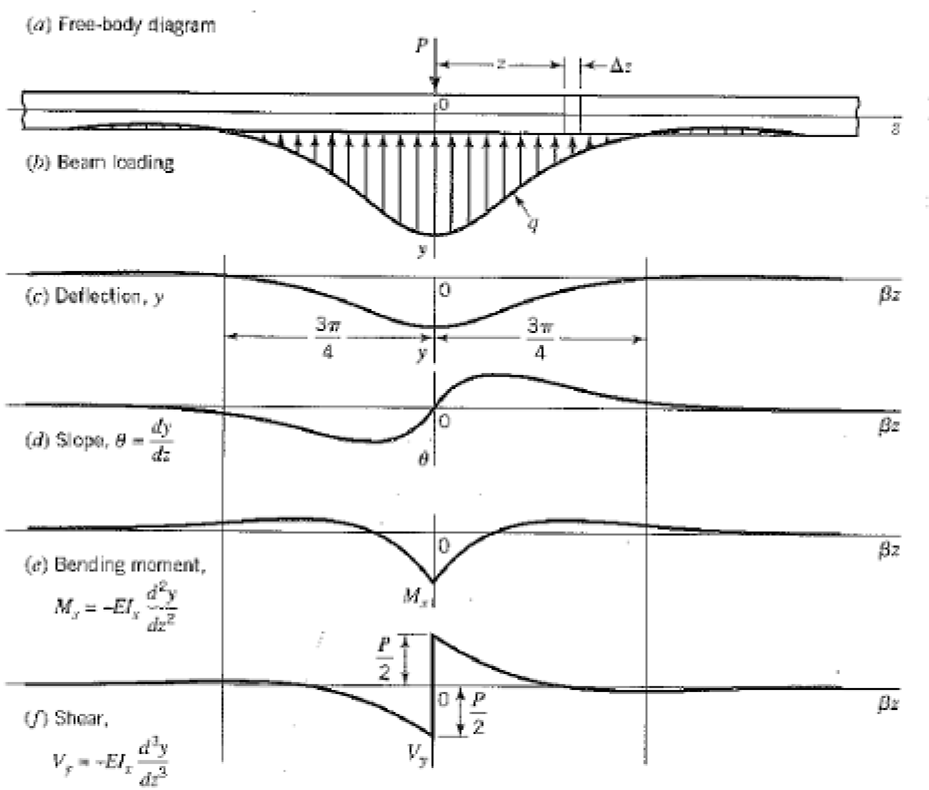
4 $\uparrow + \sum F_y = 0 \rightarrow [P(x)dx - F = 0]$

but $p(x) = k\,y = k\left(ce^{-\beta x}(\cos(\beta x) + \sin(\beta x)\right)$

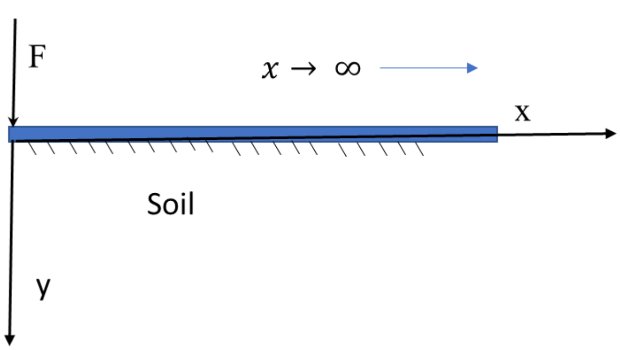
$$2\int_0^{\infty} k\left(ce^{-\beta x}(\cos(\beta x) + \sin(\beta x)\right)dx - F = 0$$

By using methods of integration, we can find $c = \frac{\beta F}{2k}$

$$y = \frac{\beta F}{2k} e^{-\beta x}(\cos(\beta x) + \sin(\beta x)$$



H.W. For the Simi Infinite beam with concentrated load shown in figure



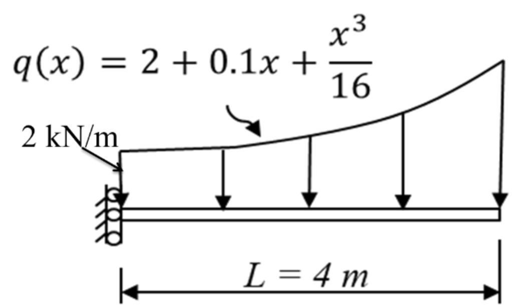
Write the deflection equation and draw reaction, shear and bending moment diagram.

Limitation of rigid and elastic foundation:

Calculate βL , $where\ \beta = \sqrt[4]{\frac{kb}{4EI}}\ \ and\ L\ is\ the\ length\ of\ the\ beam$

1. If $\beta L < \frac{\pi}{4}\quad the\ founation\ is\ rigid$
2. If $\frac{\pi}{4} \leq\ \beta L \leq \pi\ \ the\ founation\ is\ rigid\ or\ elastic$
3. If $\beta L >\ \pi\ the\ founation\ is\ elastic$

Example (2): For the beams on elastic foundation shown in the figure



1 Find the particular solution.
2 State only the boundary conditions needed.

Solution:

1- $\frac{d4y}{dx^4} + \frac{k}{EI}y = \frac{q(x)}{EI} \rightarrow \qquad \frac{d4y}{dx^4} + \frac{k}{EI}y = \frac{2+0.1x+\frac{x^3}{16}}{EI}$

$$let\ y_p = Ax^3 + Bx^2 + Cx + D\ \rightarrow \frac{d4y_p}{dx^4} = zero$$

$$zero + \frac{k}{EI}(Ax^3 + Bx^2 + Cx + D) = \frac{2 + 0.1x + \frac{x^3}{16}}{EI}$$

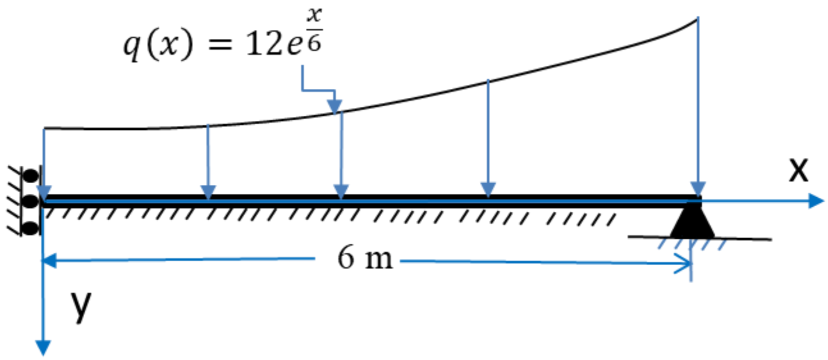
$$A\ = \frac{1}{16k}\ ,\qquad B = zero\ ,\ \ C = \frac{0.1}{k}\ \ ,\qquad D = \frac{2}{k}$$

$$y_p = \frac{1}{16k}x^3 + \frac{0.1}{k}x + \frac{2}{k}$$

2- boundary conditions $at\ x = 0,\qquad y' = 0\qquad and\qquad V = 0 = -EIy'''$

$$at\ x = 4m\ ,\ \ M = 0 = -EIy''\quad and\quad V = 0 = -EIy'''$$

Example (3): For the beams on elastic foundation shown in the figure



1 Find the particular solution.
2 State only the boundary conditions needed.

Solution:

1- $\frac{d4y}{dx^4} + \frac{k}{EI} y = \frac{q(x)}{EI} \rightarrow \qquad \frac{d4y}{dx^4} + \frac{k}{EI} y = \frac{12e^{\frac{x}{6}}}{EI}$

$=let\ y_p = Ae^{\frac{x}{6}} \rightarrow \frac{d4y_p}{dx^4} = \frac{A}{6^4} e^{\frac{x}{6}} = \frac{A}{1296} e^{\frac{x}{6}}$

$$\frac{A}{1296} e^{\frac{x}{6}} + \frac{k}{EI}\left(Ae^{\frac{x}{6}}\right) = \frac{12e^{\frac{x}{6}}}{EI}$$

$$A = \frac{\frac{12}{EI}}{\frac{1}{1296} + \frac{k}{EI}}$$

$$y_p = \left(\frac{\frac{12}{EI}}{\frac{1}{1296} + \frac{k}{EI}}\right) e^{\frac{x}{6}}$$

$$\therefore y = y_h + y_p$$

$$= e^{-\beta x}\left[(c_1 \cos(\beta x) + c_2 \sin(\beta x)) + e^{\beta x}(c_3 \cos(\beta x) + c_4 \sin(\beta x))\right] + \left(\frac{\frac{12}{EI}}{\frac{1}{1296} + \frac{k}{EI}}\right) e^{\frac{x}{6}}$$

2- boundary conditions $at\ x = 0, y' = 0\ and\ V = 0 = -EIy'''$

$$at\ x = 6\ m\ , M = 0 = -EIy''and\ y = 0$$

- للاشكال المتناظرة ممكن استعمال المعادلة ادناه للتقليل الشروط من اربعة الى اثنين ولكن يجب عدم تكرار نفس الشرط من جهتين (يمين ويسار نقطة الاصل) و عدم استعمال الشرط $(at\ x = zero, y' = zero)$ لنها تعتبر متطلبات التناظر. تكون نقطة الاصل بالمنتصف دائما.

$$y_h = A_1 \cosh(\beta x) \cos(\beta x) + A_2 \cosh(\beta x) \sin(\beta x) + A_3 \sinh(\beta x) \cos(\beta x) + A_4 \sinh(\beta x) \sin(\beta x)$$

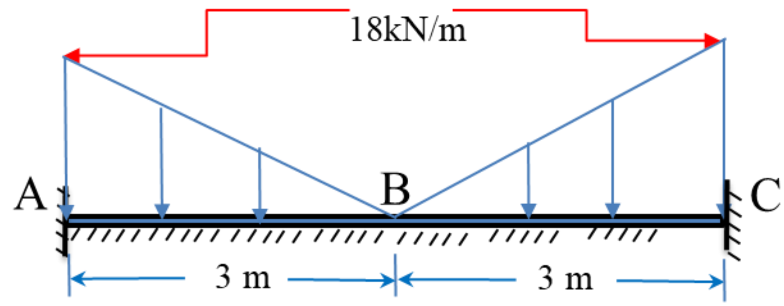
- نلاحظ ان الجزئين الاول والاخير و التي تحوي الثوابت (A1&A4)هما دوال زوجية اي تحقق التناظر حول محور y بينما الجزئين الوسطية و التي تحوي الثوابت (A2&A3) تحقق تناظر حول نقطة الاصل.
- فنحدد من السؤال و الحمل اذا كان الشكل متناظر حول محور (y) فنجعل قيمة (A2&A3) تساوي صفر و نكتب عليها من التناظر.

