5. Absorption Coefficient α And Gain Factor G

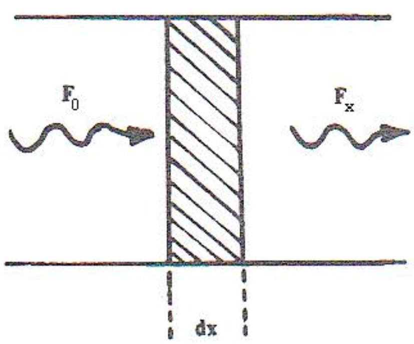
The absorption section σ is related with the absorption coefficient α by

$$\alpha = \sigma\,(N_1 - N_2)$$

The absorption coefficient α is positive when $N_1 > N_2$
In order to obtain the laser, it must be $N_2 > N_1$ and thus the absorption coefficient will be negative. In this case, the wave will be amplified due to absorption by the medium. This is called gain factor G (It is a positive amount) i.e.

$$G = -\alpha = \sigma\,(N_2 - N_1)$$

When a photon flow (F) falls on a homogeneous medium (towards x)as shown in Fig(1-4), the change in this flow dF due to the crossing of a dx distance in the medium can be expressed by equation



$$dF = -\,\alpha\, F dx$$

$$\int_{F_0}^{F_x} \frac{dF}{F} = -\,\alpha \int_0^x dx$$

$$\ln \frac{F_x}{F_0} = -\,\alpha\, x$$

Fig(1-4):

Where F_0 is the falling flow and F_x flows after passing x distance

$$F_x = F_0\; e^{-\alpha x}$$

where I : intinsity of radiation $F = \frac{I}{\hbar\omega}$

$$I_x = I_0\; e^{-\alpha x}$$

6. Einstein Calculations of Probability

Einstein assumed two levels of atomic system as shown in Fig (1-5), the population of these levels N_1 and N_2 atoms per cubic meter respectively, when electromagnetic radiation with density $\boldsymbol{\rho_{v12}}$ is passed this system, the probability of the three processes (spontaneous, absorption and stimulated emission) are as follows:

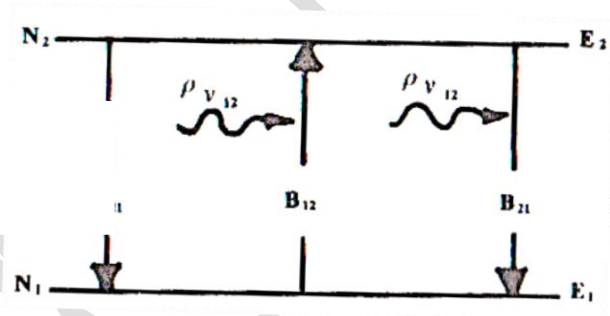


Fig (1-5)

1. Spontaneous Emission

It occurs from level 2 to level 1 with a probability of $\boldsymbol{A_{21}}$ (atoms per second) and an emission of energy equal to $\boldsymbol{hv_{12}}$. The number of such transfers per second for cubic meter of matter equals $\boldsymbol{A_{21}\,N_2}$

2. Absorption

The material atom at level 1 may absorb energy of radiation and jump to level 2 with a probability of $\boldsymbol{W_{12}}$ atom per second as:

$$\boldsymbol{W_{12} = B_{12}\,\rho_{v12}}$$

Where $\boldsymbol{B_{12}}$ is a constant (B-Einstein). The number of transfers from level 1 to level 2 per second for cubic meter of material is equal to $\boldsymbol{W_{12}\,N_1}$

3. Stimulated Emission

The material atom at level 2 may be stimulated by radiation and jump to level 1 with a probability of $\boldsymbol{w_{21}}$ atoms per second as:

$$\boldsymbol{W_{21} = B_{21}\,\rho_{v21}}$$

Where $\boldsymbol{B_{21}}$ is a constant (B-Einstein). The number of transfers from level 2 to level 1 per second for cubic meter of material is equal to $\boldsymbol{W_{21}\,N_2}$

Einstein assumed that the material was in thermodynamic equilibrium, so the number of transitions downwards should equal the number of transitions upwards, ie

$$B_{12}\,N_1\,\rho_{v12} = A_{21}\,N_2 + B_{21}\,N_2\;\rho_{v12}$$

$$\rho_{v12} = \frac{A_{21}\,N_2}{B_{12}\,N_1 - B_{21}\,N_2} = \frac{A_{21}}{B_{12}\,\frac{N_1}{N_2} - B_{21}}$$

By using Boltzmann's statistic at the thermodynamic equilibrium, the atoms of matter are distributed at energy levels:

$$\frac{N_2}{N_1} = e^{-hv/kT}$$

$$\rho_{v12} = \frac{A_{21}}{B_{12}\;e^{hv/kT} - B_{21}}$$

for the black body radiation

$$B_{12} = B_{21} = B \quad \text{and} \quad A_{21} = A$$

$$\rho_v = \frac{8\,\pi h v^3\,/c^3}{e^{hv/kT}\;-1} \qquad \text{Planck equation}$$

$$\frac{8\,\pi\,h\,v^3/c^3}{e^{hv/kT} - 1} = \frac{A}{B\;e^{hv/kT}\; - B} = \frac{A}{B(e^{hv/kT} - 1)}$$

$$\boxed{\frac{A}{B} = \frac{8\,\pi\,h\,v^3}{c^3}}$$