

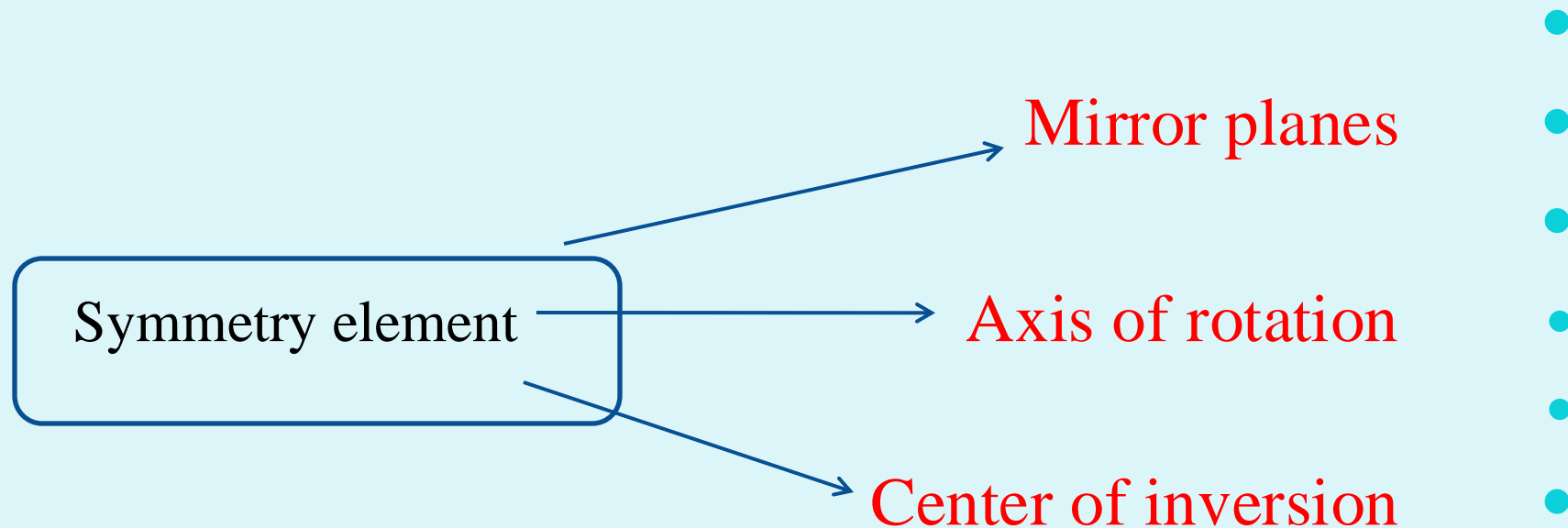
Symmetry and Group Theory

⇒ The symmetry properties of molecules and how can be used to predict vibrational spectra, hybridization, optical activity.

Point groups: Molecule with similar symmetry are put into the same point group.

A point group contains all objects that have the same symmetry.

Symmetry Element



Symmetry Element

Element	Symmetry operation	Symbol
-n-fold axis or (axis of rotation)	Rotation by $360/n$	C_n
-	Identity	E
Mirror plane	Reflection	σ
Center of inversion	Inversion	i
-n-fold axis of improper rotation or (improper rotation axis)	Rotation by $360/n$ followed by reflection perpendicular to the axis of rotation	S_n

Identity, E

All molecules have identity. This operation leaves the entire molecule unchanged.

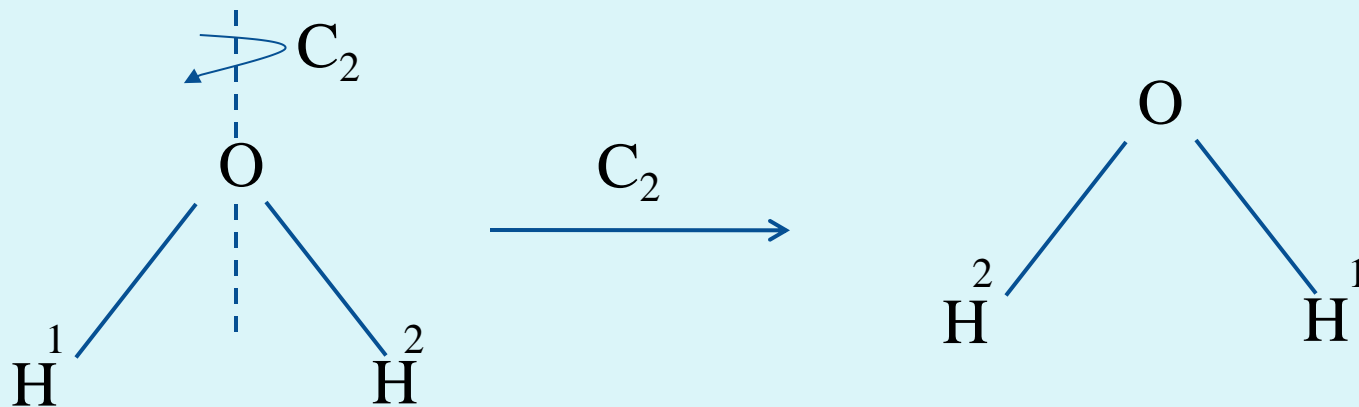
Ex.: Tetrahedral carbon with four different group (CHBrCl) has only identity and No other symmetry element.

