Linear Algebra

Second Semester

For the 1rd Class Student

Mathematics Department

College of Science for Women

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Linear Transformations التحويلات الخطية

Def :- let V and U be vector spaces over the field K . if L : V ⟹ U is function from V to U , then we say that L is ***linear transformation*** if

(1) for all U , V ∈ V ⟹ L (U + V) = L (U) + L (V)

(2) for k ∈ K , U ∈ V ⟹ L (k U) = k L (U)

Ex:- show that $L : R^3 \Longrightarrow R^2$ be defined by $L(U_1, U_2, U_3) = (U_1, U_2)$ is ***linear transformation***

Sol :-

(1) let U , V ∈ R^3 ⟹ $U = (U_1, U_2, U_3)$ and $V = (V_1, V_2, V_3)$ by def

$L(U+V) = L((U_1, U_2, U_3) + (V_1, V_2, V_3))$

$= L((U_1+V_1, U_2+V_2, U_3+V_3))$ (by sum of vector)

$= ((U_1+V_1), (U_2+V_2))$ (by def of L)

$= (U_1, U_2) + (V_1, V_2)$

$= L(U_1, U_2, U_3) + L(V_1, V_2, V_3)$

$= L(U) + L(V)$

(2) let k ∈ R

$L(kU) = L(k(U_1, U_2, U_3))$

$= L(kU_1, kU_2, kU_3)$

$= (kU_1, kU_2)$

$= k(U_1, U_2)$

$= kL(U_1, U_2, U_3)$

$= kL(U)$

by (1) and (2)

therefore , **L** is *linear transformation*

ملاحظة :-

ان عملية الجمع U + V خاصة بالفضاء V بينما عملية الجمع L (U) + L (V) هي خاصة بالفضاءU وكذلك عملية الضرب .

Ex :- (2) Let $L : R^3 \Longrightarrow R^2$ be defined by

$L(U_1, U_2, U_3) = (U_1, U_1 + U_2 + U_3)$. show that L is linear transformation

Sol :- (1) let $U, V \in R^3$

$$L(U+V) = L((U_1, U_2, U_3) + (V_1, V_2, V_3))$$
$$= L(U_1 + V_1, U_2 + V_2, U_3 + V_3)$$
$$= ((U_1 + V_1), (U_1 + V_1) + (U_2 + V_2) + (U_3 + V_3))$$
$$= (U_1 + V_1, (U_1 + U_2 + U_3) + (V_1 + V_2 + V_3))$$
$$= (U_1, U_1 + U_2 + U_3) + (V_1, V_1 + V_2 + V_3)$$
$$= L(U_1, U_2, U_3) + L(V_1, V_2, V_3)$$
$$= L(U) + L(V)$$

(2) Let $K \in R$ and $U \in R^3$

$$L(K(U)) = L(KU_1, KU_2, KU_3)$$
$$= KU_1, KU_1 + KU_2 + KU_3$$
$$= K(U_1, U_1 + U_2 + U_3)$$
$$= KL(U)$$

by (1) and (2) there for L is linear transformation

Ex :- (3) let $T : R^2 \Longrightarrow R^2$ be defined by $T(x, y) = (x, y + 1)$. determine whether T is linear transformation

Sol :-

(1) let $V, U \in R^2 \Longrightarrow$ $T(V) = (V_1, V_2 + 1)$
$T(U) = (U_1, U_2 + 1)$

$$T(V+U) = T((V_1, V_2) + (U_1, U_2))$$
$$= T((V_1 + U_1, V_2 + U_2))$$
$$= (V_1 + U_1, V_2 + U_2 + 1)$$

but $T(V) + T(U) = (V_1 + U_1, V_2 + U_2 + 2)$

$T(V+U) \neq T(V) + T(U)$

Then T is not linear transformation

The Kernal and Rang Of Linear Transformation

نواة ومدى التحويلات الخطية

Def :- let $L : V \rightarrow W$ be a linear transformation the ***kernel*** of L is the sub set of V consisting of all vectors V such that $L(v) = O_w$ and denoted by ker(L)

$$Ker(L) = \{ v \in V : L(v) = O_w \}$$

(النواة:- مجموعة كل العناصر في V بحيث صورتها تساوي المتجه الصفري في W تحت تاثير الدالة الخطية L)

Ex :- if $L : R^3 \rightarrow R^2$ be defined by $L(U_1, U_2, U_3) = (U_1, U_3)$ find ker.(L)

