The Electric Field

Both the gravitational force and the electrostatic force are capable of acting through space, without any physical contact or intervening medium. That is, electric and gravitational forces can act across an empty vacuum, with no matter to carry them. These types of forces are known as **field forces**.

Corresponding to the electrostatic force, an **electric field** is said to exist in the region of space surrounding a charged object. **The electric field exerts an electric force on any other charged object within the field.**
The field concept partially eliminates the conundrum of "force at a distance", since the force on a charged object is now said to be caused by the *electric field* at that point in space.

The electric field $\vec{E}$ produced by a charge q at the location of a small "test" charge q_0 is defined as the electric force $\vec{F}$ exerted by q on q_0, divided by the test charge q_0.

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad or \qquad \vec{F} = q_0\vec{E} \qquad (1)$$

The SI unit for electric field is **Newton per Coulomb [N/C]**. The direction of $\vec{E}$ is the direction of the force that acts on a positive test charge q_0 placed in the field.

From Coulomb law

$$\vec{F} = k\frac{qq_0}{r^2}\,\hat{r} \qquad (2)$$

Sub. eq. 2 in eq. 1

$$\vec{E} = k\frac{q}{r^2}\,\hat{r} \qquad (3)$$

The electric field produced by a charge depends only on the magnitude of that charge which sets up the field, and how far away from that charge you are. It does not depend on the presence of a hypothetical test charge.

The principle of superposition also holds for electric fields, just as it did for the electric *force*. In order to calculate the electric field from a group of charges, one may calculate the field from each charge individually, and add (as vectors) the individual fields.

$$\vec{E}(r) = \vec{E}_1(1) + \vec{E}_1(1) + \cdots \ldots.. + \vec{E}_n$$

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{ip}^2}\,\hat{r}_{ip} \qquad (4)$$

Gauss' Law

Gauss' law this is actually comes from experiment, and is true no matter WHAT the shape of the closed surface is! Ut involves measurable quantities, and as far as we know, there are NO EXCEPTIONS to it.

It state that:
The total electric flux through a closed surface is proportional to the charge enclosed by that surface.

Mathematically

$$\Phi = \frac{q}{\varepsilon_0} \qquad (5)$$

where: q is the net charge enclosed by the surface, and Φ is the flux through a enclosed surface surrounding the charge and constant ε_0 which is called the permittivity of free space.

From Eq. 5

$$q = \varepsilon_0 \, \Phi \qquad (6)$$

but the flux is $$\Phi = \oint E \cdot dA \qquad (7)$$

sub. Eq 7 in Eq.6

$$q = \varepsilon_0 \oint E \cdot dA$$

or

$$\oint E \cdot dA = \frac{q}{\varepsilon_0} \qquad (8)$$

Eq.8 is another form for **Gauss' Law**

Proof of Gauss' Law

Let we take a point charge *q* , and draw Gaussian surface as a sphere with radius *r* surrounding the charge , as shown in the Fig(1)

$$\Phi = \oint \vec{E} \bullet d\vec{A} = \oint EdA\cos\theta$$
$$\Phi = \oint EdA \quad (\theta{=}0^\circ)$$
$$\Phi = E\oint dA$$
$$\Phi = E4\pi r^2$$
$$\Phi = \frac{kq}{r^2}4\pi r^2$$
$$\Phi = 4\pi kq$$
$$\Phi = 4\pi \frac{1}{4\pi\varepsilon_o} q$$
$$\Phi = \frac{q}{\varepsilon_o}$$

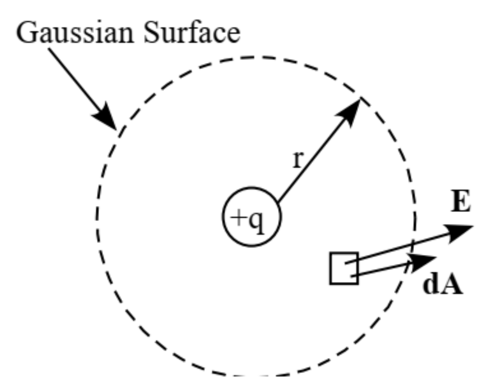


Fig. (1)

Note that this result is independent of *r* and thus holds true for any spherical surface. This result holds true for any closed surface surrounding the charge because the flux must be the same.