n-fold rotation

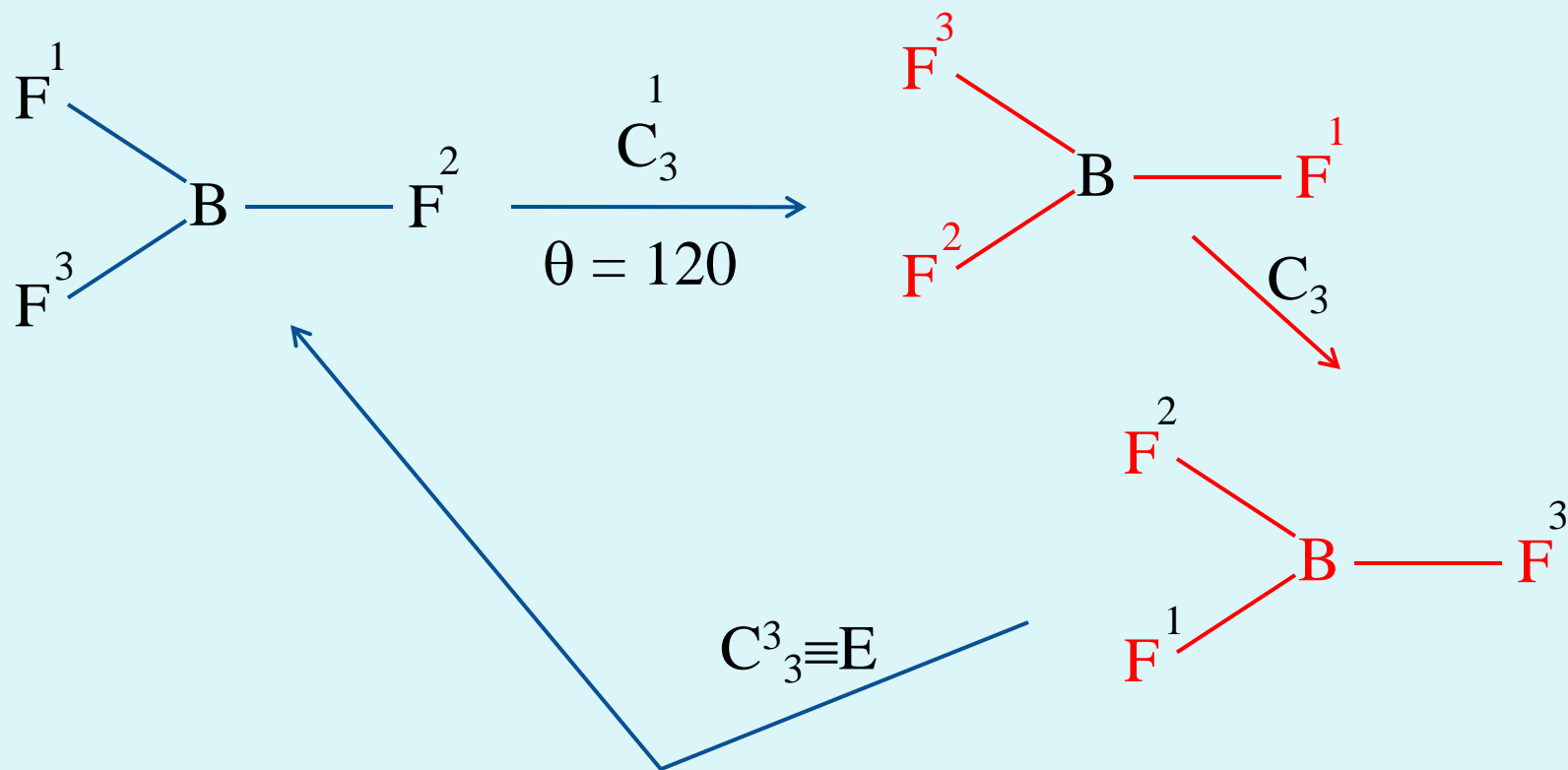
Ex: (1)

