Coulomb's Law

Coulomb's Law

As you probably already know, the interaction between electric charges at rest is described by Coulomb's law:

Coulomb's law: *state that any two stationary electric charges repel or attract one another with a force proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them.*

Thus, if two point charges q_1, q_2 are separated by a distance r in vacuum, the magnitude of the force (F) between them is given by

$$\vec{F}_2 = k\,\frac{q_1 q_2}{r_{21}^2}\,\hat{r}_{21} \qquad 1$$

where k stands for a universal constant called Coulomb's constant.

$$k = 9\times\ 10^9\ N\cdot m^2/C^2$$

Here q_1 and q_2 are numbers (scalars) giving the magnitude and sign of the respective charges, $\hat{r}_{21}$ is the unit vector in the direction from charge 1 to charge 2, and $\vec{F}_2$ is the force acting on charge 2.

The forces that two charges exert on each other always act along the line joining the charges. The two forces are always equal in magnitude and opposite in direction, even when the charges are not equal. *The forces obey Newton's third law*, that is, $F_2 = -F_1$.
As we've seen, q_1 and q_2 can be either positive or negative quantities. When the charges have the same sign (both positive or both negative), the forces are repulsive; when they are unlike, the forces are attractive. We need the absolute value bars in Equation 1 because F is the magnitude of a vector quantity. By definition, F is always positive, but the product $q_1\,q_2$ is negative whenever the two charges have opposite signs.

Instead of k, it is customary (for historical reasons) to introduce a constant ε_0 which is called the **permittivity of free space**.

defined by

$$k \equiv \frac{1}{4\pi\varepsilon_0} \;\rightarrow\; \varepsilon_0 \equiv \frac{1}{4\pi k} = 8.85\ \times 10^{-12}\ C^2/Nm^2$$

In terms of ε_0 , Coulomb's law in Eq. (1) takes the form

$$\vec{F}_2 = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{21}^2}\,\hat{r}_{21} \qquad 1$$

.

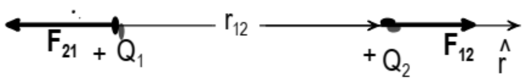


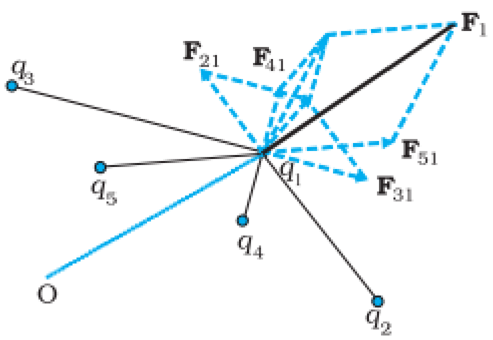
Fig. 1 Electric force between two charges.

Coulomb's Law for system of many charges

Let we have system of n charges (q_1 , q_2 , q_3, ……… , q_n), then the net force acting on the charge q_1 is obey to the principle of superposition, that is vector addition to all forces acting on q_1 by other charges in the system, i.e

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots + \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1q_3}{r_{13}^2}\hat{\mathbf{r}}_{13} + \ldots + \frac{q_1q_n}{r_{1n}^2}\hat{\mathbf{r}}_{1n}\right]$$

$$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\,\hat{r}_{1i}$$



Compare the Electrostatic to the Gravitational forces

It is useful to compare the electrostatic force with gravitational force, let we calculate the lectrostatic force and gravitational force between two electrons