و اذا كان السؤال و الحمل متناظر حول نقطة الاصل فنجعل قيمة (A1&A4) تساوي صفر و نكتب عليها من التناظر.

- و يبقى ثابتين فقط بالمسالة تحتاج الى شرطين فقط اضافة الى شرطي التناظر لايجاد ثابتين فقط بدل اربعة ثوابت حيث اثنان منها معلومة القيمة وتساوي صفر.

Example (4): For the beams on elastic foundation shown in the figure

1 Find the particular solution.
2 State only the boundary conditions needed.

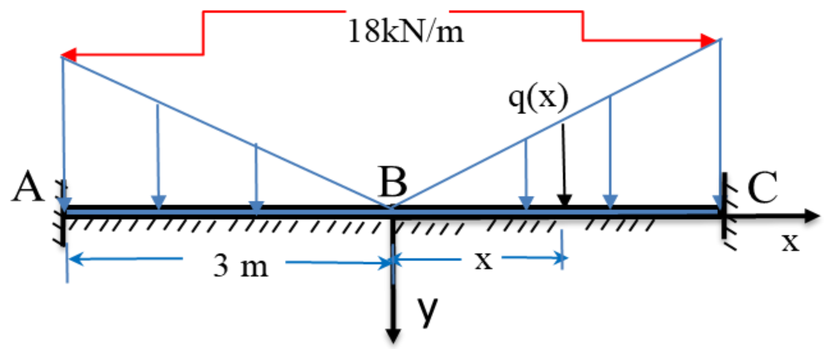


Solution:

1- $\frac{q(x)}{x} = \frac{18}{3} \rightarrow q(x) = 6x$

$$\frac{d4y}{dx^4} + \frac{k}{EI}y = \frac{q(x)}{EI}$$

$$\rightarrow \frac{d4y}{dx^4} + \frac{k}{EI}y = \frac{6x}{EI}$$



$=let\ y_p = Ax + B \rightarrow \frac{d4y_p}{dx^4} = zero$

$$0 + \frac{k}{EI}(Ax + B) = \frac{6x}{EI}$$

$$A = \frac{6}{k}\ and\ B = zero \rightarrow \qquad y_p = \frac{6}{k}x$$

$$y = A_1 \cosh(\beta x)\cos(\beta x) + A_2 \cosh(\beta x)\sin(\beta x) + A_3 \sinh(\beta x)\cos(\beta x) + A_4 \sinh(\beta x)\sin(\beta x) + \frac{6}{k}x$$

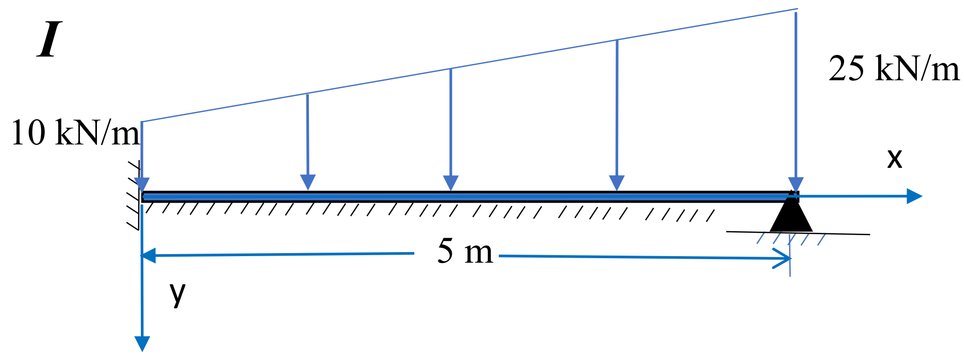
2- boundary conditions $put\ A2 = A3 = zero$ $\boldsymbol{from\ symmetry}$

$$at\ x = 3m\ , y = 0 = and\ y' = 0$$

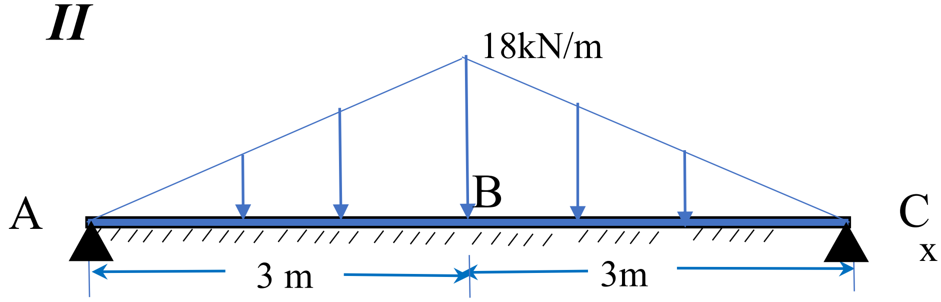
H.W.: For the beams on elastic foundation shown in the figure

1 Find the particular solution.
2 State only the boundary conditions needed.

I

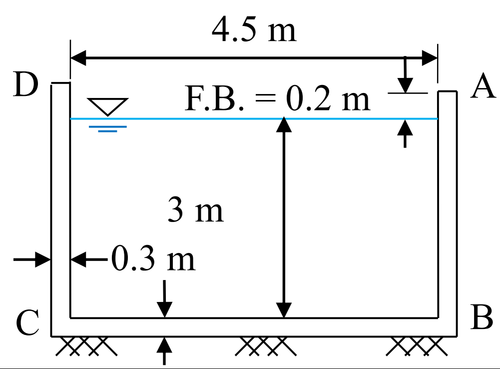


II



Example: Design and Draw the pressure distribution in the subsoil; shear force, and bending moment diagram of the base for the concrete aqueduct shown in fig having the following data:

$E_{conc.}=25\times10^6\ kn/m^2$	$\gamma_{conc.}=24\ kn/m^3$
$\gamma_{water.}=10\ Kn/m^3$	$K_{soil}=14000\ Kn/m^3$
$fc'=28$ Mpa	$Fy=410$ Mpa



Solution:

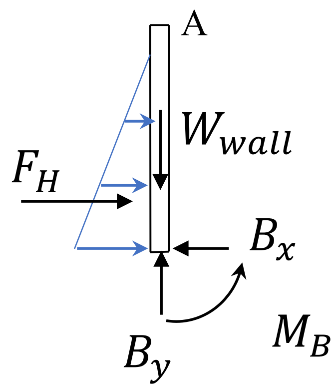
K =	14000		fc' =	28	Mpa	h =	0.3	m
Econc. =	25000000		fy =	410	Mpa	L =	4.5	m
β =	0.49944	1/m	βL =	2.2474958	mid			

For wall AB:

$$F_H = \gamma_w . h_c . A = (10)(1.5)(3\times1) = 45\ kN/m$$

$$W_{wall} = \gamma_{conc.} \times volume = (24)(3.2\times0.3\times1) = 23\ kN/m$$

$$End\ moments = F_H \times \frac{H}{3} = 45\ \times 1 = 45\ kN.m/m$$

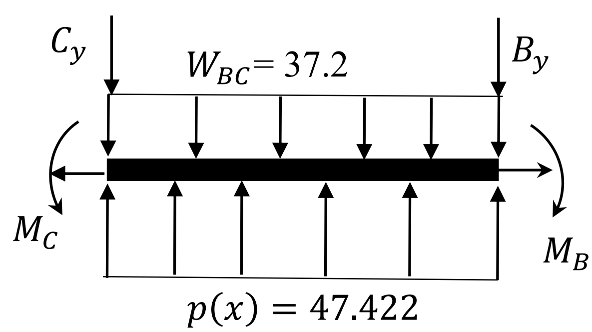


For Base BC:

$$W_{BC} = \gamma_w \times H_w + \gamma_{conc.} \times t$$

$$= (10)(3) + (24)(0.3) = 37.2\ kN/m^2$$

$$\uparrow + \sum F_y = 0 \rightarrow p(x)\times L - W_{BC}\times L - 2\times W_{wall} = 0$$



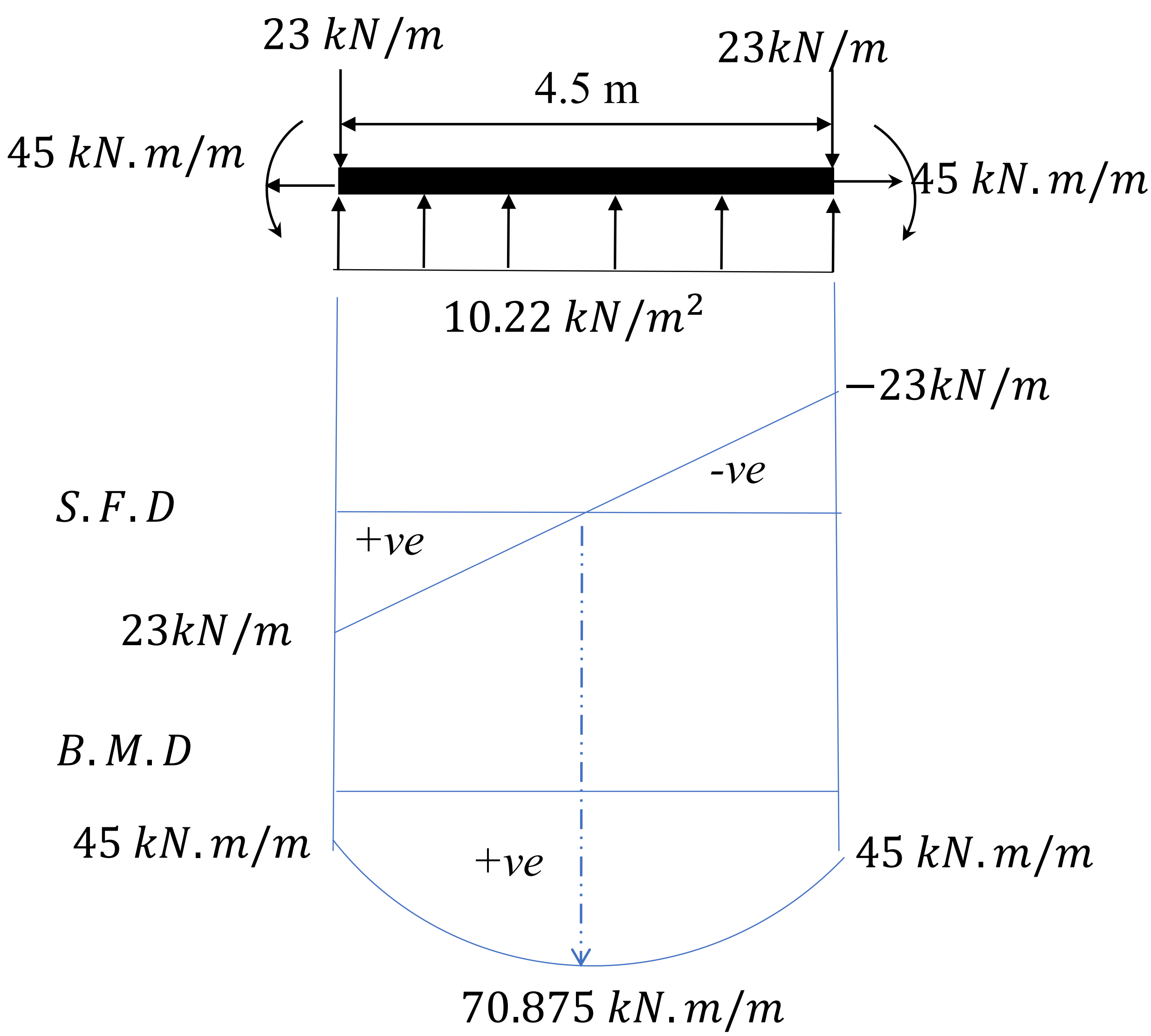
For Rigid Founadtion:

$$p(x) = \frac{W_{BC}\times L + 2\times W_{wall}}{L}$$

$$= \frac{37.2 \times 4.5 + 2 \times 23}{4.5} = 47.422\ kN/m^2$$

The net load = $p(x) - W_{BC} = 47.422 - 37.2 = 10.22 \frac{kN}{m^2}$ $upword$

$$maximum\ moment\ when\ V = zero = 45 + 23 \times \frac{2.25}{2} = 70.875\ kN.m/m$$



For Elastic Foundation:

EI = 56250 Kn.m²

B.C. :-

Put A2=A3=0 from symmetry

when x=L/2=2.25 m $y''' = -23/EI$ -0.0004089 $y'' = 45/EI$ 0.00080

$\beta L/2 =$ 1.12375 $2\beta^2 =$ 0.4988877 $2\beta^3 =$ 0.249166203

$\cosh(\beta L/2)\cos(\beta L/2) =$ 0.735227346 $\sinh((\beta L/2)\sin(\beta L/2) =$ 1.240462276

$\cosh(\beta L/2)\sin(\beta L/2) =$ 1.533576952 $\sinh((\beta L/2)\cos(\beta L/2) =$ 0.59470233

$y''/2\beta^2 = A4\ \cosh(\beta x)\cos(\beta x)+A1\ \sinh(\beta x)\sin(\beta x)$

0.001603567 = 0.735227346 ×A4 - 1.240462276 ×A1 **eq(1)**

$y'''/2\beta^3 = A4\ (\sinh(\beta x)\cos(\beta x)-\cosh(\beta x)\sin(Bx)) - A1\ (\sinh(\beta x)\cos(\beta x)+\cosh(\beta x)\sin(\beta x))$

-0.001641029 = -0.938874622 ×A4 - 2.128279283 ×A1 **eq(2)**

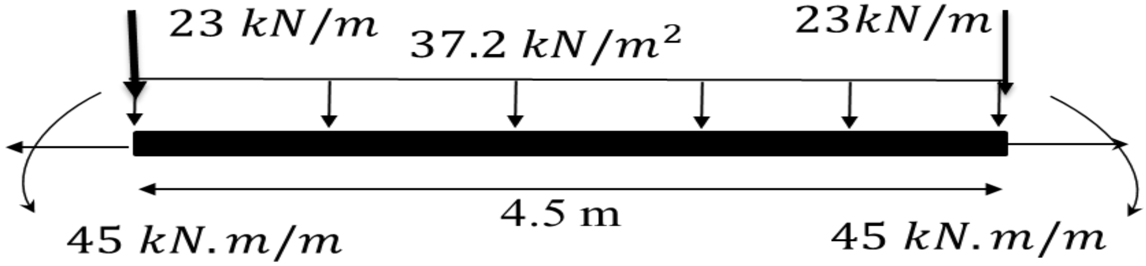
eq(1) and eq (2) we get A1 = -1.10E-04 & A4 = 2.00E-03

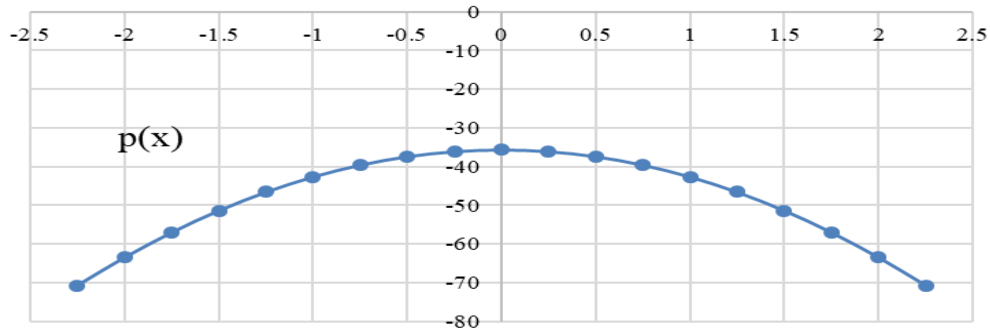
$P(x) = K\,y = K(A1\cosh(\beta x)\cos(\beta x)+\sinh(\beta x)\sin(\beta x)+37.2/K)$

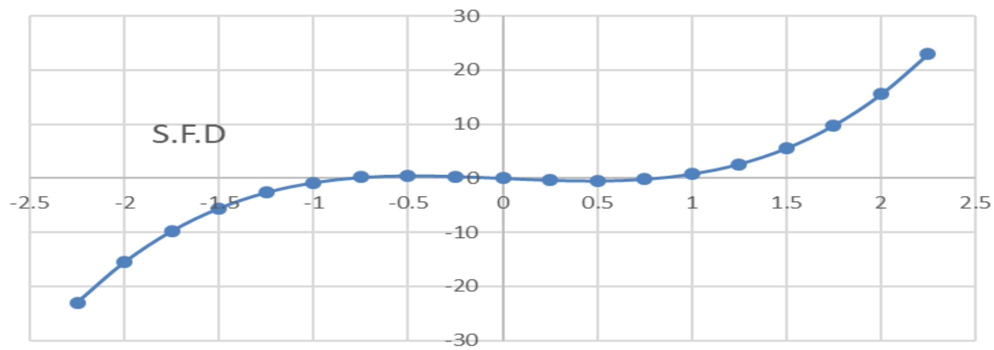
$V(x) = 2\beta^3(-(A1+A4)\cosh(\beta x)\sin(\beta x)+(A4-A1)\sinh(\beta x)\cos(\beta x))$

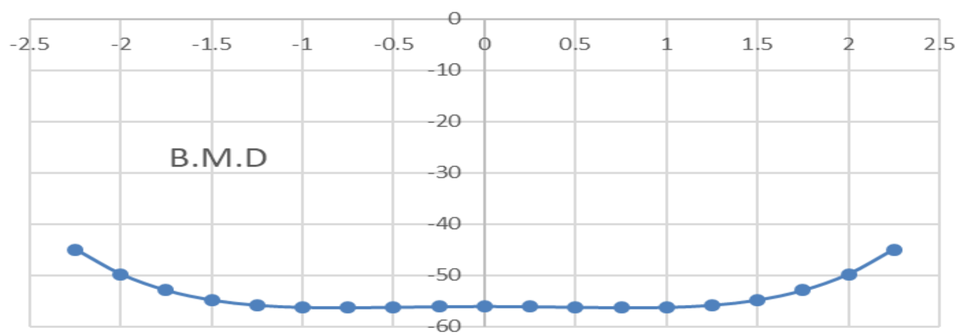
$M(x) = 2\beta^2(A4\cosh(\beta x)\cos(\beta x)-A1\sinh(\beta x)\sin(\beta x))$

X (m)	p(x)(kN/m)	v(x)(kN/m)	M(x)(kN.m/m)
-2.25	70.739475	23	45
-2	63.496692	15.53589738	49.779258
-1.75	57.026121	9.787231956	52.910944
-1.5	51.377	5.554329151	54.799214
-1.25	46.579161	2.627753394	55.796984
-1	42.648512	0.792457395	56.204038
-0.75	39.591705	-0.16933773	56.266007
-0.5	37.409951	-0.476322849	56.173936
-0.25	36.101994	-0.347130298	56.064192
0	35.666233	0	56.018532
0.25	36.101994	0.347130298	56.064192
0.5	37.409951	0.476322849	56.173936
0.75	39.591705	0.16933773	56.266007
1	42.648512	-0.792457395	56.204038
1.25	46.579161	-2.627753394	55.796984
1.5	51.377	-5.554329151	54.799214
1.75	57.026121	-9.787231956	52.910944
2	63.496692	-15.53589738	49.779258
2.25	70.739475	-23	45



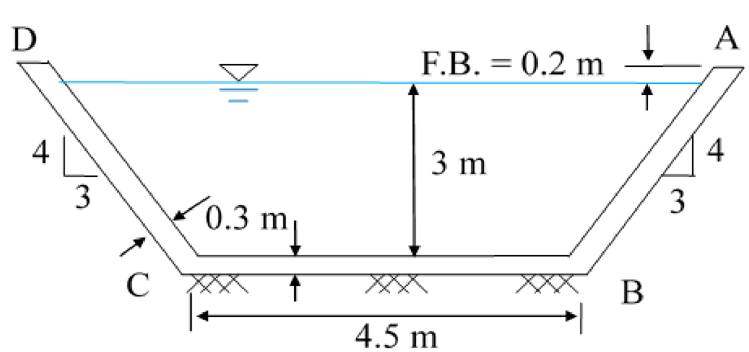




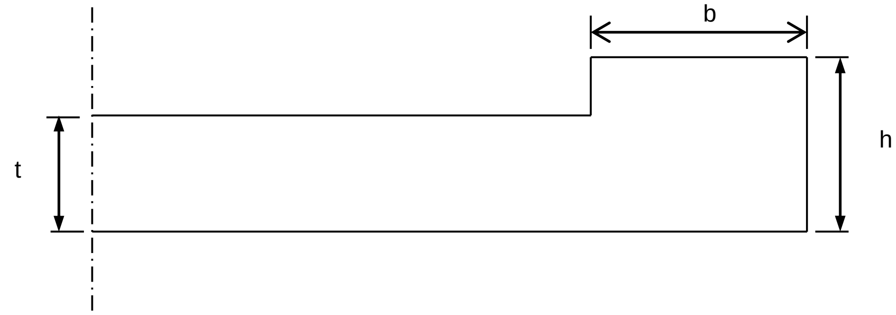


H.W.: Draw the pressure distribution in the subsoil; shear force, and bending moment diagram of the base for the concrete aqueduct shown in fig having the following data:

$E_{conc.}$=25×10^6 kn/m^2	$\gamma_{conc.}$=24 kn/m^3
$\gamma_{water.}$=10 Kn/m^3	K_{soil}=14000 Kn/m^3
fc'=28 Mpa	Fy=410 Mpa



Example: Design the simply supported reinforced concrete slab bridge having Cross-section shown in the figure and the following data:



Width = 8 m	Span c/c = 5.4 m	Clear span = 5 m	Asphalt Wt =1.5 kN/m^2
Truck: MS18	fc' = 25 MPa	fy = 300 MPa	Hand rail Wt = 1 kN/m

Solution:

1-Slab design

$$t_{min.} = \frac{l}{20} = \frac{5}{20} = 0.25\,m \qquad \text{use t} = 0.3 \text{ m}$$

Span l=min. (Span c/c, Clear span+t) = min. (5.4,5.3) = 5.3 m

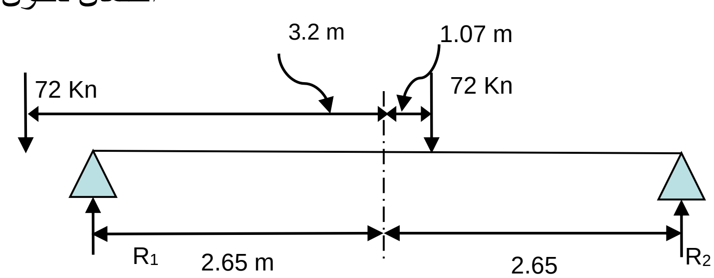
Dead moment:

$w_d = \gamma_{conc.} \times t + \text{asphalt } w_t = 24 \times 0.3 + 1.5 = 8.7 \text{ kN/m}^2$

$$M_d = \frac{w_d \times (l)^2}{8} = \frac{8.7 \times (5.3)^2}{8} = 30.55 kN.\text{m/m}$$

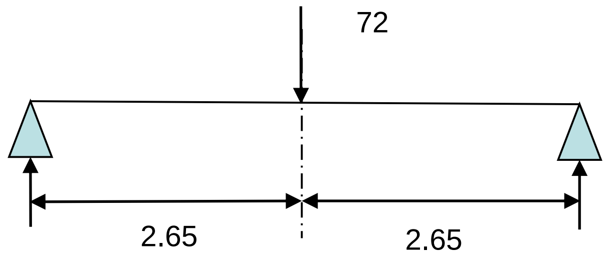
Live moment:

1- احتمال دخول ثلاث عجلات غير وارد لان طول الجسر 5.3 م و المسافة الكلية بين العجلات هو (4.27+4.27) م
2- احتمال دخول عجلتين ايضا غير وراد بعد الحسابات



3- احتمال دخول عجلة واحدة تكون بالمنتصف

$$E = 1.22 + 0.06S \le 2.14m$$
$$= 1.22 + 0.06 \times 5.3 = 1.538\,\text{m} < 2.14\,\text{m}$$
$$\therefore \text{E} = 1.538\,\text{m}$$



$$M_L = \frac{\frac{P \times L}{4}}{E} = \frac{\frac{72 \times 5.3}{4}}{1.538} = 62.03\,\text{kN.m/m}$$

Impact Moment

$$I = \frac{15.24}{l + 38.1} \le 30\% = \frac{15.24}{5.3 + 38.1} = 0.352 > 0.3$$
$$use\ I = 0.3$$

$$\text{M}_I = I \times \text{M}_l = 0.3 \times 62.03 = 18.61\ \text{Kn.m/m}$$

Total moment $M_t = M_d + M_l + M_I = 30.55 + 62.03 + 18.6 = 111.2\ \text{kN.m/m}$

$$n = \frac{E_s}{E_c} = \frac{200000}{4730\sqrt{fc}} = \frac{200000}{4730\sqrt{25}} = 8.46\ , \qquad r = \frac{f_s}{f_c\,\text{max.}} = \frac{140}{0.4 \times 25} = 14$$

$$K = \frac{n}{n + r} = \frac{8.46}{8.46 + 14} = 0.377\,, \qquad \text{J} = 1 - \frac{\text{K}}{3} = 1 - \frac{0.377}{3} = 0.874$$

$$d_{req.} = \sqrt{\frac{2 \times M_t}{f_c\,\text{max.} \times K \times J \times b}} = \sqrt{\frac{2 \times (111.2 \times 10^6)}{10 \times 0.377 \times 0.874 \times 1000}} = 259.8mm\,,$$
$$d_{\text{ava.}} = t - \text{cov}er - \phi/2 = 300 - 25 - 25/2 = 262.5\,mm > d_{req.} \qquad \text{ok}$$

$$A_{st.} = \frac{M_t}{f_s \times J \times d} = \frac{111.2 \times 10^6}{140 \times 0.874 \times 262.5} = 3462\,mm^2/m$$

$$s = 1000 \times \frac{A_{bar}}{A_{st.}} = 1000 \times \frac{491}{3462} = 141\,mm \quad , \quad \text{use}\ \phi 25@140\,mm\,,\ \text{A}_{\text{st}} = 1000 \times \frac{491}{140} = 3507mm$$

Secondary reinforcement:

$$A_{sd.} = \frac{1}{\sqrt{3.28s_c}} \times A_{st.} \le 50\% \times A_{st.} = \frac{1}{\sqrt{3.28 \times 5.3}} \times A_{st.} \le 0.5 \times A_{st.} = 0.24 \times A_{st.} \le 0.5 \times Ast.$$
$$A_{sd} = 0.5 \times 3507 = 842\,mm^2/m$$

$$s = 1000 \times \frac{A_{bar}}{A_{st.}} = 1000 \times \frac{201}{842} = 239\,mm \qquad \text{use}\ \phi 16@230\,mm$$

Curb design:

Dead moment:

$W_d = \gamma_{conc.} \times b \times h + w_{handrail} = 24 \times 0.6 \times 0.5 + 1 = 8.6$ kN/m

$$M_d = \frac{w_d \times l^2}{8} = \frac{8.6 \times (5.3)^2}{8} = 30.2 \text{ kN.m}$$

Live moment:

$M_L = 0.1 \times P \times S = 0.1 \times 72 \times 5.3 = 38.16$ kN.m

Total moment $M_t = M_d + M_l = 30.2 + 38.16 = 68.36$ *kN.m*

Design:

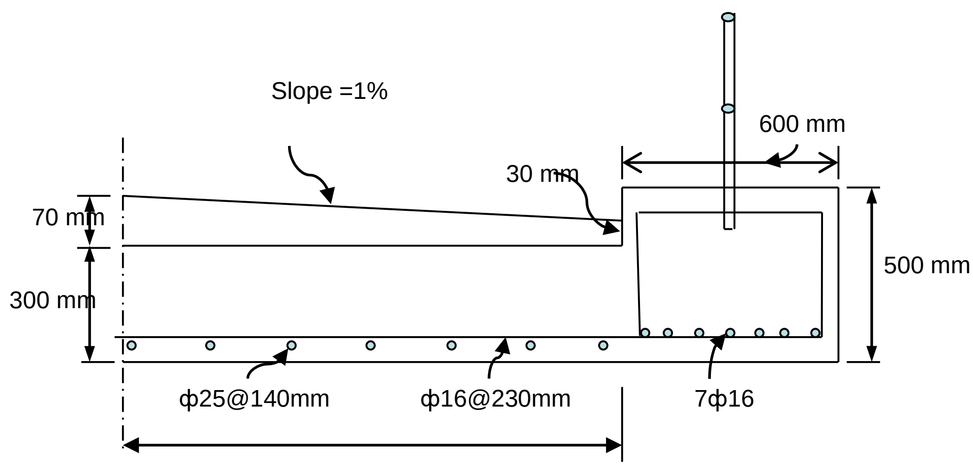
n= 8.46, r=14, k=0.377, j=0.874

$$d_{req.} = \sqrt{\frac{2 \times M_t}{f_c \max. \times K \times J \times b}} = \sqrt{\frac{2 \times (68.36 \times 10^6)}{10 \times 0.377 \times 0.874 \times 1000}} = 262.975\, mm,$$

$$d_{ava.} = t - cover - d_s - \phi/2 = 500 - 50 - 16 - 16/2 = 426\, mm > d_{req.} \quad \text{singly reinforced section}$$

$$A_{st.} = \frac{M_t}{f_s \times J \times d} = \frac{68.36 \times 10^6}{140 \times 0.874 \times 426} = 1311\, mm^2$$

NO. of $\Phi 16 = \frac{As}{A_{bar}} = \frac{1311}{201} = 6.5$ use $7\phi 16$



H.W.

Resolve the previous example with

Width = 8 m	Span c/c = 7.4 m	Clear span =7 m	Asphalt Wt =1.5 kn/m^2
Truck: MS18	fc' = 25 Mpa	fy = 300 Mpa	Hand rail Wt = 1 Kn/m

Example: Design the simply supported reinforced concrete girder-deck bridge having Cross-section shown in the figure and the following data:

Width = 8.7 m

Span c/c = 15.6 m

Clear span = 15 m

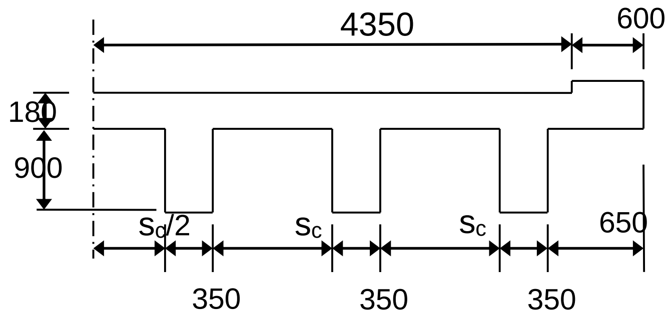
Truck: MS18

fc' = 25 Mpa & fy = 400 Mpa

Asphalt Wt =1.5 kn/m^2

Hand rail Wt = 1 Kn/m

Curb height = 150 mm



Cross-section (all dimensions in mm)

Solution:

-Slab design:

$$2.5\times s_c = \{8700/2+600-650-3\times 350\} \rightarrow s_c = 1300\,mm$$

$$t_{\min.} = \frac{l}{28} = \frac{1300}{28} = 46.4mm < t = 180mm \quad \text{ok}$$