H_2O has C_2

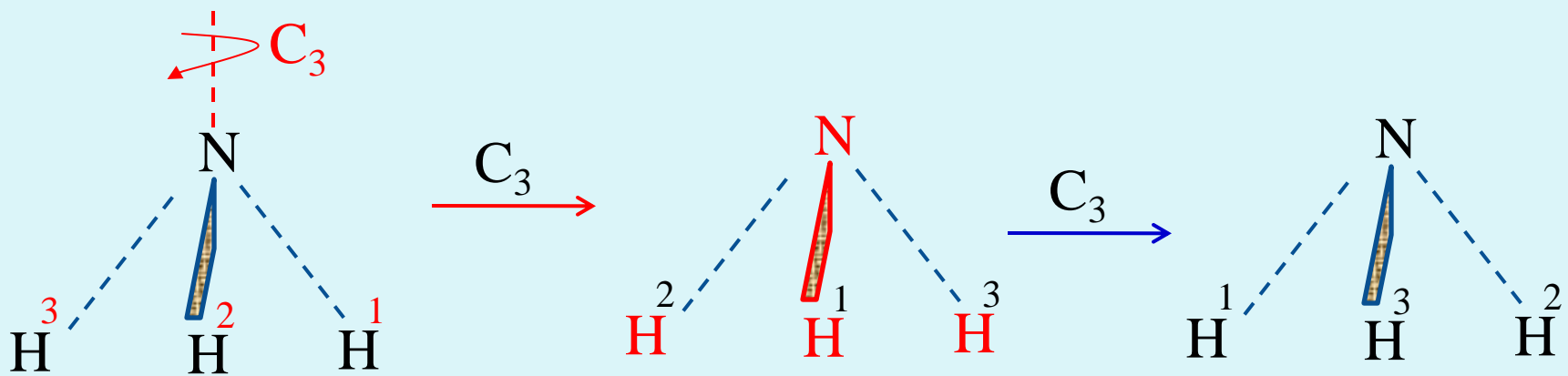
$\theta = 180$ angle of rotation



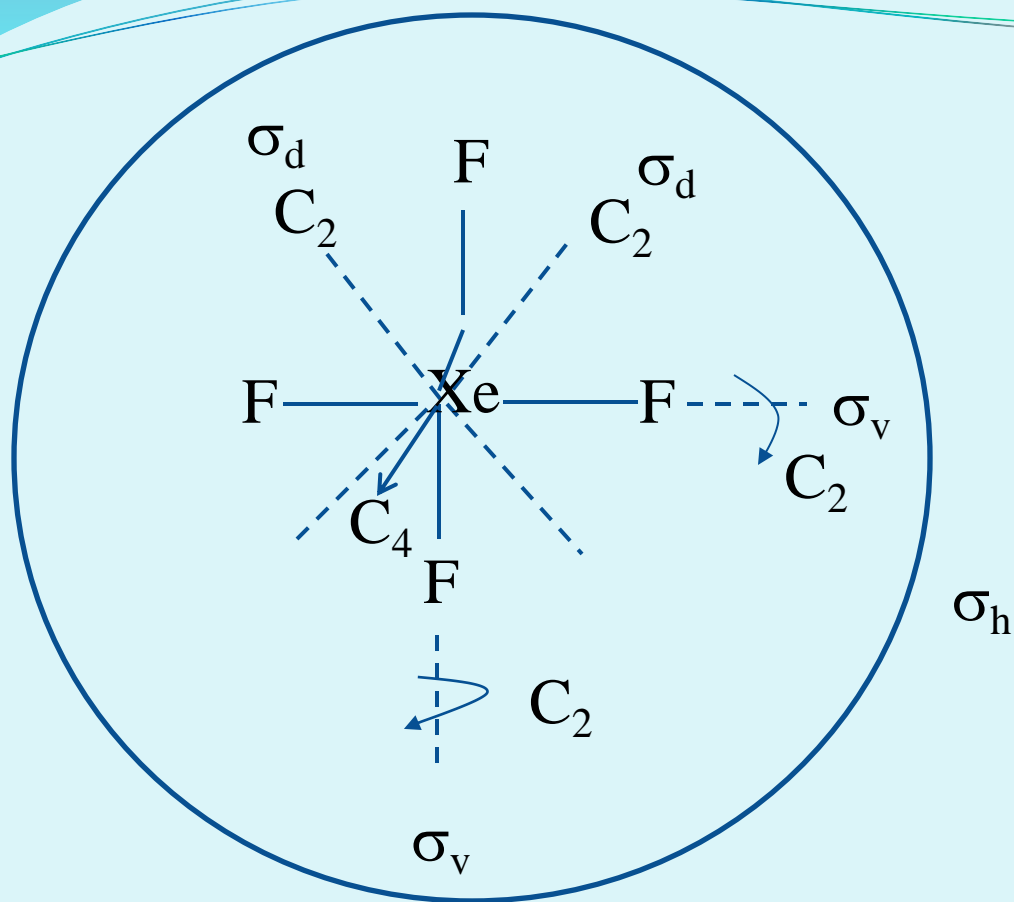
Ex: (2)



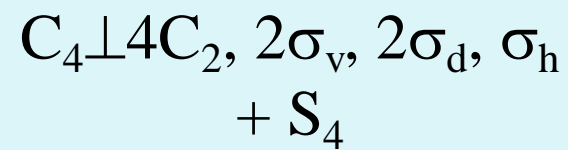
Ex: (3) Ammonia



Only C_3

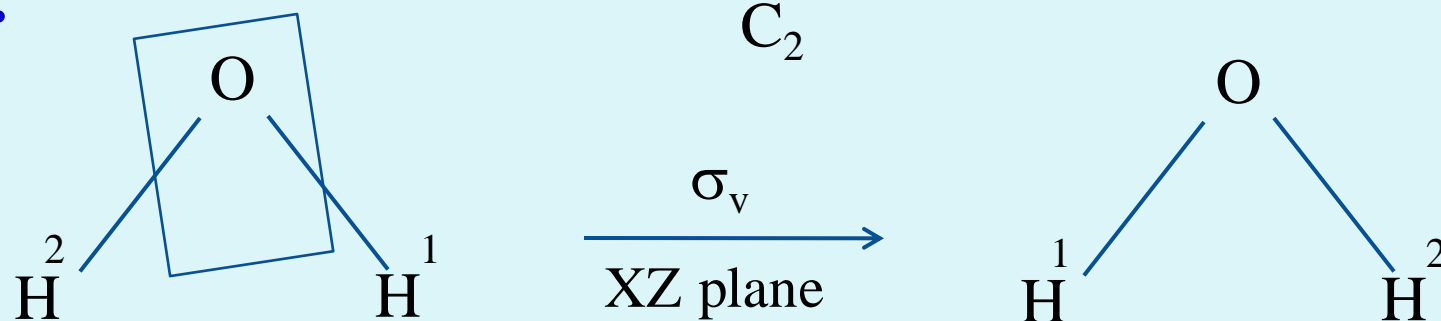


Square planar

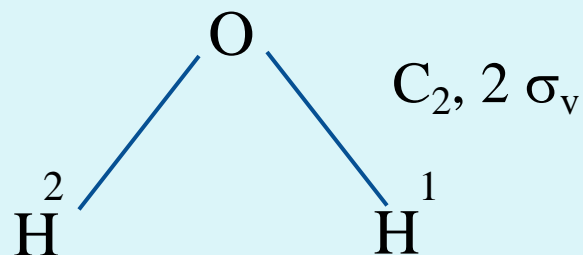


Mirror plane \Rightarrow Reflection of molecules

Ex:



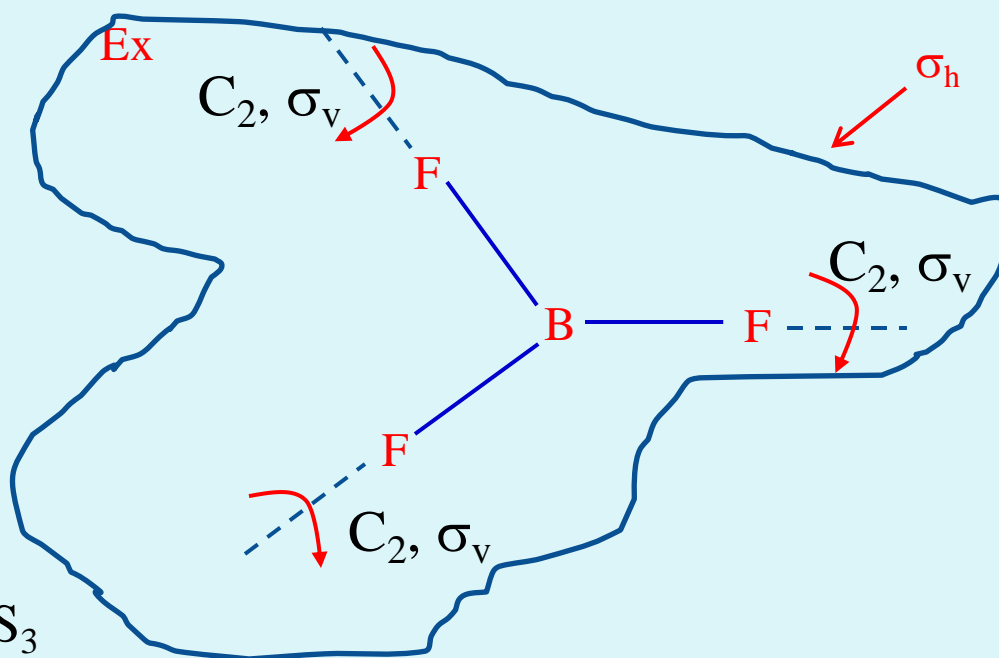
Vertical plane σ_v



Mirror plane

Horizontal plane
symmetry

$$(C_3 \perp 3C_2 + \sigma_h + 3\sigma_v) + S_3$$



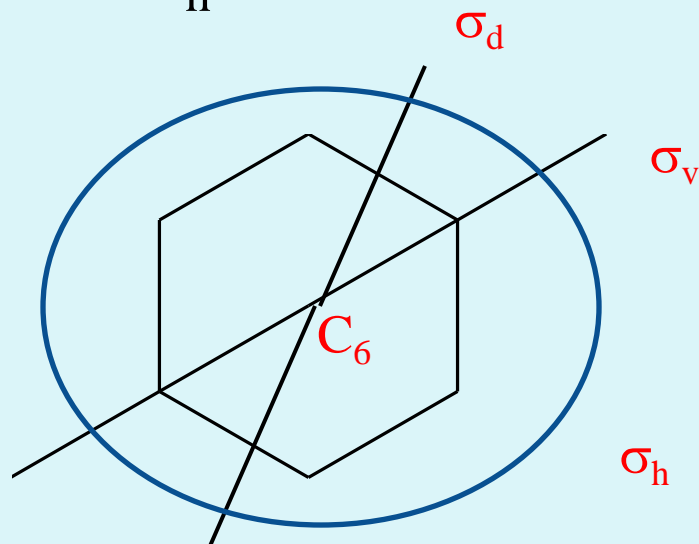
Ex.: Benzene

The benzene ring has C_6 axis which is the principle axis of rotation.

- has σ_v , σ_d

- has horizontal plane σ_h

[The plane that bisect the bonds are called dihedral plane]

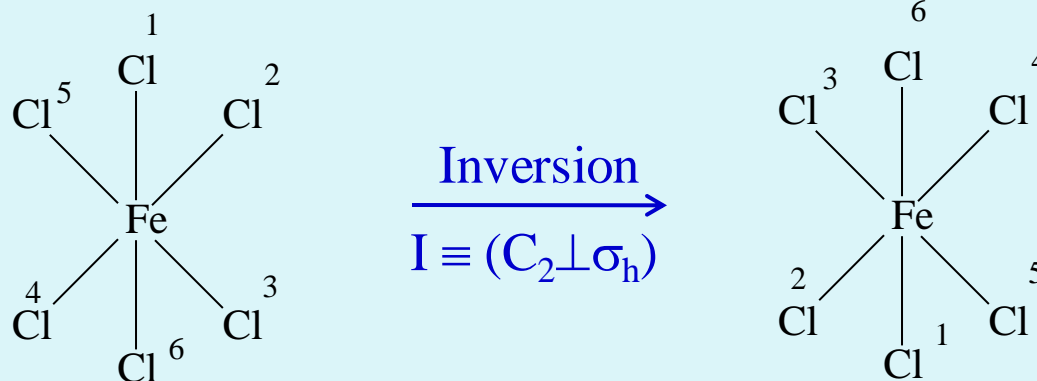
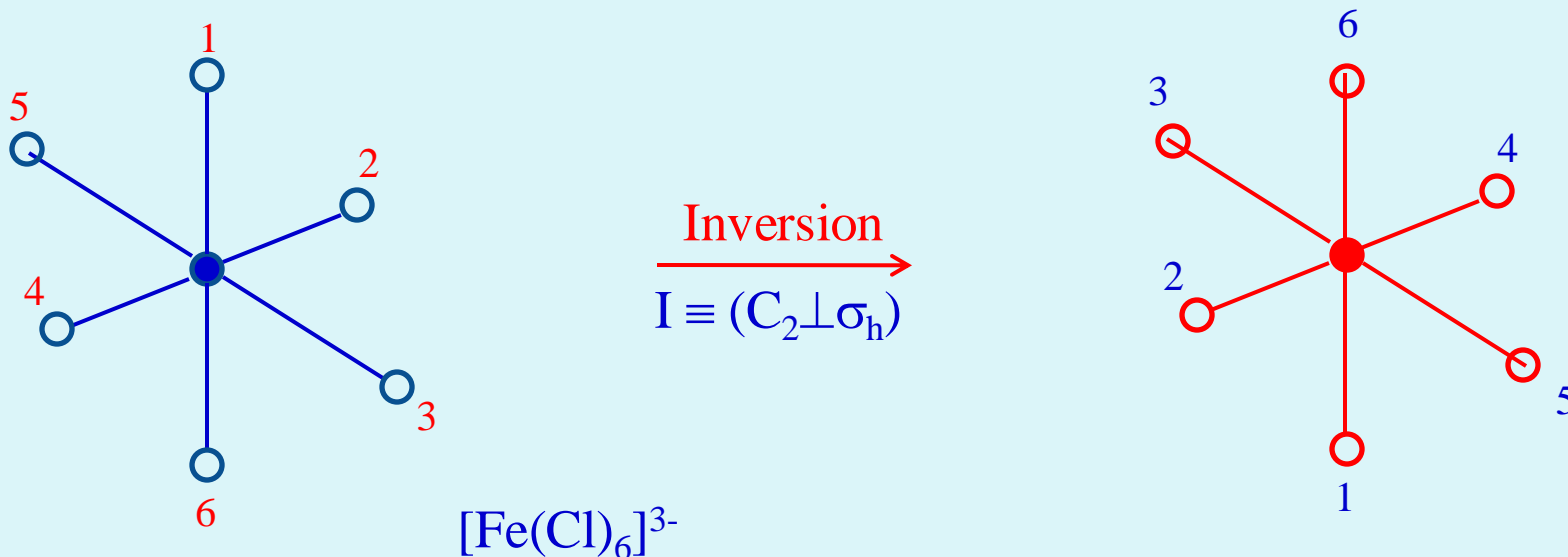


