Ch.5 Inventory Models

The inventory deals with stocking an item to meet fluctuations in demand. The inventory problem involves placing and receiving orders of given sizes periodically. The basis for the decision is a model that balances the cost of capital resulting from holding too much inventory against the penalty cost resulting from inventory shortage. The problem reduces to controlling the inventory level by devising an ***inventory policy*** that answers two questions:

1. How much to order?
2. When to order?

The basis for answering these questions is the minimization of the following inventory cost function:

$$\begin{pmatrix} Total \\ inventory \\ cost \end{pmatrix} = \begin{pmatrix} Purchasing \\ cost \end{pmatrix} + \begin{pmatrix} Setup \\ cost \end{pmatrix} + \begin{pmatrix} Holding \\ cost \end{pmatrix} + \begin{pmatrix} Shortage \\ cost \end{pmatrix}$$

1. ***Purchasing cost*** is the price per unit of an inventory item. At times the item is offered at a discount if the order size exceeds a certain amount, which is a factor in deciding how much to order.

2. ***Setup cost*** represents the fixed charge incurred when an order is placed regardless of its size. This includes salaries, transportation cost, insurance, etc.

3. ***Holding cost*** represents the cost of maintaining inventory in stock. It includes the interest on capital, the cost of storage, maintenance, and handling.

4. ***Shortage cost*** is the penalty incurred when we run out of stock. It includes potential loss of income, disruption in production, and the more subjective cost of loss in customer's goodwill.

An inventory system may be based on ***periodic review*** (e.g., ordering every week or every month). Alternatively, the system may be based on ***continuous review***, where a new order is placed when the inventory level drops to a certain level, called the ***reorder point***.

5.1 Role of Demand in the Development of Inventory Models

In general, the analytic complexity of inventory models depends on whether the demand for an item is deterministic or probabilistic. Within either category, the demand may or may not vary with time. For example, the consumption of natural gas used in heating homes is seasonal. Though this seasonal pattern repeats itself annually, the same-month consumption may vary from year to year, depending, for example, on the severity of weather.

In practical situations the demand pattern in an inventory model may assume one of four types:

1. Deterministic and constant (static) with time.
2. Deterministic and variable (dynamic) with time.
3. Probabilistic and stationary over time.
4. Probabilistic and non-stationary over time.

This categorization assumes the availability of data that are representative of future demand. Demand is usually probabilistic, but in some cases the simpler deterministic approximation may be acceptable. The complexity of the inventory problem does not allow the development of a general model that covers all possible situations.

5.2 Static Economic-Order-Quantity (EOQ) Models

5.2.1 Classic EOQ Model (Constant-Rate Demand, no Shortage)

The simplest of the inventory models involves constant-rate demand with instantaneous order replenishment and no shortage. Define:

y = Order quantity (number of units)

D = Demand rate (units per unit time)

t_0 = Ordering cycle length (time units)

The inventory level follows the pattern explained if Figure (5.1). When the inventory reaches zero level, an order of size y units is received instantaneously. The stock is then depleted uniformly at the constant demand rate D.

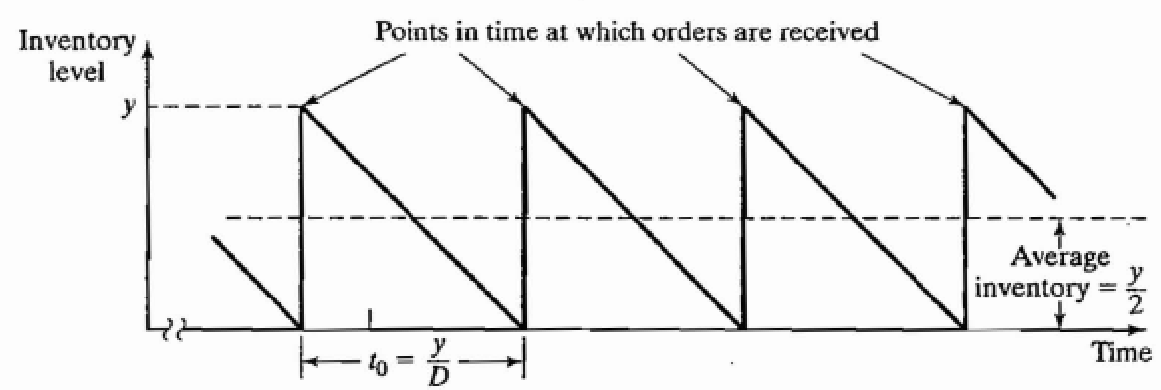

Figure (5.1)