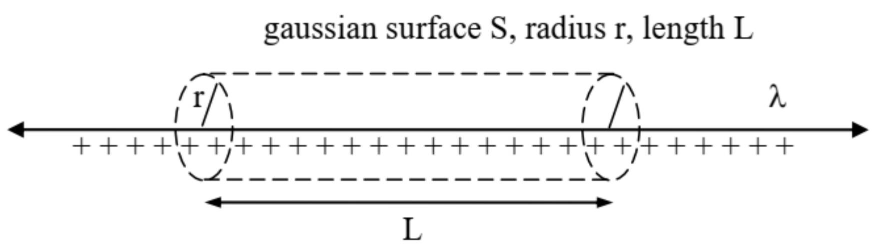
APPLICATIONS OF GAUSS'S LAW

Ex-1: Infinitely Long Rod of Uniform Charge Density

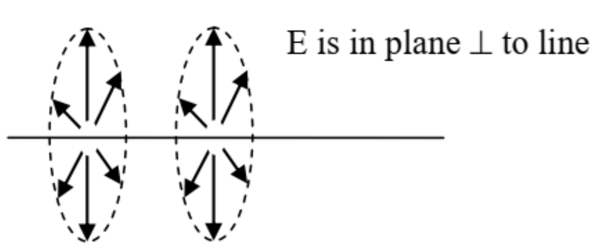
An infinitely long rod of negligible radius has a uniform charge density λ . Calculate the electric field at a distance *r* from the wire.

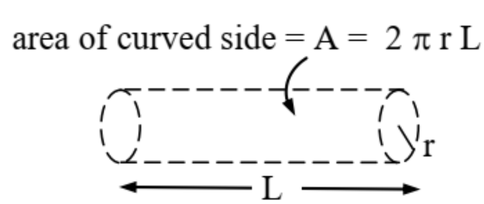
Solution

To calculate the field, imagine a cylindrical Gaussian surface with radius r, as shown in the Fig. 1.
Since the field is everywhere radial, flux through the two ends of the cylindrical Gaussian surface is zero. At the cylindrical part of the surface, E is normal to the surface at every point, and its magnitude is constant, since it depends only on *r*. The surface area of the curved part is $2\pi r L$, where *L* is the length of the cylinder.



By symmetry, **E** is in the cylindrically radial direction and E = E(r).





$$\oint_S \vec{E}\cdot d\vec{a} = \underbrace{\int_{ends} \vec{E}\cdot d\vec{a}}_{0} + \int_{side} \vec{E}\cdot d\vec{a} = E\int_{side} da = EA_{side}$$

$$= E(2\pi r L)$$

The charge inside the gaussian surface is

$$q_{ecl} = \lambda L$$

so Gauss gives

$$\int_S E \cdot dA = E(r)\int_S \hat{r} \cdot dA = E(r)A = E(r)2\pi r L = \frac{\eta L}{\varepsilon_0}$$

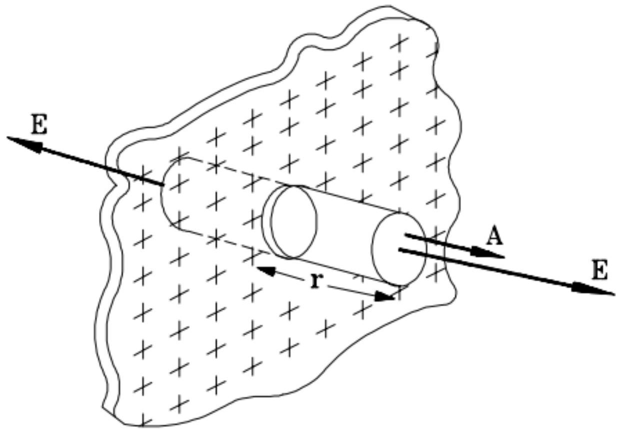
where $A = 2\pi r L$ is a curved surface of the cylinder

$$\boxed{E(r) = \frac{\eta}{2\pi\epsilon_0 r}}$$

Ex 2- A uniform sheet of charge (2 dimension)

For a uniform charged sheet with a charge density of σ (charge per unit area) it is useful to be able to calculate the field **E** at any distance r from the sheet.