]The vertical plane σ_v , through carbon atom, and include C_6 axis

[horizontal plane perpendicular to C_6 axis called σ_h]

(i) Inversion

The inversion operation projects each atom through the center of inversion, and across to the other side of the molecules



Improper axis of rotation

$S_1 \Rightarrow S_1$ axis doesn't exist it is the same as mirror plane.

$S_2 \Rightarrow S_2$ axis is center of inversion.

Improper rotation (S_n): is rotation followed by reflection in the plane perpendicular to the axis of rotation.

Point group:

Molecules with the same symmetry elements are placed into point group.

Types of point groups

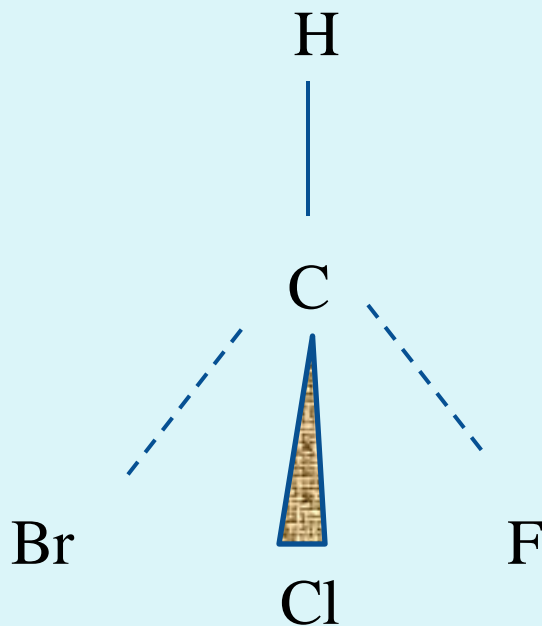
1- C₁ point group:

Molecules with no symmetry element.

Ex.: CHBrFCl

1- C1 point group:

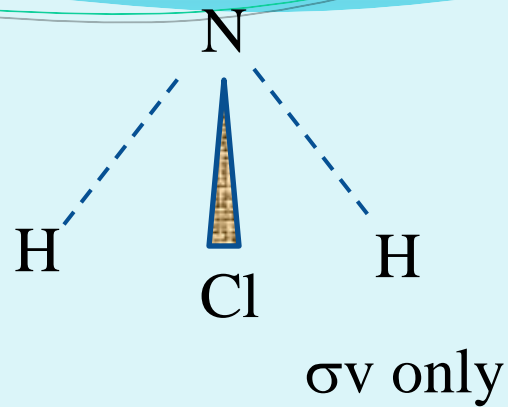
- Molecules with no symmetry element.
- Ex.: CH Br F Cl



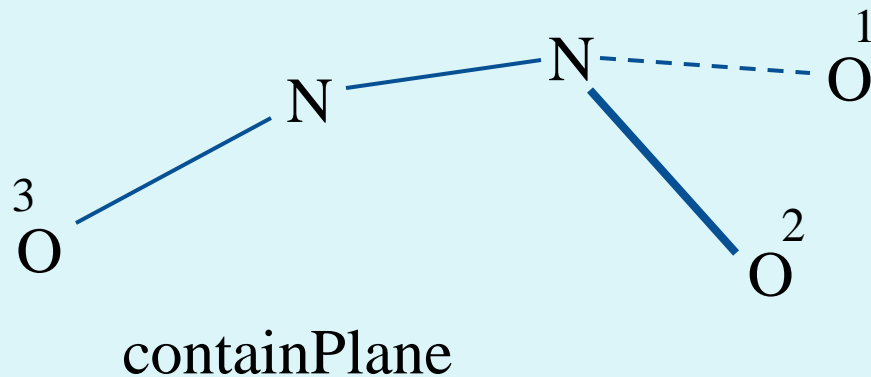
2- Cs-point group:

Molecules with only σ (plane).