$w_d = \gamma_{conc.} \times t + \text{asphalt } w_t$ = 24×0.18 + 1.5 = 5.82 Kn/m^2

$$M_d = \frac{w_d \times (s)^2}{10} = \frac{5.82\times(1.3)^2}{10} = 0.984\,\text{Kn.m/m}$$

$$M_l = 0.8\times\frac{3.28\times s_c + 2}{32}\times p = 0.8\times\frac{3.28\times 1.3+2}{32}\times 72 = 11.275\,\text{Kn.m/m}$$

$$I = \frac{15.24}{l+38.1} \le 30\% = \frac{15.24}{1.3+38.1} = 0.387 > 0.3$$

$use\ I = 0.3$

$M_I = I\times M_l$ = 0.3 × 11.275 = 3.383 Kn.m/m

<u>*Total moment*</u> $M_t = M_d+M_l+M_I$ = 0.984+11.275+3.383 = 15.642 Kn.m/m

$$n = \frac{E_s}{E_c} = \frac{200000}{4730\sqrt{fc}} = \frac{200000}{4730\sqrt{25}} = 8.46\ , \qquad r = \frac{f_s}{f_c\max.} = \frac{170}{0.4\times 25} = 17$$

$$K=\frac{n}{n+r}=\frac{8.46}{8.46+17}=0.332, \qquad J=1-\frac{K}{3}=1-\frac{0.332}{3}=0.889$$

$$d_{req.}=\sqrt{\frac{2\times M_t}{f_c \max.\times K\times J\times b}}=\sqrt{\frac{2\times(15.642\times10^6)}{10\times0.332\times0.889\times1000}}=102.94\,mm,$$

$$d_{ava.}=t-cover-\phi/2=180-25-16/2=147\,mm>d_{req.} \qquad \text{ok}$$

$$A_{st.}=\frac{M_t}{f_s\times J\times d}=\frac{15.642\times10^6}{170\times0.889\times147}=704.1\,mm^2/m$$

$$s=1000\times\frac{A_{bar}}{A_{st.}}=1000\times\frac{201}{704.1}=285.5\,mm \quad , \quad \text{use } \phi16@280\,mm \text{ top \& bottom, } A_{st}=1000\times\frac{201}{280}=717.8mm$$

Secondary reinforcement:

$$A_{sd.}=\frac{2.2}{\sqrt{3.28s_c}}\times A_{st.}\le 67\%\times A_{st.}=\frac{2.2}{\sqrt{3.28\times1.3}}\times A_{st.}\le 0.67\times A_{st.}=1.06\times A_{st.}\le 0.67\times Ast.$$

$$A_{sd}=0.67\times717.8=481\,mm^2/m$$

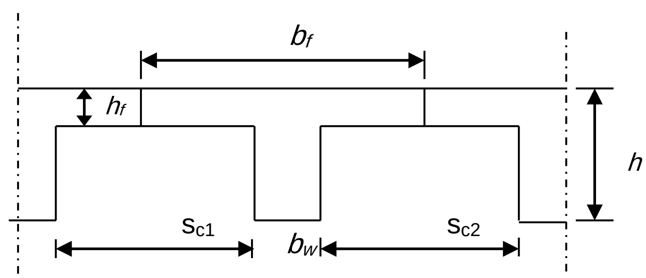
$$s=1000\times\frac{A_{bar}}{A_{st.}}=1000\times\frac{113}{481}=235\,mm \qquad \text{use } \phi12@230\,mm \text{ top \& bottom}$$

-Interior Girder:

h = 900+180= 1080 mm = 1.08 m

span l = min. $[l_{c/c}\ ,\ l_c+h]$

= min. [*15.6* , *15+1.08*]= 15.6 m



$$b_f=\min.\{\frac{l}{4},\ 16\times h_f+b_w,\ \frac{s_{c1}+s_{c2}}{2}+b_w\}$$

$$=\min.\{\frac{15.6}{4},\ 16\times0.18+0.35,\ \frac{1.3+1.3}{2}+0.35\}$$

$$=\min.\{3.9,\ 3.23,\ 1.65\}=\ 1.65m$$

Dead moment:

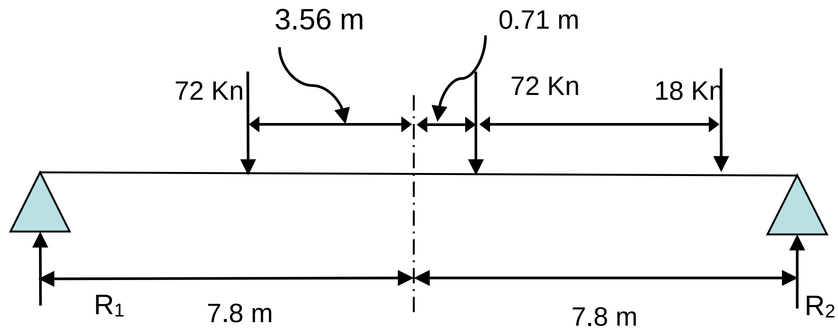
$$W_d=W_{slab}\times b_f+\gamma_{conc.}\times b_w\times(h-h_f)$$

$$=5.82\times1.65+24\times0.35\times0.9$$

$$=17.163\,\text{Kn/m}$$

$$M_d = \frac{W_d \times (l)^2}{8} = \frac{17.163 \times (15.6)^2}{8} = 522.1 \text{ Kn.m}$$

Live moment:



$$\sum F_y = R_y$$

$$72 + 72 + 18 = R \quad \Rightarrow \quad R_y = 162 kN$$

$$\sum M_{72} = R_y \times X$$

$$72 \times 4.27 - 18 \times 4.27 = 162 \times X$$

$$X = 1.42 \qquad \Rightarrow \quad \text{X/2} = 0.71 \text{m}$$

$$\sum M_{R2} = 0$$

$$R_1 = \frac{162 \times (7.8 + 0.71)}{15.6} = 88.373 \text{ Kn}$$

$$M_{\text{max.}} = R_1 \times (7.8 + 0.71) - 72 \times 4.27$$

$$M_{\text{max.}} = 444.615 \text{ Kn.m}$$

$f_{\text{int.}} = 0.66 \times s$ & $s = s_c + b_w = 1.3 + 0.35 = 1.65$ m

$= 0.66 \times 1.65 = 1.089$

$$\therefore \quad M_l = f_{\text{int.}} \times M_{\text{max.}} = 1.089 \times 444.615 = 484.186 \ \text{Kn.m}$$

Impact moment:

$$I = \frac{15.28}{l + 38.1} = \frac{15.28}{15.6 + 38.1} = 0.284 < 0.3$$

$$\therefore I = 0.284$$

$M_I = I \times M_l = 0.284 \times 484.186 = 137.51$ kN.m

$M_t = M_d + M_l + M_I = 522.1 + 484.186 + 137.51 = 1143.8$ *kN.m*

Use three layers and Φ36

$$d = h - \text{cover} - d_s - \phi - 40 - \frac{\phi}{2} = 1080 - 50 - 14 - 36 - 40 - \frac{36}{2} = 922\,\text{mm}$$

$$A_s = \frac{M_t}{fs \times (d - \frac{h_f}{2})} = \frac{1143.8 \times 10^6}{170 \times (922 - \frac{180}{2})} = 8086.8\ mm^2$$

$$No.\ \text{of}\ \varphi 36\ = \frac{A_s}{A_{bar}} = \frac{8086.8}{1018} = 7.9 \Rightarrow \quad use \quad 8\varphi 36$$

b_{req}=2×cover+2×d_s+n×Φ+(n-1) s=2×50+2×14+3×36+2×36

=308 mm < b_w=350 mm ok

Shear design:

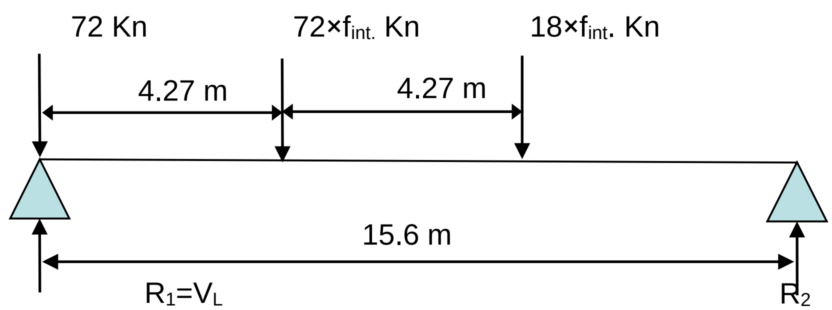
$$V_d = \frac{W_d \times l}{2} = \frac{17.163 \times 15.6}{2} = 155.87\,\text{Kn}$$

ملاحظة : اكبرقوة قص تكون عادة في المساند وتوضع اثقل عجلة على المسند و لا تضرب بمعامل التوزيع

$$\sum M_{R2} = 0$$

$$V_L = \frac{72 \times (15.6) + 72 \times f_{int.} \times (15.6 - 4.27) + 18 \times f_{int.} \times (15.6 - 2 \times 4.27)}{15.6} = 137.82\ \text{Kn}$$

$$V_t = V_d + V_L \times (1 + I)$$
$$= 133.87 + 137.82 \times (1.284)$$
$$= 310.83\,\text{Kn}$$



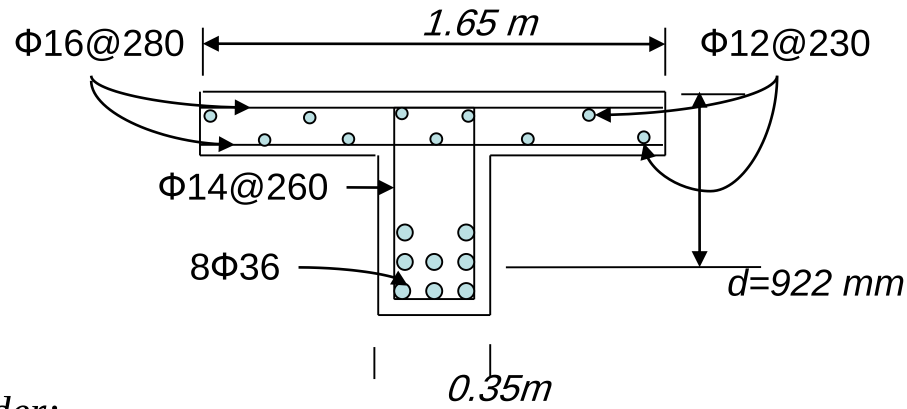
$$V_c = 0.079\sqrt{f_c'} \times b_w \times d$$
$$= 0.79\sqrt{25} \times 350 \times 922 \times 10^{-3}$$
$$= 127.5\ \ \text{Kn} \qquad \& \quad 3V_c = 382.4\ \ \text{Kn}$$

$$V_c < V_t < 3V_c$$

$$\therefore\ Spacing = \min.\{\frac{d}{2}, 600, \frac{3.Av.fy}{b_w}, \frac{Av.fs.d}{V - V_c}\} \qquad \text{Av} = 2 \times \frac{\pi}{4} \times 14^2\ = 307.9\,\text{mm}^2$$

$$= \min.\{\frac{922}{2}, 600, \frac{3 \times 307.9 \times 400}{350_w}, \frac{307.9 \times 170 \times 922}{(310.83 - 127.5) \times 10^3}\}$$

