Computer Science Department
1st Class : Mathematics

Lecture 1. Set Theory

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A **set** is a collection of distinct objects.

For example: {1, 2, 3} is a set but **{1, 1, 3}** is not because 1 appears twice in the second collection. The second collection is called a multiset.

The set of natural numbers

$$\mathbf{N = \{1, 2, 3, 4 \ldots\ldots\}}.$$

The set of even integers can be written

$$\mathbf{\{2n : n \text{ is an integer}\}}.$$

The integers are the set of whole numbers, both positive and negative

$$\mathbf{Z = \{0, \pm 1, \pm 2, \pm 3, \ldots\}}.$$

The set of rational numbers Q which is the set of all quotients of integers that can be written by

$$Q = \{x : x = p/q, \text{ where } p,q \text{ are integers and } q \neq 0\}.$$

Whereas, the irrational numbers is the set of not rational numbers.
For example, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$.

Also, there is a set of real numbers R that is

$$R = \text{set of all rational and irrational numbers.}$$

Definition. The empty set is a set containing no objects. It is written as a pair of curly braces with nothing inside {} or by using the symbol $\emptyset$.

Definition. The set membership symbol $\in$ is used to say that an object is a member of a set. It has a partner symbol $\notin$ which is used to say an object is not in a set.

Definition. We say two sets are equal if they have exactly the same members.

Example. If

$$S = \{1, 2, 3\}$$

then $3 \in S$ and $4 \notin S$. The set

$$T = \{2, 3, 1\}$$

is equal to S because they have the same members.

Definition. The cardinality of a set is its size. For a finite set, the cardinality of a set is the number of members it contains which is written $|S|$.

Example. For the set $S = \{1, 2, 3\}$ we show cardinality by writing $|S| = 3$.

Definition. The intersection of two sets S and T is the collection of all objects that are in both sets. It is written $S \cap T$.

$$S \cap T = \{x : (x \in S) \text{ and } (x \in T)\} \text{ or}$$

$$S \cap T = \{x : (x \in S) \wedge (x \in T)\}.$$

Example. Suppose $S = \{1, 2, 3, 5\}$, $T = \{1, 3, 4, 5\}$, and $U = \{2, 3, 4, 5\}$. Then

$$S \cap T = \{1, 3, 5\},\ S \cap U = \{2, 3, 5\},\ \text{and } T \cap U = \{3, 4, 5\}.$$

Definition. If A and B are sets and $A \cap B = \emptyset$ then we say that A and B are disjoint, or disjoint sets.

Definition. The union of two sets S and T is the collection of all objects that are in either set. It is written $S \cup T$ which is defined by

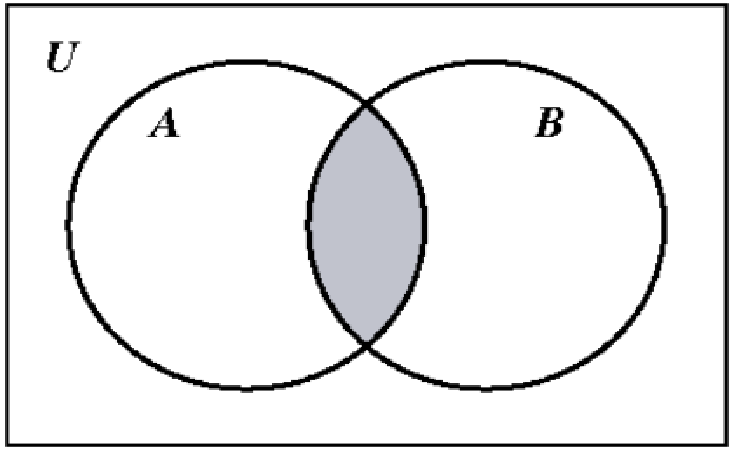
$$S \cup T = \{x : (x \in S) \text{ or } (x \in T)\} \text{ or}$$

$$S \cup T = \{x : (x \in S) \vee (x \in T)\}.$$

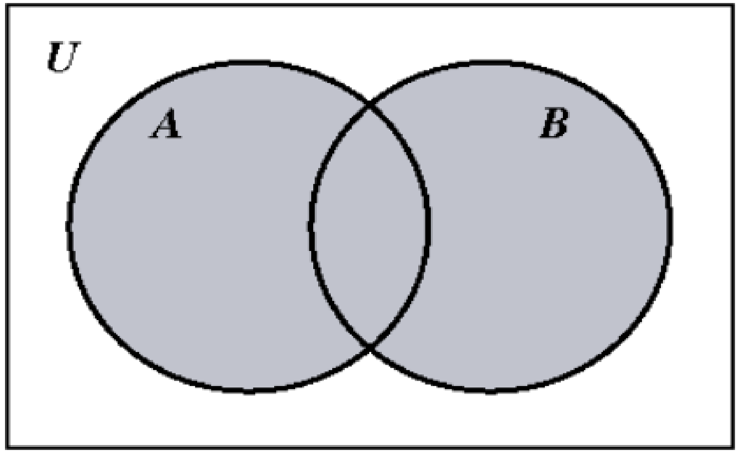
Example. Suppose S = {1, 2, 3}, T = {1, 3, 5}, and U = {2, 3, 4, 5}. Then: $S \cup T = \{1, 2, 3, 5\}$, $S \cup U = \{1, 2, 3, 4, 5\}$, and $T \cup U = \{1, 2, 3, 4, 5\}$

Definition. The universal set, at least for a given collection of set theoretic computations, is the set of all possible objects.