Sol :-

$$\begin{aligned} Ker(L) &= \{ U \in R^3 : L(U) = O_{R^2} \} \\ &= \{ (U_1, U_2, U_3) : T(U_1, U_2, U_3) = (0, 0) \} \\ &= \{ (U_1, U_2, U_3) : U_1 = 0, U_2 = 0 \} \\ &= \{ ((0, 0, U_3) \in R^3, U_3 \in R \} \end{aligned}$$

Ex :- (2) let $T : R \rightarrow R^2$ be defined by $T(U) = (U, 2U)$, find ker (T)

Sol :-

$$\begin{aligned} Ker(T) &= \{ U \in R : T(U) = O_{R^2} \} \\ &= \{ U \in R : (U, 2U) = (0, 0) \} \\ &= \{ U \in R : U = 0 \} \\ &= \{ 0 \} \end{aligned}$$

Ex :- (3) let $T : R^3 \rightarrow R^3$ be defined by $T(U_1, U_2, U_3) = (U_1 + U_2, U_2, U_1 - U_3)$, find ker (T)

Sol :-

$$\begin{aligned} Ker(T) &= \{ U \in R^3 : T(U) = O_{R^3} \} \\ &= \{ (U_1, U_2, U_3) \in R^3 : T(U_1, U_2, U_3) = (0, 0, 0) \} \\ &= \{ (U_1, U_2, U_3) \in R^3 : (U_1 + U_2, U_2, U_1 - U_3) = (0, 0, 0) \} \\ &= \{ (U_1, U_2, U_3) \in R^3 : U_1 + U_2 = 0, U_2 = 0, U_1 - U_3 = 0 \} \\ &= \{ (U_1, U_2, U_3) \in R^3, U_1 = 0, U_2 = 0, U_3 = 0 \} \\ &= \{ (0, 0, 0) \} \end{aligned}$$

Def:- let $L: V \longrightarrow W$ is a linear transformation the ***Range*** of L is the set of all vectors in W that are images under L. of vectors in V.

Range (T) = $\{ \; w \in W : v \in V \quad s.t \quad T(v) = w \}$

المدى :- مجموعة المتجهات في W والتي تكون صور لمتجهات من V تحت تأثير الدالة الخطية L

Theorm :- if $T: V \longrightarrow W$ is linear transformation then

(1) Ker (T) is ***subspace*** of V .

(2) Range (T) is ***subspace*** of W .

Proof :- (1)

(a) let $U, V \in \text{Ker}(T)$

$T(U) = 0 \;, T(V) = 0$

$T(U+V) = T(U) + T(V)$ (T. linear tr.)