The ordering cycle for this pattern is:

$$t_0 = \frac{y}{D} \text{ time units}$$

The cost model requires two cost parameters:

K = Setup cost associated with the placement of an order (monetary units per order)

h = Holding cost (monetary units per inventory unit per unit time)

Given that the average inventory level is $\frac{y}{2}$, the total ***cost per unit time*** (***TCU***) is thus computed as

$$TCU(y) = \text{Setup cost per unit time} + \text{Holding cost per unit time}$$

$$= \frac{\text{Setup cost} + \text{Holding cost per cycle } t_0}{t_0}$$

$$= \frac{K + h(\frac{y}{2})t_0}{t_0} = \frac{K}{(\frac{y}{D})} + h\left(\frac{y}{2}\right) = \frac{KD}{y} + \frac{hy}{2}$$

The optimum value of the order quantity y is determined by minimizing (y). Assuming y is continuous, a necessary condition for optimality is:

$$\frac{dTCU(y)}{dy} = -\frac{KD}{y^2} + \frac{h}{2} = 0$$

The condition is also sufficient because $TCU(y)$ is convex.

The solution of the equation yields the EOQ y^* as

$$y^* = \sqrt{\frac{2KD}{h}}$$

Thus, the optimum inventory policy for the proposed model is

$$\text{Order } y^* = \sqrt{\frac{2KD}{h}} \text{ units every } t_o^* = \frac{y^*}{D} \text{ time units}$$

Actually, a new order need not be received at the instant it is ordered. Instead, a positive ***lead time***, L, may occur between the placement and the receipt of an order. In this case, the ***reorder point*** occurs when the inventory level drops to LD units. Sometimes, it is assumed that the lead time L is less than the cycle length t_0^* ,which may not be the case in general. To account for this situation, we define the ***effective lead time*** as

$$L_e = L - nt_0^*$$

where n is the largest integer not exceeding $\frac{L}{t_0^*}$. The reorder point occurs at L_eD units, and the inventory policy can be restated as:

Order the quantity y^* whenever the inventory level drops to L_eD units

Example (5.1):

Neon lights on the U of A campus are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs 100 $ to initiate a purchase order. A neon light kept in storage is estimated to cost about 0.02 $ per day. The lead time between placing and receiving an order is 12 days. Determine the optimal inventory policy for ordering the neon lights.

Solution:

From the data of the problem, we have:

D = 100 units per day

K = 100 $ per order

h = 0.02 $ per unit per day

L = 12 days

Thus,

$$y^* = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2\times100\times100}{0.02}} = 1000 \text{ neon light}$$

The associated cycle length is:

$$t_0^* = \frac{y^*}{D} = \frac{1000}{100} = 10\ days$$

Because the lead time $L = 12$ days exceeds the cycle length t_0^* (= 10 days) , we must compute L_e. The number of integer cycles included in L is

$$n = \left(Largest\ integer\ \le \frac{L}{t_0^*}\right) = \left(Largest\ integer\ \le \frac{12}{10}\right) = 1$$

Thus,

$$L_e = L - nt_0^* = 12 - 1\times10 = 2 \text{ days}$$

The reorder point thus occurs when the inventory level drops to

$$L_eD = 2\ X\ 100 = 200 \text{ neon lights}$$

The inventory policy for ordering the neon lights is:

Order 1000 units whenever the inventory level drops to 200 units.

The daily inventory cost associated with the proposed inventory policy is:

$$TCU(y) = \frac{KD}{y} + \frac{hy}{2} = \frac{100\times100}{1000} + 0.02\left(\frac{1000}{2}\right) = 20\ \$/\text{ day}$$

Exercise 5.1 (in addition to text book exercises)

A carpenter orders 48000 unit of an item yearly. The order costs 800$ and the holding cost is 10 cents per item monthly. The lead time between placing and receiving an order is 4 month. Determine the optimal inventory policy.