2-2 Interval Estimation

Def: An (1-α) confidence interval (C. I.) estimator is an interval whose end points are functions of the sample statistics such that if we could generate indefinitely samples, the interval should contain the true parameters (1-α) % of the times.

Constructing C. I.: -

The following steps are necessary to construct the C.I.

step (1): obtain the probability distribution of the point estimator for the unknown parameter.

Step (2): Standardize the estimator such that we get a r.v with completely known distribution.

Step (3): Construct C.I. for standardized r.v. then

1 - Solve for the unknown parameter.

2- 2-1 C.I for means of normal population

i- if σ^2 is know:-

Let $x_1, x_2, ..., x_n$ be a r.s from normal population with unknown mean μ and known Variance of σ^2 Applying the above steps:-

1- the sample mean $\bar{x}$ is a point estimate of μ with probability distribution N (μ,$\frac{\sigma^2}{n}$).

2- standardizing z=$\frac{\bar{x}-M}{\delta/\sqrt{n}}$ $\sim N(0,1)$

3- The values $-z_{\alpha/2}$, $z_{\alpha/2}$ place $\frac{1}{2}\alpha$ in each tail of normal distribution

Therefor

$p_r[-z_{\alpha/2} < \frac{\bar{x}-M}{\delta/\sqrt{n}} < z_{\alpha/2}]$ =1-α i.e C.I $z_{\alpha/2} =$

$pr(-c < M < c)$= N(c) -N(-c)=N(c)- N(c) +1

=2N(c)-1

=N(c)=1-α/2

solving for a we obtain

$p_r[\bar{x} - z_{\alpha/2}\,\delta/\sqrt{n} < \mu < \bar{x} + z_{\alpha/2}\,\delta/\sqrt{n}]$ =1-α

Where 0< α < 1 and selected often to be 0.1,0.01 or 0.5

Ex: Find 95% c.I for the mean of normal population N(μ,25) if it is known that normal $\bar{x} = 10\,, n = 100.$

Solution: we have 1-α= 0.95 , α=0.05

$\frac{\alpha}{2} = 0.025.$ from tables of standard distribution, we get $z_{\alpha/2}$ = Z 0.025 = 1.96

$p_r[-z_{\alpha/2} < \frac{\bar{x}-M}{\delta/\sqrt{n}} < z_{\alpha/2}]$ =1-α

$p_r[\bar{x} - z_{\alpha/2}\,\delta/\sqrt{n} < \mu < \bar{x} + z_{\alpha/2}\,\delta/\sqrt{n}]$ =1-α

Pr [10- (1.96) $\frac{5}{\sqrt{100}} < \mu < x10 - (1.96)\frac{5}{\sqrt{100}}] = 0.95$

Pr [9.022< μ $< 10.98] = 0.95$

lower bound E: 9.02

upper bound Cu= 10.98

2) **if σ^2 is unknown**

W=$\frac{\bar{x}-M}{\delta/\sqrt{n}}$ ~ N(0,1)

V= $\frac{ns^2}{\sigma^2}$ ~ x^2(r), r =n-1

T=$\frac{w}{\sqrt{v/r}}$

T=$\frac{\frac{\bar{x}-M}{\delta/\sqrt{n}}}{\sqrt{\frac{\frac{ns^2}{\sigma^2}}{n}-1}}$

T=$\frac{w}{\sqrt{v/r}}$

$= \frac{\bar{x}-M}{s/\sqrt{n-1}}$

a) For small samples (n < 30)

- In this case the r.v $\frac{\bar{x}-M}{\delta/\sqrt{n-1}}$ ~ (n-1) Applying the steps stated earlier we get

$$p_r[\bar{x} - t\alpha_{/2}\, s/\sqrt{n} - 1 \ < \ \mu \ < \bar{x} + t\alpha_{/2}\, s/\sqrt{n-1}] \ =1-\alpha$$

Ex: let $\bar{x}$=20, s^2 =29 denote the means and variance of a r.s of size 16 is from N(μ, σ^2) Find 95% c.I. form.

Solution: we have 1- α = 0.95 α= 0.05 , $\frac{\alpha}{2}$ =0.025 from tables of t distribution we get $t\alpha_{/2}$(n-1)= $t_{0.025}$= 2.145 from table.

as another way to represent C.I we write C. I for

$M=\bar{x}\mp t_{\alpha/2}\frac{\delta}{\sqrt{n-1}}$ $= 20 \mp (2.145)\frac{\sqrt{29}}{\sqrt{15}}$

CL =18.338 , CU=21.661

(18.338 , 21.661)

b) For Large Samples (n > 30)

In this case and form statistical inference: theory the distribution of the r.v

.: $t=\frac{\sqrt{n}(\bar{x}-M)}{s}$ will converge to N(0,1).

which means that we can use the standard normal tables instead of t distribution table and hence.

c. I for $M=\bar{x}\mp Z_{\alpha/2}\frac{\delta}{\sqrt{n}}$

Ex: let $\bar{x}$=20, s^2 =16 denote the means and variance of a r.s of size 100. Find 99% c.I. for μ.

Solution: we have 1- α = 0.99 α= 0.01 , $\frac{\alpha}{2}$ =0.005 from tables of Normal

we get $z_{\alpha/2} = z_{0.005} = 2.58$

C.I= $\bar{x}\mp Z_{\alpha/2}\frac{s}{\sqrt{n}}$ $=20\mp(2.58)\frac{\sqrt{16}}{\sqrt{100}}$

= 20 ∓ (1.38). 4 / 5

= (18.968 , 21.032)

2-2-2: C.I for difference between two means Dafoe,

or are known let X, Xr denote the means of two independent random samples of size noun from normal

populations with variances &?, de respectively A (1-4); col for more is le. I form - Mn = (*) I Za/2 + 1

af d?, du are unknown @ for large samples (nigh 2 730)

A(1) % c. I for M.-Me is given by (C.I. for M, M = (x1-x2) I tan seht so

I

where si? ,S

denote

the variances of

the two

Samples.

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