Ex1: NH_2Cl



Ex2: N_2O_3



3- Ci-point group:

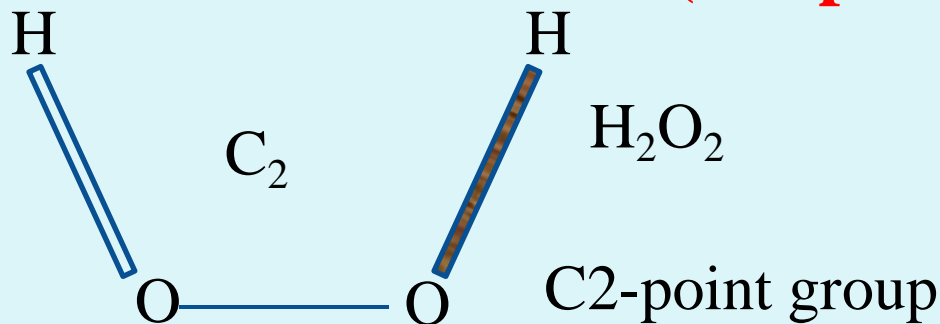
Molecules with only E & i.

4- Molecules with only C_n $n > 1$

C_n -point group

Ex: H_2O_2 & N_2O_2 & N_2H_2

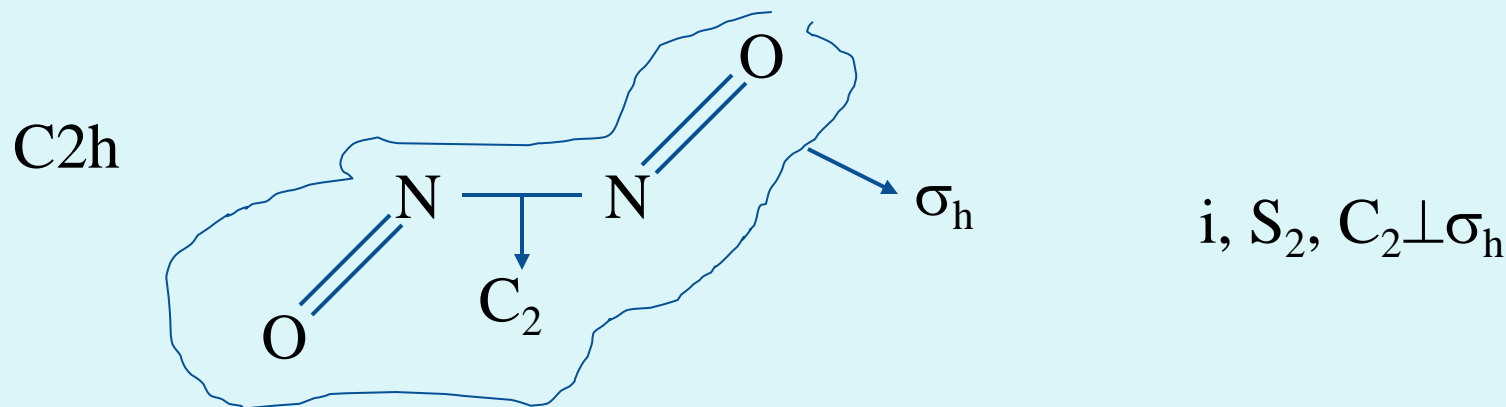
These molecules have book structure (not planar)



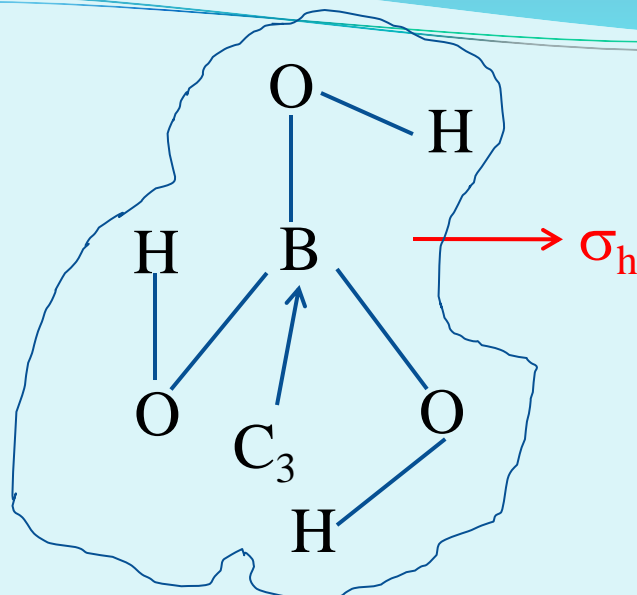
5- Molecules with $C_n + \sigma_h$

belong to C_{nh} – point group

Ex: H_2O_2 , N_2O_2 in planar structure



Ex: $\text{B}(\text{OH})_3$

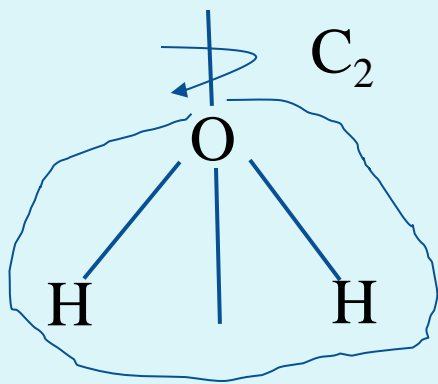


These molecule have C_3 and no C_2 due to (O-H bond)

- All atom are in σ_h

C_{3h}

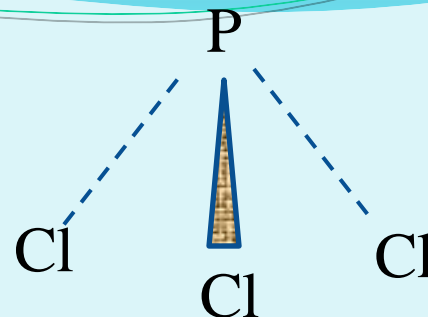
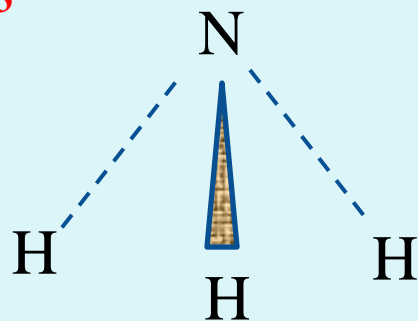
6) Molecules with only $(\text{C}_n + n \sigma_v)$ belong to C_nv