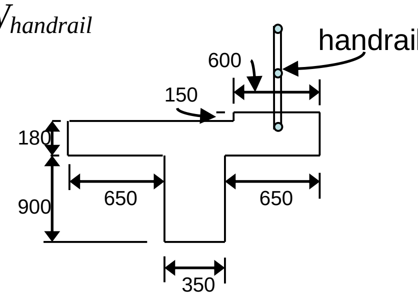
$$= \min.\{461, 600, 1056, 263\} \qquad \therefore\ \text{use}\ \phi 14@260$$



-Exterior Girder:

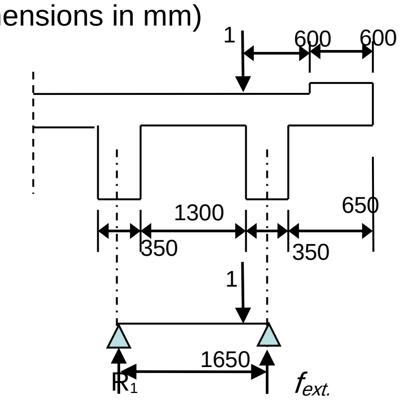
$$W_d = \gamma_{conc.}\times(0.9\times0.35+0.18\times1.65+0.15\times0.6)+W_{asphalte.}\times b_f+W_{handrail}$$

$$=20.323\ \text{Kn/m}$$



$$M_d = \frac{W_d\times(l)^2}{8} = \frac{20.323\times(15.6)^2}{8} = 618.226\ \text{Kn.m}$$

(all dimensions in mm)



$$\sum M_{R1} = 0$$

$$f_{ext,} = \frac{1\times1.275}{1.65} = 0.773$$

$$\therefore\ M_l = f_{ext.}\times M_{max.} = 0.773\times444.615 = 343.7\ \text{Kn.m}$$

$$M_t = M_d+M_l+M_I = 618.226+343.7\times(1+I)$$

$$= 1059.5\ \text{Kn.m} < M_t\ \text{interior}$$

يوصي ال(code) بان تسليح (exterior girder) اكبر او يساوي تسليح(interior girder)

لذلك يعاد نفس تسليح ال (interior girder).

Shear design:

$$V_d = \frac{W_d\times l}{2} = \frac{20.323\times15.6}{2} = 158.5\ \text{Kn}$$

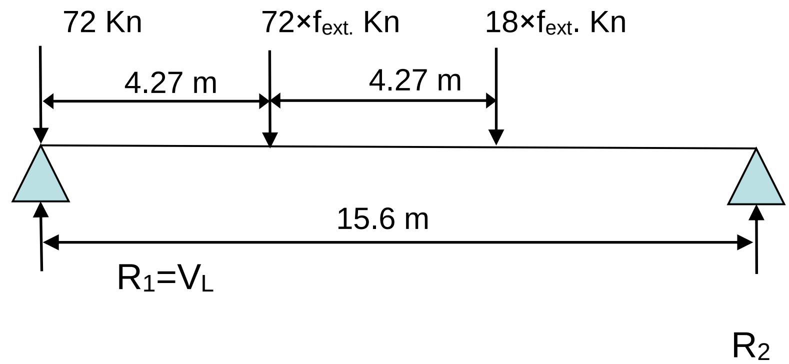
$$\sum M_{R2=0}$$

$$V_L = \frac{72\times(15.6)+72\times f_{ext.}\times(15.6-4.27)+18\times f_{ext}.\times(15.6-2\times 4.27)}{15.6} = 118.72\,\text{Kn}$$

$$V_t = V_d + V_L\times(1+I)$$

$$= 158.5 + 118.72\times(1.284)$$

$$= 310.96\,\text{Kn} \approx \text{V}_\text{t}\ \text{interior}$$

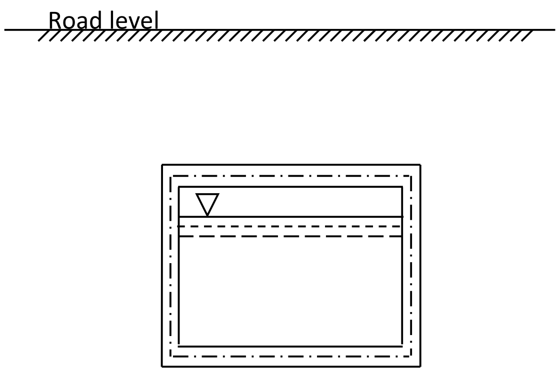


نستعمل نفس التسليح للعزم والقص بالنسبة لل (exterior girder) و ال (interior girder).

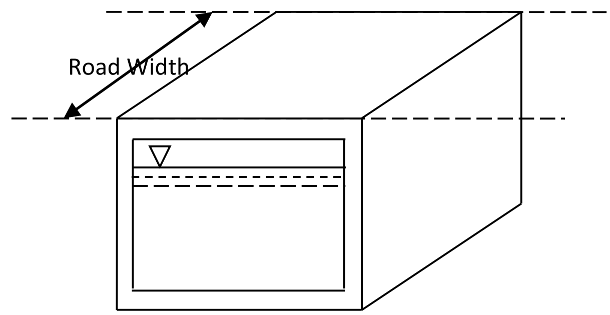
Box Culverts

These are provided for conveying water to serve the following requirements:

- To serve as means for a cross drain.
- To provide a supporting slab for a roadway, under which the cross-drainage flows.



Box Culvert



Box Culvert

The culvert should be designed to remain safe for the following cases:

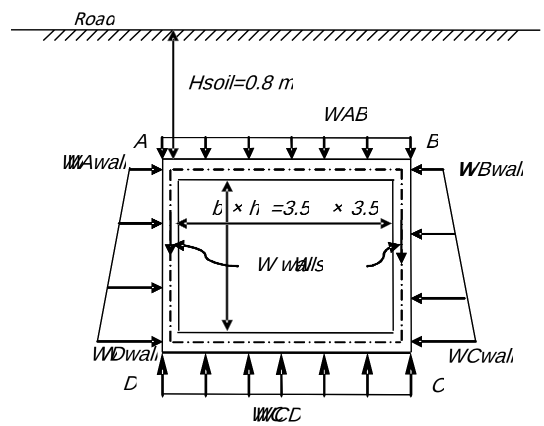
- **Case I.** When the top slab carries the dead and the live load and the culvert is empty.
- **Case II.** When the top slab carries the dead and the live load and the culvert is full of water.
- **Case III.** When the sides of the culvert do not carry the live load and the culvert is full.

Example: Design a box culvert having inside dimensions *b×h = 3.5m × 3.5m. The* live load on the culvert is *50 Kn/m²*. The soil at the site weight *18 Kn/m³* having an angle of repose of *30 º*. *The* culvert is 0.8 m below the road.

Solution:

Case I. When the top slab carries the dead and the live load and the culvert is empty.

Loads:



$$total\ load\ on\ top\ slab = \gamma_{conc.} \times t_{AB} + \gamma_{soil} \times h_{soil} + W_{live}$$

Weight of vertical wall = $\gamma_{conc.} \times t \times h_{c/c}$

$$+\uparrow \sum F_y = 0$$

Reaction of the bottom salb (W_{CD})= $\frac{(2\times W_{wall} + W_{AB} \times b_{c/c})}{b_{c/c}}$

At any depth from the level of the road, lateral pressure

$$W_{B(wall)} = W_{A(wall)} = (W_L + \gamma_{soil} \times (H_{soil} + t/2)) \times K_a$$

$$W_{C(wall)} = W_{D(wall)} = (W_L + \gamma_{soil} \times (H_{soil} + t/2 + h_{c/c})) \times K_a$$

WL=	50	Kn/m^2		H_{soil} =	0.8	m	Φ =	30
γconc.=	24	Kn/m^3		b =	3.5	m		
γsiol=	18	Kn/m^3		h =	3.5	m		
γwater=	10	Kn/m^3		use t =	0.35	m		
K_0 =	0.500		$h_{c/c}$ = $b_{c/c}$=		3.85	m		
WAB =		72.8	kN/m^2					
W_{wall}		32.34	kN/m					
WA_{wall} = WB_{wall} =		33.774928	kN/m^2					
WC_{wall} = WD_{wall} =		68.424855	kN/m^2					
WCD =		89.6	kN/m^2					

joint	A		B		C		D	
member	AD	AB	BA	BC	CB	CD	DC	DA
D.F.	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
F.E.M.	-58.83902	89.923167	-89.92317	58.839024	-67.399	110.67467	-110.6747	67.399
Bal.	-15.54207	-15.54207	15.542071	15.542071	-21.63783	-21.63783	21.637833	21.637833
CO.	10.818917	7.7710356	-7.771036	-10.81892	7.7710356	10.818917	-10.81892	-7.771036
Bal.	-9.294976	-9.294976	9.2949762	9.2949762	-9.294976	-9.294976	9.2949762	9.2949762
M total	-72.85715	72.857155	-72.85715	72.86	-90.56077	90.560774	-90.56077	90.560774

RA =RB=	140.14	Kn/m	on the top slab AB					
MAB midspan=		62.0275951	Kn.m/m	cut of points @ x =			0.62	m
							3.23	m
RC =RD=	172.48	Kn/m	on the bottom slab CD					
MCD midspan=		-75.451226	Kn.m/m	cut of points @ x =			0.63	m
							3.22	m
RB=RA=	82.652097	Kn/m	on the side walls					
RC=RD=	114.08249	Kn/m	on the side walls					

$VBC = RB - Wbwall*X - (WC-WB)*X^2/2$ put VBC =zero find X X= 1.94 m

$M_{BC} = -M_{bwall} + R_{Bwall} \times X - W_{BWALL} \times X^2/2 - (W_C - W_B) \times X^3/(6 \times hc/c)$

$M_{BC} = -72.86 + 82.65X - 16.8875X^2 - 1.5X^3$ for X=1.94 M=12.97 kN.m/m

Case II:

WL=	50	Kn/m^2		H_{soil} =	0.8	m
γconc.=	24	Kn/m^3		b =	3.5	m
γsiol=	18	Kn/m^3		h =	3.5	m
γwater=	10	Kn/m^3		use t =	0.35	m
K_0 =	0.500		$h_{c/c} = b_{c/c}$=		3.85	m
WAB =		72.8	Kn/m^2			
W_{wall}		32.34	Kn/m^2			
$WA_{wall} = WB_{wall}$ =		33.775	Kn/m^2			
$WC_{wall} = WD_{wall}$ =		29.925	Kn/m^2			
WCD =		89.6	Kn/m^2			

joint	A		B		C		D	
member	AD	AB	BA	BC	CB	CD	DC	DA
D.F.	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
F.E.M.	-39.81694	89.923167	-89.92317	39.816941	-38.86583	110.67467	-110.6747	38.86583
Bal.	-25.05311	-25.05311	25.053113	25.053113	-35.90442	-35.90442	35.904418	35.904418
CO.	17.952209	12.526557	-12.52656	-17.95221	12.526557	17.952209	-17.95221	-12.52656
Bal.	-15.23938	-15.23938	15.239383	15.239383	-15.23938	-15.23938	15.239383	15.239383
M total	-62.15723	62.157227	-62.15723	62.157227	-77.48307	77.483075	-77.48307	77.483075

RA =RB=	140.14	Kn/m	the top slab AB			
MAB midspan=	72.7275227	Kn.m/m	cut of points @ x =		0.511	m
					3.338	m
RC =RD=	172.48	Kn/m	on the bottom slab CD			
MCD midspan=	-88.528925	Kn.m/m	cut of points @ x =		0.52	m
					3.33	m
RB=RA=	58.565719	Kn/m	n the side walls			
RC=RD=	64.056781	Kn/m	n the side walls			

$V_{BC} = R_c - W_c \times X - (WB-Wc)/2hc/c \times X^2$

$V_{BC} = 64.06 - 29.925X - 0.5X^2$ put V=0 find X= 2.07 m

$M_{BC} = -M_{cwall} + R_{cwall} \times X - W_{cWALL} \times X^2/2 - (W_B - W_C) \times X^3/(6 \times hc/c)$

$M_{BC} = -77.48 + 64.06X - 14.96X^2 - 0.1667X^3$ at x=2.07 m M=-10.456 kN.m/m

Case III:

WL=	50	Kn/m²		H_{soil} =	0.8	m
γconc.=	24	Kn/m³		b =	3.5	m
γsiol=	18	Kn/m³		h =	3.5	m
γwater=	10	Kn/m³		use t =	0.35	m
Ka=	0.500		$h_{c/c}$ = $b_{c/c}$=		3.85	m

WAB =	72.8	Kn/m²
W_{wall}	32.34	Kn/m²
WA_{wall} = WB_{wall} =	8.775	Kn/m²
WC_{wall} = WD_{wall} =	4.925	Kn/m²
WCD =	89.6	Kn/m²

joint	A		B		C		D	
member	AD	AB	BA	BC	CB	CD	DC	DA
D.F.	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
F.E.M.	-8.936732	89.923167	-89.92317	8.9367323	-7.99	110.67467	-110.6747	7.99
Bal.	-40.49322	-40.49322	40.493217	40.493217	-51.34452	-51.34452	51.344522	51.344522
CO.	25.672261	20.246609	-20.24661	-25.67226	20.246609	25.672261	-25.67226	-20.24661
Bal.	-22.95943	-22.95943	22.959435	22.959435	-22.95943	-22.95943	22.959435	22.959435
M total	-46.71712	46.717123	-46.71712	46.717123	-62.04297	62.042971	-62.04297	62.042971

RA =RB=	140.14	Kn/m	the top slab AB		
MAB midspan=	88.1676268	Kn.m/m	cut of points @ x =	0.37	m
				3.48	m
RC =RD=	172.48	Kn/m	on the bottom slab CD		
MCD midspan=	-103.96903	Kn.m/m	cut of points @ x =	0.4	m
				3.45	m
RB=RA=	10.440719	Kn/m	n the side walls		
RC=RD=	15.931781	Kn/m	n the side walls		

$V_{BC} = R_c - W_c \times X - (WB-WC)/2hc/c \times X^2$

$V_{BC} = 15.93 - 4.925X - 0.5X^2$ put V=0 find X= 2,566 m

$M_{BC} = -M_{cwall} + R_{cwall} \times X - W_{cWALL} \times X^2/2 - (W_B - W_C) \times X^3/(6 \times hc/c)$

$M_{BC} = -62.04 + 15.93X - 2.46X^2 - 0.1666X^3$ at x=2.087 m M=-40.177 Kn.m/m

Circular Tanks

Example: Find the maximum reinforcement for the circular tank having the following data:

Base : fixed	Tensial strength of concrete = 2 Mpa		H= 6 m
f'_c=20 Mpa	*fs (hoop)=95Mpa*	*fs (vertical)=135Mpa*	D(inside) =16 m
γ_w=10 kn/m^3	$C=300\times10^{-6}$	*n =10*	E_s =200 Gpa

Solution:

Assume t= 250 mm

$$\frac{H^2}{D.t}=\frac{(6)^2}{16\times0.25}=9$$

$T=c_1.\gamma HR$ (c_1 from table A 3.1) $c_1=0.5915$

$M=c_2.\gamma H^3$ (c_2 from table A 3.3) $c_2=0.00335$

$V=c_3.\gamma H^2$ (c_3 from table A 3.12) $c_3=0.166$

$T_{max.}=0.5915(480)=283.92$ kn/m @0.6H

$M_{max.}+ve=0.00335(2160)=7.236$ kn.m/m @0.7H

$M_{max.}-ve=0.0134(2160)=28.944$ kn.m/m @the base

$V_{max.}=0.166(360)=59.76$ kn/m

Design of hoop reinforcement (horizontal reinforcement)

$$A_s)_{hoop}=\frac{T_{max.}}{f_s)_{hoop}}=\frac{283.92\times10^3}{95}=2988.63\text{ mm}^2/m>A_s)_{min}=0.0025Ag$$

$$=0.0025(1000\times250)$$

$$=625\text{ mm}^2/m$$

$$use\ \phi16\text{m3m s}=1000\times\frac{\pi/4(16)^2}{2988.63}=134.5mm<s_{max.}=\min.(3t,500)=500mm$$

use ϕ16@125 mm c/c each face

cheack stress in concrete

$$f_{ct}=\frac{c\times Es\times A_s)_{hoop}+T}{Ag+nA_s)_{hoop}}=\frac{300\times10^{-6}\times200000\times2988.63+283.92\times10^3}{1000\times250+10\times2988.63}=1.655\text{ Mpa}<2\text{Mpa}$$

Design of flexure reinforcement (vertical reinforcement)

$$r=\frac{fs}{fc}=\frac{135}{0.45\times 20}=15$$

$$k=\frac{n}{n+r}=\frac{10}{10+15}=2/5\ , \quad \text{j=1-}\frac{k}{3}=1-\frac{2/5}{3}=13/15$$

use $\phi 20$

$$\text{d=250-25-}\frac{20}{2}=215\ mm$$

$$d_{req.}=\sqrt{\frac{2M}{fc.k.j.b}}=\sqrt{\frac{2\times(28.944\times 10^6)}{9\times(\frac{2}{5})\times(\frac{13}{15})\times(1000)}}=136.5\ \text{mm} < \text{d} \quad \text{o.k}$$

$$\text{As}_1=\frac{M}{fs.j.d}=\frac{28.944\times 10^6}{135\times(\frac{13}{15})\times 215}=1150.626\ \text{mm}^2/m$$

$$As_2=\frac{M}{fs.j.d}=\frac{7.236\times 10^6}{135\times(\frac{13}{15})\times 215}=287.66\ \text{mm}^2/m$$

$$A_{s.\min}=0.0015Ag=0.0015(1000\times 250)=375\ mm^2/m$$

use $\text{As}_2=A_{s.\min}=375\ \text{mm}^2/m$

$$s_1=1000\times\frac{314}{115.626}=272.9\ mm < S_{\max.}=\min(3t,500)=500\ \text{mm}$$

use $\phi 20$@250 mm c/c (inner face)

$$s_1=1000\times\frac{113}{375}=301.33\ mm < S_{\max.}=\min(3t,500)=500\ \text{mm}$$

use $\phi 12$@300 mm c/c (outer face)

Check Shear:

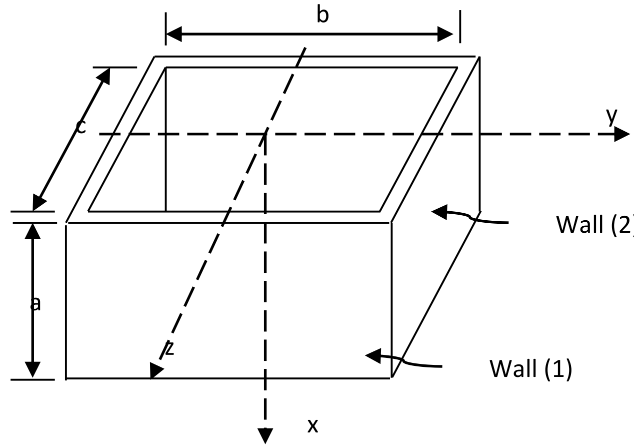
$\text{V}_{\text{base}}=59.76\ \text{kn}$

$$\text{V}_c=0.09\sqrt{f_c'}b.d=0.09\sqrt{20}\times 1000\times 215\times 10^{-3}=86.5358\ \text{kn/m}$$

$\text{V}_{base} < V_c$ o.k

<u>H.W. Re-design the tank of previous example assuming hinge base</u>

Example : Design the critical sections for the rectangular tank shown in figure , use the following data:



a = 6 m	*b = 15 m*	*c = 7.5 m*
fy=300 Mpa	f'_c *=20 Mpa*	*n = 9*
top : free	*bottom:hinged*	$\gamma_w = 10\ kn/m^3$

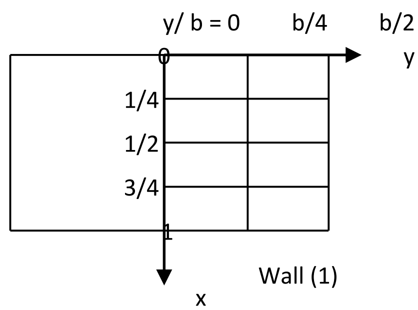
Solution :