Venn Diagrams. A Venn diagram is a way of depicting the relationship between sets.



$A \cap B$



$A \cup B$

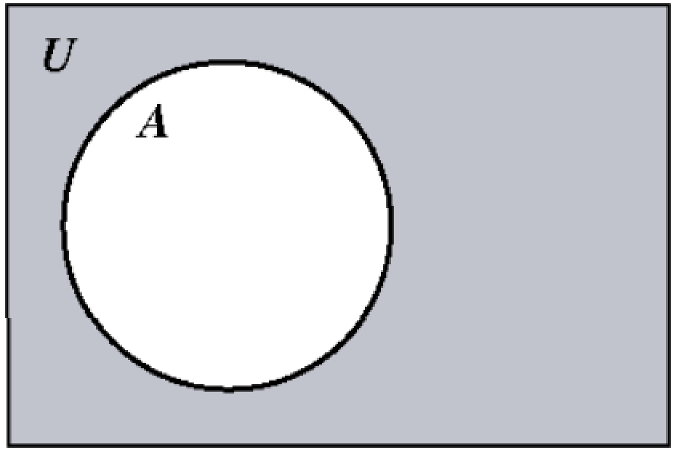
Definition. The compliment of a set S is the collection of objects in the universal set that are not in S. The compliment is written S^c.

$$S^c = \{x : (x \in U) \wedge (x \notin S)\}.$$

Example.

1. Let the universal set be the integers. Then the compliment of the even integers is the odd integers.
2. Let the universal set be {1, 2, 3, 4, 5}, then the compliment of S = {1, 2, 3} is S^c = {4, 5} while the compliment of T = {1, 3, 5} is T^c = {2, 4}.
3. Let the universal set be the letters {a, e, i, o, u, y}. Then $\{y\}^c$ = {a, e, i, o, u}.

The Venn diagram for A^c is



A^c

Definition. The difference of two sets S and T is the collection of objects in S that are not in T . The difference is written S − T.

Definition. For two sets S and T we say that S is a subset of T if each element of S is also an element of T . In formal notation $S \subseteq T$ if for all $x \in S$ we have $x \in T$.

If $S \subseteq T$ then we also say T contains S which can be written $T \supseteq S$. If $S \subseteq T$ and $S \neq T$ then we write $S \subset T$ and we say S is a proper subset of T.

Example. If $A = \{a, b, c\}$ then A has eight different subsets:

$$\emptyset \ \{a\} \ \{b\} \ \{c\} \ \{a, b\} \ \{a, c\} \ \{b, c\} \ \{a, b, c\}.$$

Notice that $A \subseteq A$ and in fact each set is a subset of itself. The empty set $\emptyset$ is a subset of every set.

Proposition. Two sets are equal if and only if each is a subset of the other. In symbolic notation:

$$(A = B) \Leftrightarrow (A \subseteq B) \wedge (B \subseteq A).$$

Proposition. De Morgan's Laws Suppose that S and T are sets. De Morgan's Laws state that

(i) $(S \cup T)^c = S^c \cap T^c$, and

(ii) $(S \cap T)^c = S^c \cup T^c$.

H.W. 1

1. Suppose that the set U = {n : $0 \le n < 100$} of whole numbers as an universal set. Let P be the prime numbers in U, let E be the even numbers in U, and let F = {1, 2, 3, 5, 8, 13, 21, 34, 55, 89}. Compute

i. E^c,

ii. $P \cap F$,

iii. $P \cap E$,

iv. $(F \cap E) \cup (F \cap E^c)$, and

v. $F \cup F^c$.

H.W. 2

Compute the subsets of S = {a, b, c, d} with cardinality 2.

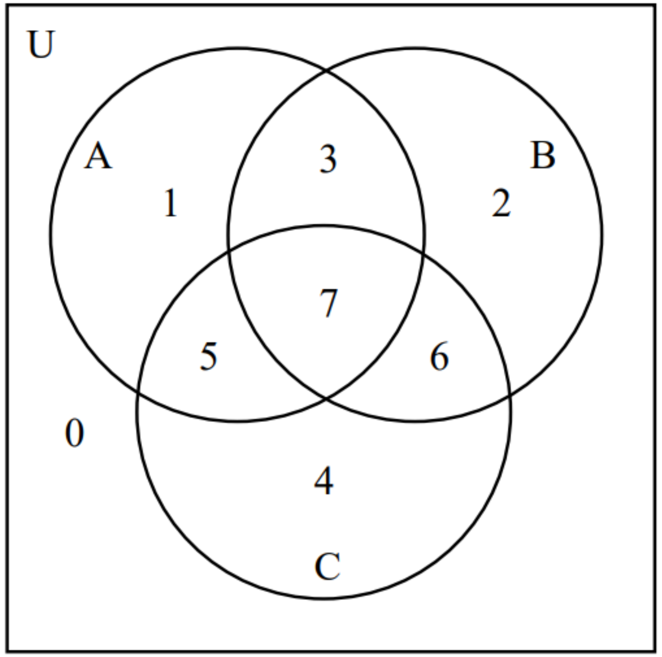
H.W. 3

Choose any finite sets S and T and show that

$$|S \cup T| = |S| + |T| - |S \cap T|.$$

H.W. 4

If the Venn diagrams for the following sets is given by



Compute

1. $A - B$
2. $B - A$
3. $A^c \cap B$
4. $A^c \cup B^c$.

Thank You Very Much for Your Attention