Conjunction الوصل

Any two statemets can be combined by the word "and" to form a composi statement which is called the conjunction of the original statements.

Symbolically, the conjunction of the two statements p and q is denoted by $p \wedge q$

Example:- Let p be "It is raining", and Let q be "The sun is shining."

Then $p \wedge q$ denotes the statement "It is raining and the sun is shining"

The truth value of the composite statement $p \wedge q$ satisfies the following property:-

If p is true and q is true, then $p \wedge q$ is true; otherwise $p \wedge q$ is false.

In other words, the conjunction of two statements is true only if each component is true.

Example:- consider the following four statements

1) Paris is in France and $2+2=5$.

2) Paris is in England and $2+2=4$.

3) Paris is in England and $2+2=5$.

4) Paris is in France and $2+2=4$.

It's clear that only (4) is true. Each of the other statements is false since at least one of its substatements is false.

truth table of "$p \wedge q$" can be written in the form

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

الفصل

Any two statements can be combined by the word "or" to form a new statement which is called the disjunction of the original two statements. Symbolically, the disjunction of statements p and q is denoted by

$$p \vee q$$

Example :- Let p be " He studied French at the university ", and Let q be "He lived in France".

Then $p \vee q$ is the statement " He studied French at the university or he lived in France.

The truth value of the composite statement $p \vee q$ ~~sti~~ satisfies the following property:- If p is true or q is true or both p and q are true, then $p \vee q$ is true ; otherwise, $p \vee q$ is false. In other words, the disjunction of two statements is false only if each component is false

The truth table of "$p \vee q$" can be written in the form

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example:- consider the following four statements

1) Paris is in France or $2+2=5$.

2) Paris is in England or $2+2=4$.

3) Paris is in France or $2+2=4$.

4) Paris is in England or $2+2=5$.

only (4) is false. Each of the other statements is true since at least one of its components is true.

Negation
النفي

Given any statement p, another statement, called the negation of p, can be formed by writing "It is false that ----" before p or, if possible, by inserting in p the word "not".

Symbolically, the negation of p is denoted by $\sim p$

Example ①:- consider the following three statements,

1) Paris is in France.

2) It is false that Paris is in France.

3) Paris is not in France.

Then (2) and (3) are each the negation of (1).

Example ②:- consider the following statements:

1) $2+2=5$

2) It is false that $2+2=5$

3) $2+2\neq 5$

Then (2) & (3) are each the negation of (1).

The truth value of the negation of a statement satisfies the following property:

If p is true, then $\sim p$ is false; if p is false, then $\sim p$ is true.

Example:- consider the statements in Example (1). Notice that (1) is true while (2) & (3) its negations, are false.

Example:- consider the statements in Example (2). Notice that (1) is false while (2) & (3) are true.

p	$\sim p$
T	F
F	T

Conditional الشرط

Many statements, especially in mathematics, are of the form "If p then q". Such statements are called conditional statements and are denoted by

$$p \longrightarrow q$$

The truth value of the conditional statement $p \longrightarrow q$ satisfies the following property :-

The conditional $p \to q$ is true unless p is true and q is false.

The truth table of "$p \longrightarrow q$" can be written in the form

p	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Remark:- consider the conditional proposition $p \longrightarrow q$ and other simple conditional propositions which contain p and q, i.e

$q \longrightarrow p$, $\sim p \longrightarrow \sim q$, and $\sim q \longrightarrow \sim p$, called, respectively, the converse معكوس inverse, and contrapositive propositions.

نظير

ضد موجب

The truth tables of these four propositions are as follows :-

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Example :- Let p :- Noor at home.

q :- Noor answer to the phone.

$p \to q$:- if Noor at home then she will answer to the phon

$q \to p$:- if Noor answer to the phone then she is at home.

$\sim p \to \sim q$:- if Noor is not at home then she isnot answer to the phone.

$\sim q \to \sim p$:- if Noor is not answer to the phone then she is not at home.

Biconditional ثنائية الشرط

Another common statement is of the form "p if and only if q" or, simply, "p iff q". such statements are called biconditional statements and are denoted by $p \longleftrightarrow q$

The truth value of the biconditional statement $p \longleftrightarrow q$ satisfies property:

If p and q have the same truth value, then $p \longleftrightarrow q$ is true;

if p and q have opposite truth values, then $p \longleftrightarrow q$ is false.

Example:- consider the following statements

1) Paris is in France iff 2+2=5.
2) paris is in England iff 2+2=4.
3) paris is in France iff 2+2=4.
4) paris is in England iff 2+2=5.

According, (3) and (4) are true while (1) and (2) are false.

The truth table written as follows

p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T