Linear Algebra

Second Semester

For the 1^{rd} Class Student

Mathematics Department

College of Science for Women

Prepared by :

Dr. Nagham .M.N

Vector Space

Definition

Let **V** be nonempty set of vectors then **V** called ***Vectors Space*** over R if and only if its satisfy the following conditions

(1)

(a) $U , V \in V \Rightarrow U + V \in V$

V is closed under +

(b) $U + V = V + U$

(c) $U + (V + W) = (U + V) + W$

(d) يوجدعنصر وحيد هو المتجه الصفري ينتمي الى V بحيث

$U + O = O + U = U$

(e) $U + (-U) = O$ لكل U ينتمي الى V فانه يوجد -U ينتمي الى V بحيث انه

(2) if $v , u \in V$ and $a , b \in R$ then

(a) $a . U \in$

(b) $a . (V + U) = a . V + a U$

(c) $(a + b) . U = a U + b U$

(d) $a . (b . U) = a b . U$

(e) $1 . U = U$

Example

show thet R^n is vectors space over R ,where

Solution

By theorem

If U ,V ,W are vectors in R^n and K , C are scalars numbers then

(1) $U+V=V+U$

(2) $(U+V)+W=U+(V+W)$

(3) $U+0=0+U$

(4) $U+(-U)=0$

(5) $(CK)U=C(KU)$

(6) $K(U+V)=KU+KV$

(7) $(C+K)V=CV+KV$

(8) $1.U=U$

Therefore , R^n is vectors space over R .

Ex:- Let V is the set of all vectors of the form $(U_1,0,U_3)$ and defined the operations of addition and multiplication by scalar number as following

$U+V=(U_1,O,U_3)+(V_1,O,V_2)=(U_1+V_1,O+O,U_3+V_3)$

$c.U=C.(U_1,O,U_2)=(CU_1,O,CU_2)$. Is V vectors space over R .

Sol :-

لكي نثبت بان V فضاء متجهات يجب ان تحقق لشروط تعريف فضاء المتجهات وكما يلي :-

(1)

(a) $U+V=(U_1,O,U_3)+(V_1,O,V_2)=(U_1+V_1,O,U_3+V_3)$

$U+V \quad V$

Then V is closed under + .

(b) $U+V=(U_1,O,U_3)+(V_1,O,V_3)$

$=(U_1+V_1,O,U_3+V_3)$

$=(V_1+U_1,O,V_3+U_3)$

$=(V_1,O,V_3)+(U_1,O,U_3)$

$=V+U$

(c) $U+(V+W)=(U+V)+W$

Let $U=(U_1,O,U_3), V=(V_1,O,V_3), W=(W_1,O,W_3)$

$U+(V+W)=(U_1,O,U_3)+((V_1,O,V_3)+(W_1,O,W_3))$

$=(U_1,O,U_2)+(V_1+W_1,O,V_3+W_3)$

$=(U_1+V_1+W_1,O,U_3+V_3+W_3)$

$=(U_1+V_1)+W_1,O,(U_3+V_3)+W_3)$

$=((U_1+V_1),O,(U_3+V_3))+(W_1,O,W_3)$

$=((U_1,O,U_3)+(V_1,O,V_3))+W$

$=(U+V)+W$

(d) $U+O=O+U=U$

$U+O=(U1,O,U3)+(O,O,O)$

$=(U1+O,O+O,U3+O)$

$=(U1,O,U3)$

$=U$

(e) $a.(U+V)=a(U1+V1,O,U3+V3)$

$=(a(U1+V1),\ a.O,a(U3+V3))$

$=(aU1+aV1,O,aU3+aV3)$

$=(aU1,O,aU3)+(aV1,O,aV3)$

$=aU+aV$

وهكذا بالنسبة لبقية الشروط

اذن المجموعة V هي فضاء متجهات

Ex:- If $V = M_{2*3}(R)$ ={ set of all matrices of order 2*3 which defined over R} with the addition and multiplication by scalar number of matrices , then show that V is vectors space

Let $U = A = \begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \end{pmatrix}$, $V = B = \begin{pmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \end{pmatrix}$

(a) $U + V = A + B = \begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \end{pmatrix} + \begin{pmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \end{pmatrix}$

$$= \begin{pmatrix} a11 + b11 & a12 + b12 & a13 + b13 \\ a21 + b21 & a22 + b22 & a23 + b23 \end{pmatrix}$$

$$= \begin{pmatrix} b11 + a11 & b12 + a12 & b13 + a13 \\ b21 + a21 & b22 + a22 & b23 + a23 \end{pmatrix}$$