$= 0 + 0$

$= 0$

Then , $U + V \in \text{Ker}(T)$

(2) let $K \in R \;, U \in V$

$T(U) = 0$

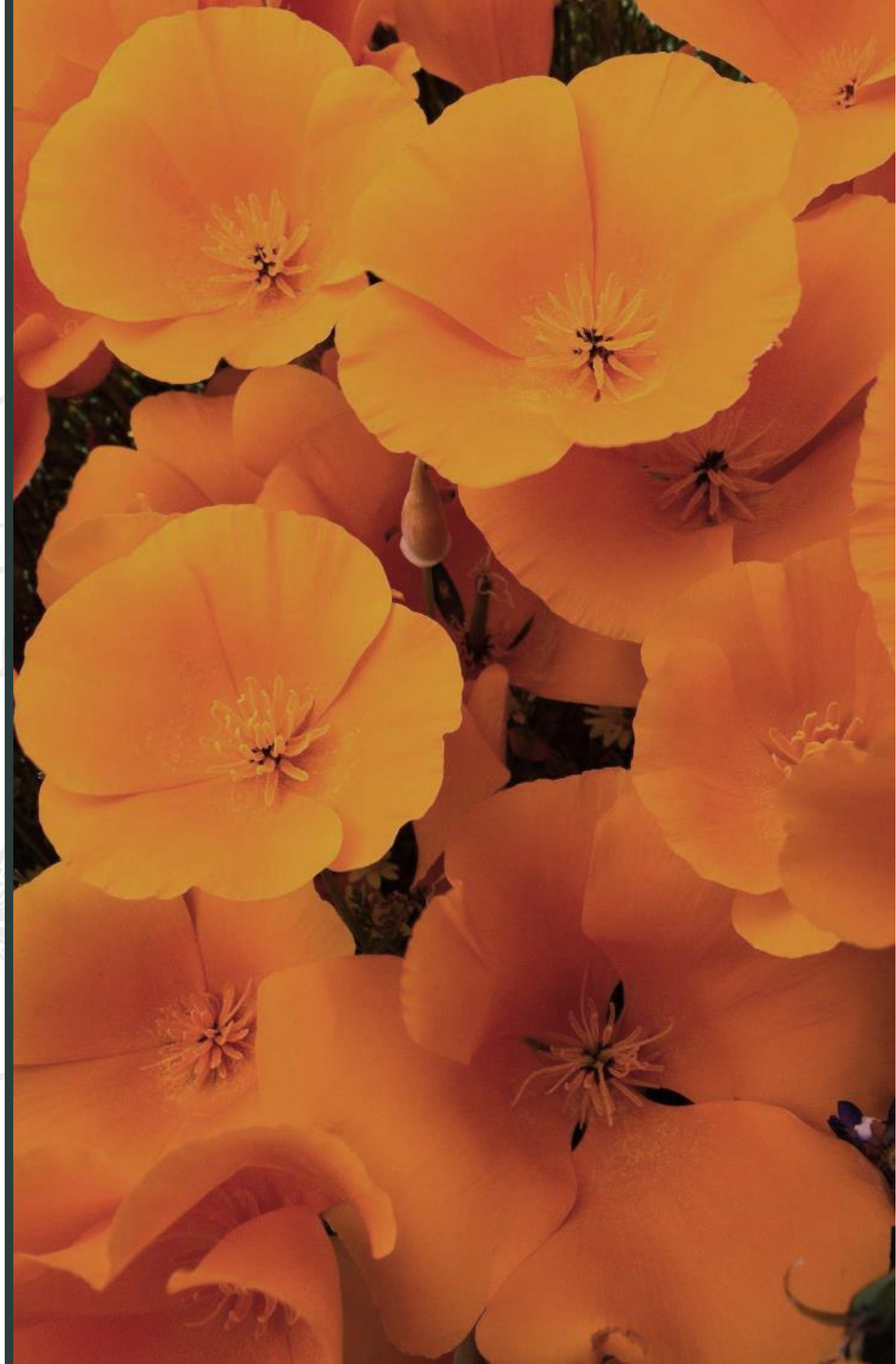
$T(KU) = K\,T(U)$

$= K.0$

$= 0$

Thus , $KU \in \text{Ker}(T)$

By (1) and (2) Ker (T) is subspace .



Ex :- let $T: R^2 \rightarrow R^2$ such that
$T(0,1)=(1,2), T(1,1)=(2,-2)$, then find $T(3,-2)$ and find $T(a,b)$?
Such that $S=((0,1),(1,1))$ is spans of R^2

Sol :-

S spans R^2

Then , every vector in R^2 is linear comb . of element of S .