$$Electron\ charge = (e) = 1.60217663 \times 10^{-19} \text{ Coulombs}$$

$$electron\ mass = m_e = 9.1 \times 10 - 31\ kilograms$$

The electrostatic force

$$F_e = k\ \frac{q_1 q_2}{r^2} = \frac{e^2}{4\pi\varepsilon_0 r^2} \qquad (1)$$

the gravitational force

$$F_G = G\ \frac{m_1 m_2}{r^2} = \frac{G m_e^2}{r^2} \qquad (2)$$

Devide eq. 1 to 2

$$\frac{F_e}{F_G} = \frac{k}{G}\ \frac{e^2}{m_e^2}$$

where $G \approx 6.7 \times 10^{-11}\ m^3 Kg^{-1} s^2$

$$\frac{F_e}{F_G} \approx 10^{42}$$

Applicable Eexamples

Ex.1

A point charge $(-1\ \mu C)$ located at otigin, another point charge $(-100\ \mu C)$ putted on the positive x-axiss at distance $500\ mm$ from the origin, find magnitude and direction of the electrostatic force on the second charge.

Solution

From Coulomb's Law

$$\vec{F}_2 = k\,\frac{q_1\,q_2}{r_{12}^2}\,\hat{r}$$

$$F_{12} = \hat{x}\,\frac{(-10^{-6})(-10)^{-4}}{4\pi(8.85\times10^{-12})(500\times10^{-3})^2}$$

$$= 3.6\hat{x}\quad N$$

the two charges have same signs so the electric force between them is repulsive, then the direction of the force is in minus direction of x-axiss.

Ex-2

A point charge q_1 = 2 μC located at origin and another point charge q_2 = -5 μC is on the coordinate (x = 3, y = 4) m.
(a) Find the electric force on charge q_1.
(b) Is the force attractive or repulsive?

Solution

The distance between two charges d

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{3^2 + 4^2} = 5 \quad \text{m}$$

From Coulomb's Law

$$\vec{F}_2 = k\,\frac{q_1\,q_2}{r_{12}^2}\,\hat{r}$$

$$= (9\times10^9)\,\frac{|(2\times10^{-6})(-5\times10^{-6})|}{5^2}$$

$$= 3.6\times10^{-3}\quad \text{N}$$

b- Becouse the two charges have opposite signs so the electric force between them is attractive.

Ex-3

Two spheres located at distance of d = 5 cm attract one another with a force of F = 3 mN. If one of them has three times more charges than the other, find the electric force between them?

Solution

Let one of charges be q_1 =? and the other $q_2 = 3q_1$. Then using Coulomb's law formula and solving for the unknown charges, we have

$$\vec{F}_2 = k\,\frac{q_1\,q_2}{r_{12}^2}\,\hat{r}$$

$$3\times 10^{-3} = \frac{9\times 10^{9}(\,q_1\times 3q_1)}{(0.05)^2}$$

$$q_1^2 = \frac{3\times 10^{-3}\times 25\times 10^{-4}}{27\times 10^{9}}$$

$$q_1^2 = 2.77\times 10^{-16}$$

$$q_1 = 2.77\times 10^{-8}\,C = 27.7\,\mu C$$

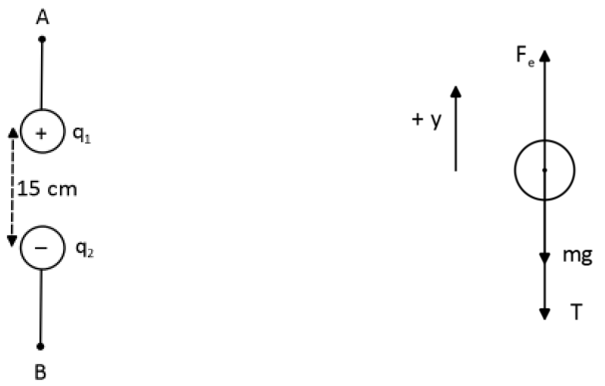
$$q_2 = 3q_1 = 3\times 27.7\,\mu C = 83.1\mu C$$

Ex-4

Two small insulating spheres are attached to silk threads and aligned vertically as shown in the figure. These spheres have equal masses of 40 g, and carry charges q_1 and q_2 of equal magnitude 2.0 μC but opposite sign.The spheres are brought into the positions shown in the figure, with a vertical separation of 15 cm between them.

Note that you cannot neglect gravity. What is the tension in the lower threads?