$$\text{C}_2 + 2\sigma_v$$

C_2V – point group

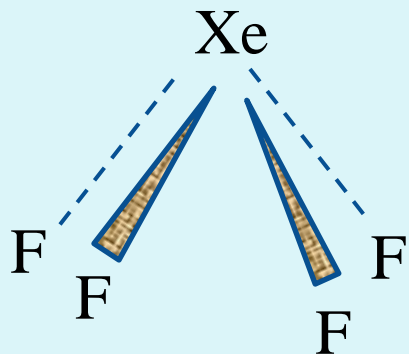
Ex2: NH_3 , PCl_3



C_3 and 3 σ_v

C_{3v} -point group

Ex3: XeF_4 square pyramide

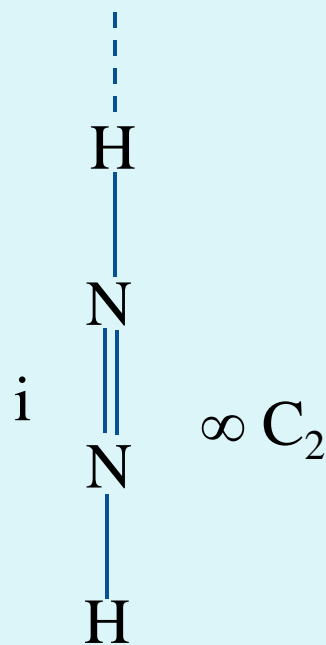


C_4 , 4 σ_v

C_{4v}

7) Molecules with $C_n \perp n C_2$ belong to D_n if then are σ_h it become dnh point group

Ex: Linear molecules



$$C_\infty \perp \infty C_2 + \sigma_h$$

$$\infty \sigma_v + \infty S_2 + i$$

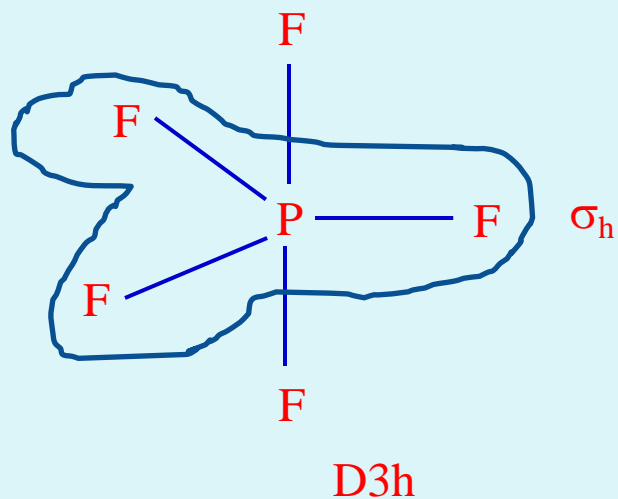
$D_{\infty h}$



The same

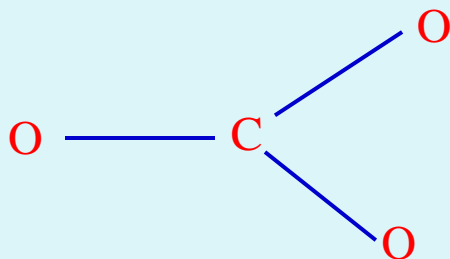


Ex: PF₅ trigonal bipyramide



$$C_3 \perp 3C_2 \sigma_h \\ S_3 + 3\sigma_v$$

Ex: CO₃²⁻



$$C_3 \perp 3C_2 \\ + \sigma_h \\ D_{3h}$$

Ex: Benzene

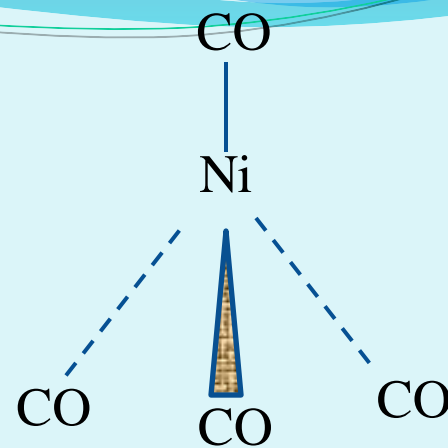
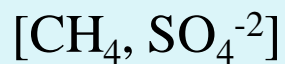
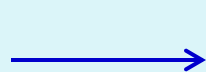
$$C_6 \perp 6C_2 + \sigma_h \equiv D_{6h}$$

8- Special groups

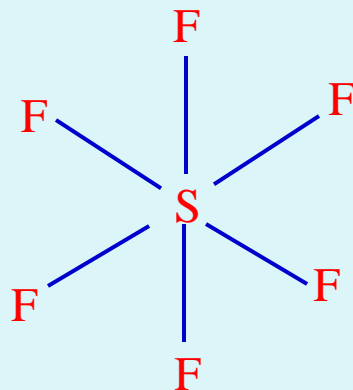
(a) Octahedral $[M(L)_6]$

Oh P.G.

(b) tetrahedral



Td.P.G.