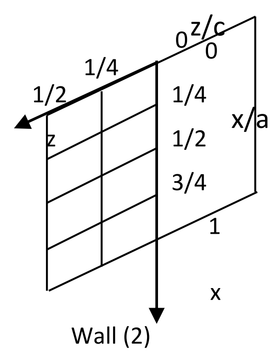
$$\frac{b}{a} = \frac{15}{6} = 2.5$$

$$\frac{c}{a} = \frac{7.5}{6} = 1.25$$

Moment Coefficient: *top: free*

Bottom: hinge





Wall (1)

	y=0		*y =b/4*		*y=b/2*		*z=c/4*		*z=0*	
x /a =	M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
0	*0*	*0.069*	*0*	*0.035*	*0*	*-0.092*	*0*	*-0.030*	*0*	*-0.010*
1/4	*0.025*	*0.059*	*0.015*	*0.034*	*-0.018*	*-0.089*	*-0.005*	*-0.024*	*-0.002*	*-0.003*
1/2	*0.045*	*0.048*	*0.031*	*0.031*	*-0.016*	*-0.082*	*0.003*	*-0.012*	*0.008*	*0.007*
3/4	*0.044*	*0.029*	*0.034*	*0.02*	*-0.012*	*-0.059*	*0.011*	*-0.002*	*0.018*	*0.008*

$M = C_1 \times \gamma_w \times a^3 = C_1 \times 10 \times 6^3 = C_1 \times (2160)$

Shear Coefficient: *top: free*

Bottom: hinge

b /a =	*1*	*2*	*3*
top of fixed side edge	*0.01*	*0.1*	*0.165*
mid-point of fixed side edge	*0.258*	*0.375*	*0.406*
lower third-point of fixed side edge	*0.311*	*0.406*	*0.416*
lower quarter point	*0.315*	*0.39*	*0.398*

$V = C_2 \times \gamma_w a^2 = C_2 \times 10 \times 6^2 = C_2 \times 360$

To find wall thickness:

For moment:

n = 9 : $r = \frac{f_s}{0.45 f_c'} = \frac{140}{0.45 \times 20} = 15.556$: $k = \frac{9}{9 + 15.556} = 0.3667$: $j = 1 - \frac{k}{3} = 1 - \frac{0.3667}{} = 0.877$

$M_{max.}$ = -0.092×*2160 =198.75 kn.m/m* (M_y @ x /a =0 & y = b /2)

$$d_{req.} = \sqrt{\frac{2M}{f_c.k.j.b}} = \sqrt{\frac{2 \times 198.75 \times 10^6}{9 \times 0.3667 \times 0.877 \times 1000}} = 370.5 \text{ mm}$$

For shear:

Maximum coeff. (@ lower third)

$$V_{max.} = \left(\frac{0.406 + 0.416}{2}\right) \times 360 = 147.96 \text{ kn/m}$$

$$V_c = \frac{\sqrt{f_c^{'}}}{11} \times b \times d_{req.} \rightarrow 147.96 \times 1000 = \frac{\sqrt{20}}{11} \times 1000 \times d_{req.}$$

$$d_{req.} = 363.93 \text{ mm}$$

Use *d = 400 mm → h=d+50 = 450 mm*

Flexural design:

A) Design for horizontal reinforcement (due to M & p)

1- For negative moment @ corner (junction between wall(1) & wall (2)). (due to M_y & P_2)

Shear force on wall (1) = axial force on wall (2)

$V_1 = P_2$ $\left[\frac{b}{a} = 2.5\right]$ for shear calculation

@ Top: $\left(\frac{x}{a} = 0, y = \frac{b}{2}\right)$

$$V_1 = P_2 = \frac{0.1 + 0.165}{2} \times 360 = 47.7 \text{ kn/m}$$

$$M_y = 0.092 \times 2160 = 198.75 \text{ kn.m/m}$$

@ Mid-point $\left(\frac{x}{a} = \frac{1}{2}, y = \frac{b}{2}\right)$

$$V_1 = P_2 = \frac{0.375 + 0.406}{2} \times 360 = 140.58 \text{ kn/m}$$

$$M_y = 0.082 \times 2160 = 177.12 \text{ kn.m/m}$$

@ Lower-quarter-point $\left(\frac{x}{a} = \frac{3}{4}, y = \frac{b}{2}\right)$

$$V_1 = P_2 = \frac{0.39 + 0.398}{2} \times 360 = 141.84 \text{ kn/m}$$

$$M_y = 0.059 \times 2160 = 127.44 \text{ kn.m/m}$$

Design:

$$d'' = d - \frac{h}{2} = 400 - \frac{450}{2} = 175\,\text{mm}, \quad M_s = M + P \times d'', \quad A_{st} = \frac{M_s}{f_s jd} - \frac{P}{f_s},$$

use $\phi 25$, $A_{bar} = 490\ mm^2$, $A_{st\ min.} = 0.0025 A_g = 0.0025 \times b \times t = 0.0025 \times 1000 \times 450 = 1125\ \text{mm}^2/m$

location	M (kn.m/m)	p (kn)	M_s (kn.m/m)	$A_{st.}$ (mm^2/m)	S (mm)
top $\left(\frac{x}{a} = 0, y = \frac{b}{2}\right)$	*198.75*	*47.7*	*190.4025*	*3876.9+340.7* *= 4217.6*	***116. 3 use*** ***Φ25@110***
Mid $\left(\frac{x}{a} = \frac{1}{2}, y = \frac{b}{2}\right)$	*177.12*	*140.58*	*152.5185*	*3105.5+1004.1* *=4109.7*	***119.23 use*** ***Φ25@110***
Lower $\left(\frac{x}{a} = \frac{3}{4}, y = \frac{b}{2}\right)$	*127.44*	*141.84*	*102.618*	*2089.5+1013.1* *=3102.6*	***157.93 use*** ***Φ25@150***

2. For positive moment @ wall(1) (due to M_y & $P_1 = V_2$)
   Shear force on wall (2) = axial force on wall (1)

$$V_2 = P_1 \left[\frac{b}{a} = 1.25\right] \text{ for shear calculation}$$

@ Top: $\left(\frac{x}{a} = 0, y = 0\right)$

$$V_2 = P_1 = (0.01 + \frac{0.1 - 0.01}{4}) \times 360 = 11.7\ \text{kn/m}$$
$$M_y = 0.069 \times 2160 = 149.04\ \text{kn.m/m}$$

@ Mid-point $\left(\frac{x}{a} = \frac{1}{2}, y = 0\right)$

$$V_2 = P_1 = (0.258 + \frac{0.375 - 0.258}{4}) \times 360 = 103.41\ \text{kn/m}$$
$$M_y = 0.048 \times 2160 = 103.68\ \text{kn.m/m}$$

@ Lower-quarter-point $\left(\frac{x}{a} = \frac{3}{4}, y = 0\right)$

$$V_2 = P_1 = (0.319 + \frac{0.39 - 0.315}{4}) \times 360 = 120.15\ \text{kn/m}$$
$$M_y = 0.029 \times 2160 = 62.64\ \text{kn.m/m}$$

location	M (kn.m/m)	p (kn)	M_s (kn.m/m)	$A_{st.}$ (mm^2/m)	S (mm)
top $\left(\frac{x}{a}=0, y=0\right)$	*149.04*	*11.7*	*146.99*	2993+83.6 = 3076.6	***159.3 use Φ25@150***
Mid $\left(\frac{x}{a}=\frac{1}{2}, y=0\right)$	*103.68*	*103.41*	*85.58*	1742.6+738.6 = 2481.3	***197.5 use Φ25@190***
Lower $\left(\frac{x}{a}=\frac{3}{4}, y=0\right)$	*62.64*	*120.15*	*41.61*	847.3+858.2 = 1705.5	***287.3 use Φ25@280***

3. For positive moment @ wall (2) (due to M_z & $P_2=V_1$)

Shear force on wall (1) = axial force on wall (2)

$$V_1 = P_2 \left[\frac{b}{a} = 2.5\right] \text{ for shear calculation}$$

@ Top: $\left(\frac{x}{a}=0, z=0\right)$

$$V_1 = P_2 = \frac{0.1+0.165}{2} \times 360 = 47.7 \text{ kn/m}$$

$$M_z = 0.01 \times 2160 = 21.6 \text{ kn.m/m}$$

@ Mid-point $\left(\frac{x}{a}=\frac{1}{2}, z=0\right)$

$$V_1 = P_2 = \frac{0.375+0.406}{2} \times 360 = 140.58 \text{ kn/m}$$

$$M_z = 0.007 \times 2160 = 15.2 \text{ kn.m/m}$$

@ Lower-quarter-point $\left(\frac{x}{a}=\frac{3}{4}, z=0\right)$

$$V_1 = P_2 = \frac{0.39+0.398}{2} \times 360 = 141.84 \text{ kn/m}$$

$$M_z = 0.008 \times 2160 = 17.28 \text{ kn.m/m}$$

location	M (kn.m/m)	p (kn)	M_s (kn.m/m)	$A_{st.}$ (mm^2/m)	S (mm)
top $\left(\frac{x}{a}=0, z=0\right)$	*21.6*	*47.7*	*13.25*	*269.8+340.7 = 610.5 use 1125*	***435.5 use Φ25@430***
Mid $\left(\frac{x}{a}=\frac{1}{2}, z=0\right)$	*15.2*	*140.58*	*9.4*	*191.4+1004 = 1195*	***409 use Φ25@400***
Lower $\left(\frac{x}{a}=\frac{3}{4}, z=0\right)$	*17.28*	*141.84*	*7.5*	*152.7+1013 = 1165*	***420.3 use Φ25@420***

$$S_{max.} = \min[3t, 500] = 500 \text{ mm}$$

B) Design for vertical reinforcement (due to M_x)

Use Φ16 , $A_{bar.} = \frac{\pi}{4} \times 16^2 = 201\,mm^2$, $A_{st.min.} = 0.0015 A_g = 0.0015 \times 1000 \times 450 = 675\,mm^2/m$

	location	Coefficient	M (kn.m/m)	$A_{st.}$ (mm^2/m)	S (mm)
wall (1) long wall	$\left(\frac{x}{a}=\frac{1}{2}, y=0\right)$	*+0.045*	*97.2*	*1979.15*	***101.5 use Φ16@100***
wall (1) long wall	$\left(\frac{x}{a}=\frac{1}{4}, y=\frac{b}{2}\right)$	*-0.018*	*-38.88*	*791.66*	***253.9 use Φ16@250***
wall (2) short wall	$\left(\frac{x}{a}=\frac{3}{4}, z=0\right)$	*0.018*	*38.88*	*791.66*	***253.9 use Φ16@250***
wall (2) short wall	$\left(\frac{x}{a}=\frac{1}{4}, z=\frac{c}{4}\right)$	*-0.005*	*-10.8*	*220<675 use 675*	***297.78 use Φ16@290***