Solution



There are three forces acting on q_2. The attractive electrostatic force F_e due to q_1, tension force in the thread, and gravity. Thus, its free body diagram is as shown in above diagram.

At equilibrium

$$(\Sigma F_y)_2 = 0$$

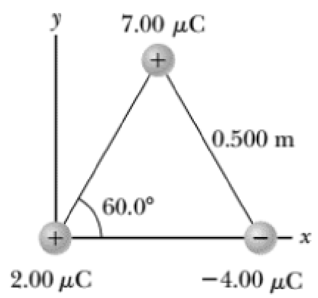
$$\Rightarrow F_e - T - mg = 0$$

$$\Rightarrow T = \frac{k\,|q_1|\,|q_2|}{(15)^2} - mg$$

$$\Rightarrow T = 9 \times 10^9 \frac{(2 \times 10^{-6})\,(2 \times 10^{-6})}{(0.15)^2} - (0.040 \times 9.8) = 1.208 \text{ N}$$

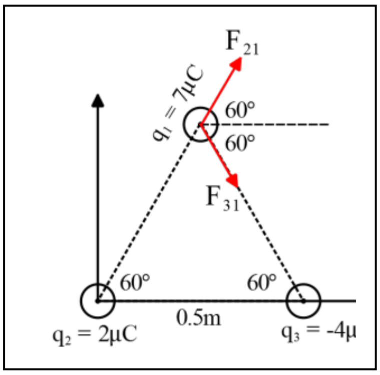
Ex-5

Three point charges are located at the corners of an equilateral triangle an in the figure. Find the magnitude and direction of the net electric force on the 7 μC charg.



Solution

First we must calculate each of the electric forces due to the $q_2 = 2\mu C$, $q_3 = -4\mu C$ charges exerted on the third charge then use the superposition principle to determine the net electric force on it.



$$\vec{F}_{21} = k\,\frac{q_1\,q_2}{r_{21}^2}\,\hat{r}_{21}$$

$$\vec{F}_{21} = 9\times 10^9\,\frac{7\times 10^{-6}\times 2\times 10^{-6}}{(0.5)^2}$$

$$\vec{F}_{21} = 0.504\,\hat{r}_{21}\;N$$

$\hat{r}_{21}$ is the unit vector points from q_2 toward q_1 so if one decomposes it, we get

$$\vec{F}_{21} = 0.504\,(cos60\,\hat{x} + \;sin60\,\hat{y}$$

$$\vec{F}_{21} = 0.504\left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right)\text{N}$$

(Notation: F_{12} is the force exerted by point charge q_1 on point charge q_2)

$$\begin{aligned}\vec{F}_{31} &= k\frac{|q_1 q_3|}{r_{13}^2}\hat{r}_{31}\\ &= 9\times 10^9\frac{|7\times 10^{-6}\times(-4)\times 10^{-6}|}{(0.5)^2}\,(\cos 60^\circ\;\hat{x}+\sin 60^\circ\;(-\hat{y}))\;\text{N}\\ &= 1.008\left(\frac{1}{2}\hat{x}+\frac{\sqrt{3}}{2}(-\hat{y})\right)\;\text{N}\end{aligned}$$

Using superposition principle: $\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31}$

, we obtain

$$\vec{F}_1 = 0.504\left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right) + 1.008\ \left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}(-\hat{y})\right)$$
$$= 0.756\ \hat{x} - 0.437\ \hat{y}\ \ (\text{N})$$

And its magnitude is

$$\left|\vec{F}_1\right| = \sqrt{(0.756)^2 + (-0.437)^2} = 0.873\,\text{N}$$

And also the direction of the resultant force with the horizontal axis (x) is

$$\alpha = \tan^{-1}\left(\frac{|-0.437|}{|0.756|}\right)\ = 30.02°$$

Since $\vec{F}_{1x}\ > 0$ and $\vec{F}_{1y}\ < 0$ so the net force lies in the fourth quadrant.