$K_1(0,1)+K_2(1,1)=(3,-2)$

$(0,K_1)+(K_2,K_2)=(2,-2)$

$(K_2, K_1+K_2)=(3,-2)$

$K_2=3$

$K_1+K_2=-3 \Rightarrow K_1=-5$

$(3,-2)=-5(0,1)+3(1,1)$

$T(3,-2)=T(-5(0,1)+3(1,1))$
$=T(-5(0,1))+T(3(1,1))$
$=-5T(0,1)+3T(1,1)$
$=-5(1,2)+3(2,-3)$
$=(-5,-10)+(6,-9)$
$=(1,-19)$

(2) to find $T(a,b)$

$C_1(0,1)+C_2(1,1)=(a,b)$

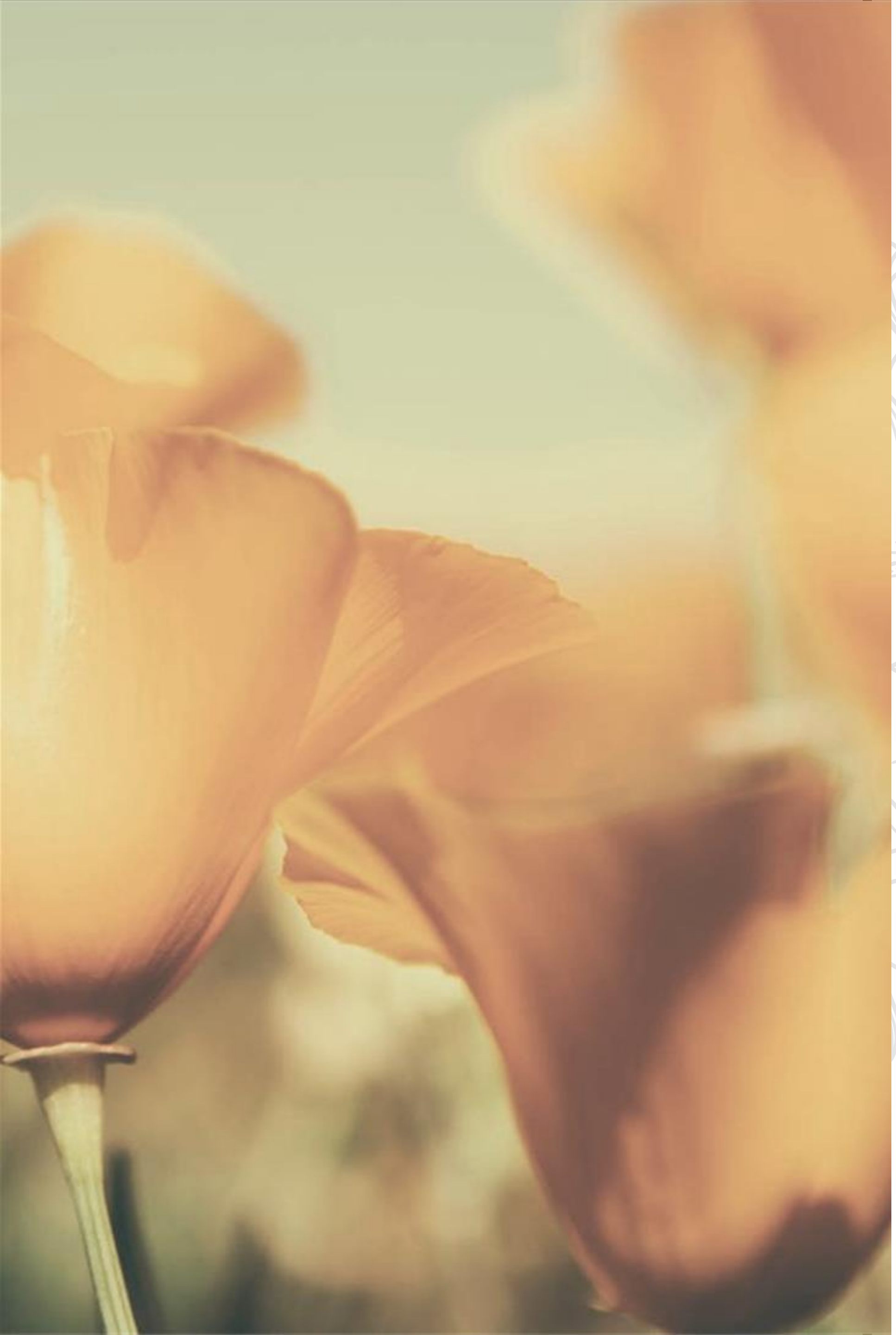
بعد حل هذه المعادلة نحصل على

$C_1 = b-a,\ C_2 = a$

Then

$(a,b)=(b-a)(0,1)+a(1,1)$

$T(a,b)= T((b-a)(0,1)+a(1,1))$
$= T(b-a)(0,1)+ T(a(1,1))$
$=(b-a)\ T(0,1)+aT(1,1))$
$=(b-a)(1,2)+a(2,-3)$
$=(b-a, 2(b-a)+(2a,-3a)$



Matrix Of Linear Transformation مصفوفة التحويل

ملاحظة :- لايجاد مصفوفة التحويل (. ***Matrix Of Linear Tran***) ذات السعة $m * n$ بحيث $T(U) = AU$ حيث

$T : R^n \Longrightarrow R^m$

نفرض ان $e_1 , e_2 , \ldots\ldots , e_n$ قاعدة اعتيادية فان

$T(e_1), T(e_2), \ldots\ldots, T(e_n)$

تمثل اعمدة المصفوفة A .

Ex : - let $T = R^2 \Longrightarrow R^2$ is L . transformation such that

$$T\begin{pmatrix} x_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} x + 2x_2 \\ x_1 - x_2 \end{pmatrix}$$

Find a matrix A such that $T(x) = A\ x$.