Solution



A convenient Gaussian surface is a cylinder of cross section A and length 2r. This pierces the plane so that its length is perpendicular to it.

The electric field **E** points perpendicularly away from the plane in both directions and is therefore perpendicular to the end cap surface vectors.

From Gauss' law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\frac{q}{\varepsilon_0} = \int_{Top} EdA + \int_{Bottom} EdA + \int_{Side} EdA$$

$$\frac{q}{\varepsilon_0} = \int_{Top} EdA + \int_{Bottom} EdA$$

$$\frac{q}{\varepsilon_0} = E(A + A) = 2EA$$

The net charge in Gauss' surface $q = \sigma A$

$$E = \frac{\sigma}{2\epsilon_0}$$

Therefore E is the same magnitude for all points on each side of the plain.

Ex-3: Spherical Shell

A thin spherical shell of radius *a* has a charge +*Q* evenly distributed over its surface. Find the electric field both inside and outside the shell.

Solution

The situation has obvious spherical symmetry. The field at any point P, outside or inside, can depend only on *r* (the radial distance from the center of the shell to the point) and must be radial (i.e., along the radius vector) Fig. ()

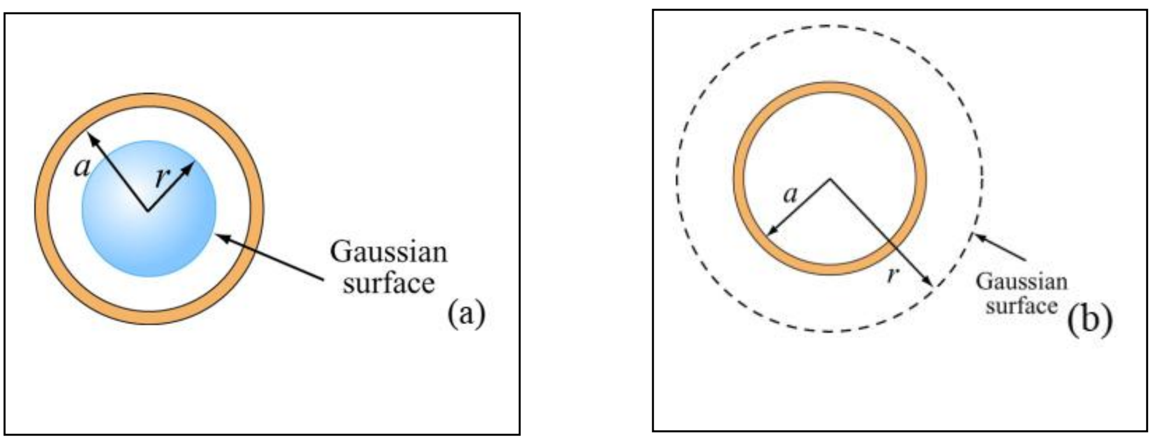


Figure 4.2.13 Gaussian surface for uniformly charged spherical shell for (a) inside shell (b) outside shell

Case 1: inside shell $r \le a$

We choose our Gaussian surface to be a sphere of radius $r \le a$, as shown in

Fig. 1(a).

The charge enclosed by the Gaussian surface is $q_{enc} = 0$ since all the charge is located on the surface of the shell. Thus, from Gauss's law,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} = 0$$

$$\vec{E} = 0, \qquad r < a$$

Case 2 outside shell $r \geq a$

In this case, the Gaussian surface is a sphere of radius $r \geq a$, as shown in Fig.3(b). Since the radius of the "Gaussian sphere" is greater than the radius of the spherical shell, all the charge is enclosed:

$$q_{\text{enc}} = Q$$

by applying Gauss's law, we obtain

$$E \oint d\vec{A} = E(A) = E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2} = k\frac{Q}{r^2} , \quad r > a}$$

Ex-4: Non-Conducting Solid Sphere

An electric charge +Q is uniformly distributed throughout a non-conducting solid sphere of radius a . Determine the electric field everywhere inside and outside the sphere.