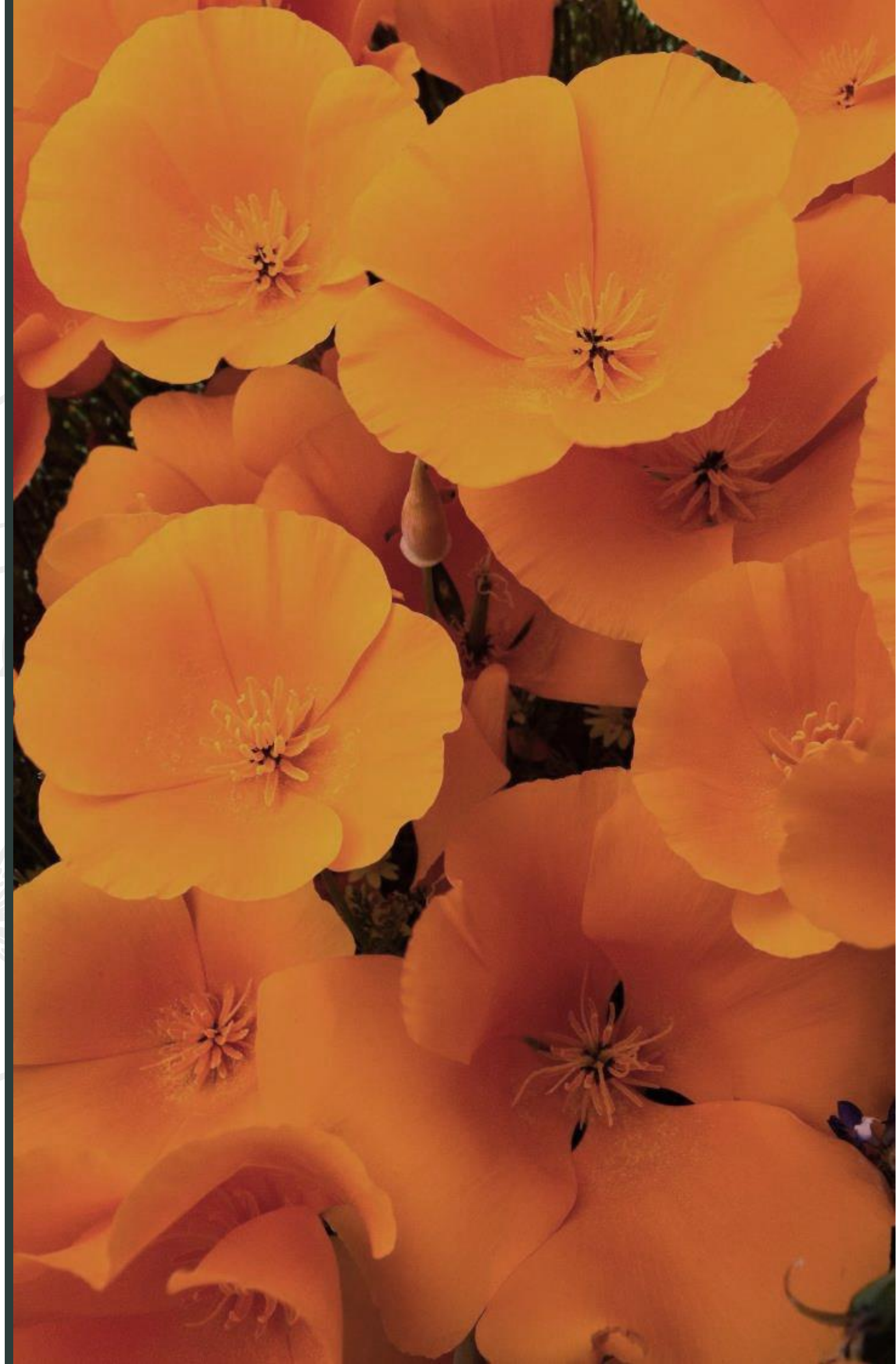
$$= \begin{pmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \end{pmatrix} + \begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \end{pmatrix}$$

$= B + A$

$= V + U$

وهكذا بالنسبة لبقية الشروط (برهان بقية الشروط واجب H . W)

اذن المجموعة V هي فضاء متجهات على حقل الاعداد الحقيقية R



Homework

Exc:- Determine whether the sets are vectors space

(1) $V = R^2$,with two operations (U1 , U2) + (V1 , V2) = (U1 + V1 , U2 + V2)

k(U.V) = (U.kV)

(2) $M = R^3$,with two operations (x_1, y_1,z_1) + (x_1, y_2,z_2) = ($x_1 + x_2$.,y_1+y_2, $z_1 + z_2$)

k(U.V.W) = (0,0,0)

(3) $S = M_{2*2}(R) = \{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} , a,b \in R \}$

with the addition and multiplication by scalar number of matrices.