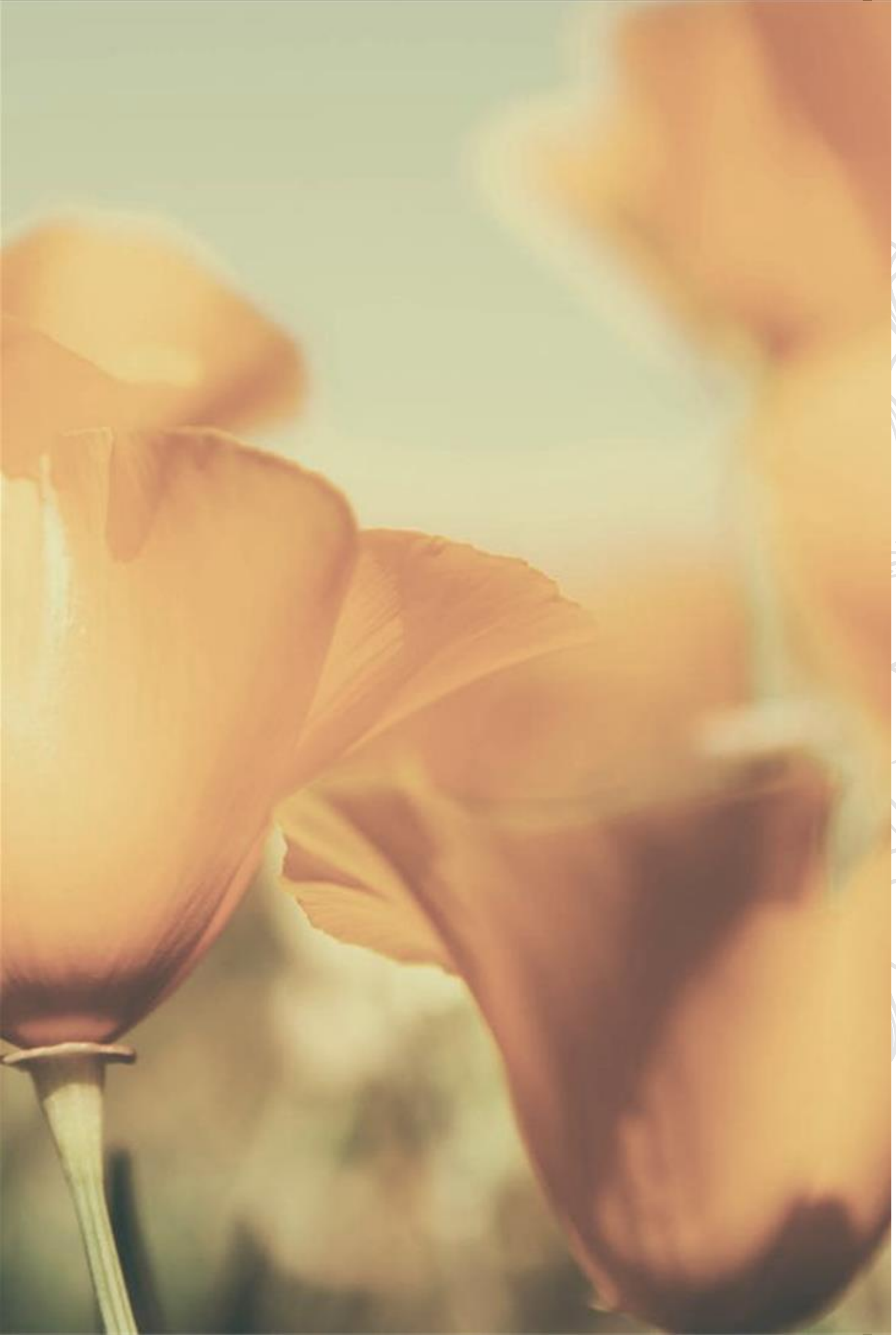
Sol :-

Let $S = ((1,0),(0,1))$ be a basis of R^3 then

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$$



Ex :- find (***Matrix Of Linear Tran .***) of $T : R^3 \Rightarrow R^4$ defined by

$$T\begin{pmatrix} x_1 \\ x_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} x_1 + 2x_2 \\ 3x_1 - x_2 \\ x_2 - x_3 \\ x_1 \end{pmatrix}$$

Sol :- let $S = \{(1,0,0),(0,1,0),(0,0,1)\}$ be a basis of R^3

Then

$$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 1 \end{pmatrix}, T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix}, T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

اذن حسب تعريف مصفوفة التحويل الخطي تصبح مجموعة الصور لدالة T اعمده للمصفوفة A (مصفوفة التحويل الخطي T)

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{pmatrix}_{4*3}$$