Solution:

The charge distribution is spherically symmetric with the charge density given by

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi a^3}$$

where V is the volume of the sphere. In this case, the electric field **E** is radially symmetric and directed outward. The magnitude of the electric field is constant on spherical surfaces of radius r.

Case 1: inside shell $r \leq a$
We choose our Gaussian surface to be a sphere of radius $r \leq a$, as shown in Fig. 4.(a).

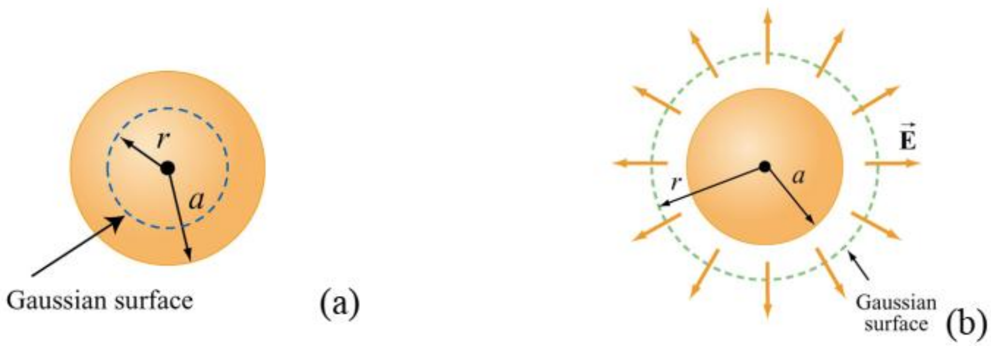


Figure 4.2.15 Gaussian surface for uniformly charged solid sphere, for (a) $r \leq a$, and (b) $r > a$.

With uniform charge distribution, the charge enclosed is

$$q_{\text{enc}} = \int_V \rho \, dV = \rho V = \rho \left(\frac{4}{3} \pi r^3 \right) = Q \left(\frac{r^3}{a^3} \right)$$

by applying Gauss's law,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

we obtain

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0} = \frac{\rho}{\varepsilon_0} (\frac{4}{3} \pi \, r^3)$$

$$E(r^2) = \frac{\rho r}{3\varepsilon_0}$$

$$\rho = \frac{Q}{\frac{4}{3}\,\pi\, a^3}$$

or

$$\boxed{E = \frac{\rho r}{3\varepsilon_0} = \frac{Qr}{4\pi\varepsilon_0 a^3}, \quad r \le a}$$

Case 2: $r \ge a$.

In this case, our Gaussian surface is a sphere of radius $r > a$, as shown in Figure 4(b). Since the radius of the Gaussian surface is greater than the radius of the sphere all the charge is enclosed in our Gaussian surface: $q_{\text{enc}} = Q$.

Applying Gauss's law, we obtain

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0} = \frac{\rho}{\varepsilon_0}\left(\frac{4}{3}\pi\, r^3\right)$$

or

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2} = k\frac{Q}{r^2}\ , \quad r > a}$$

The field outside the sphere is the same as if all the charges were concentrated at the center of the sphere.

The qualitative behavior of as a function of is plotted in Fig.4.

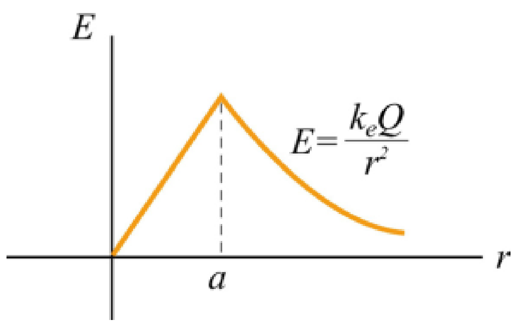


Figure Electric field due to a uniformly charged sphere as a function of r .