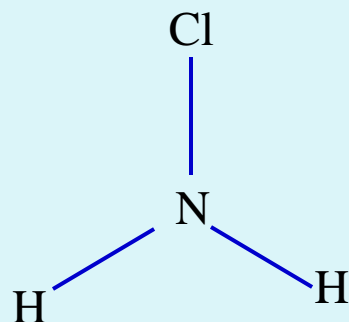


Oh

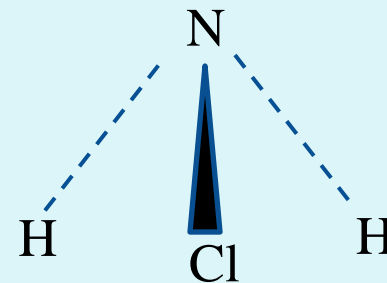
Q: Determine all symmetry elements and point group for each possible structure



Answer: [1]

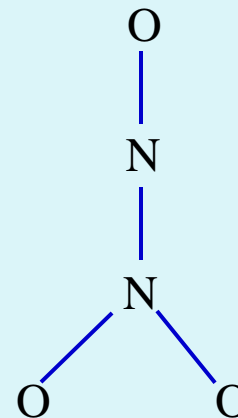
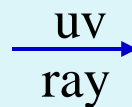
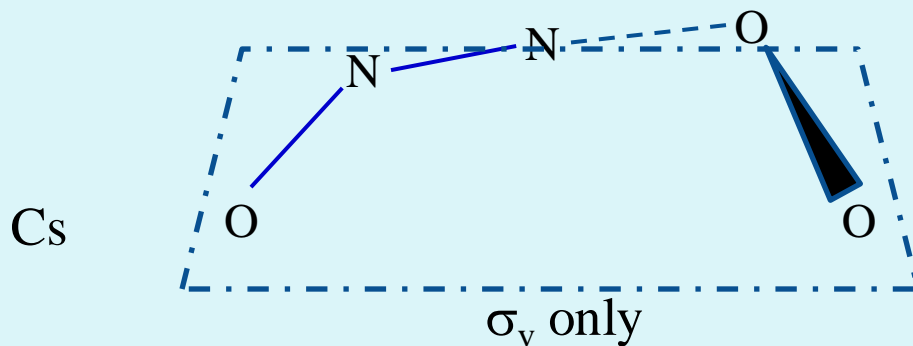


Planar
 $C_2 + \sigma_v + \sigma_v'$
 C_{2v} point group



Trigonal pyramid
 σ_v only
 C_s point group

[2]



$C_2 + \sigma_v + \sigma_v'$
 C_{2v}

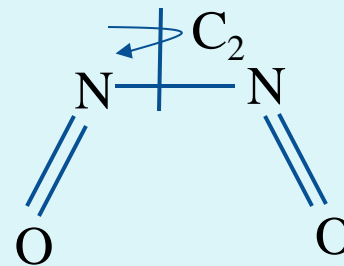
[3] N_2O_2

(i) Linear st

$C_\infty + \infty\sigma_v$
 $S_2 \sigma_h, \infty\sigma_v$
 $\equiv D_{\infty h}$

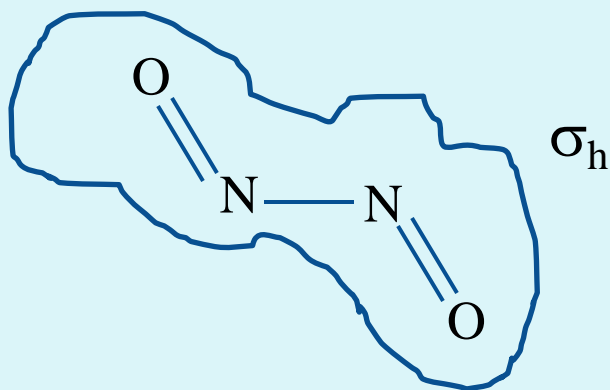


(ii) Non Linear st



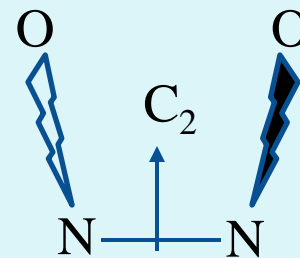
C_2, σ_v
 C_{2v}

(ii) Planar trans



$C_2 + S_2, I + \sigma_h$
 C_{2h}

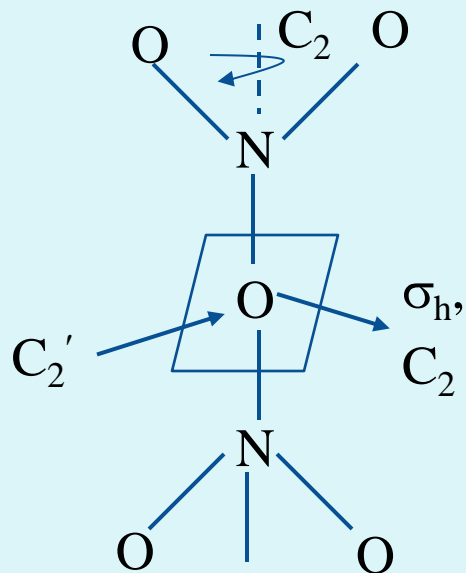
(iv) Book st



C_2 only
 C_2 point group

[4] N_2O_5

(1) Two possible

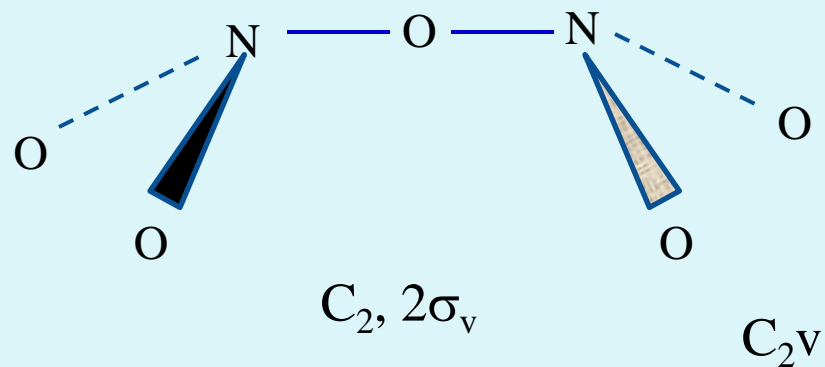


$$C_2 \perp 2C_2' + \sigma_h, 2\sigma_v$$

$$3S_2 + I$$

$$D_{2h}$$

(2) Donkey st



$$C_2, 2\sigma_v$$

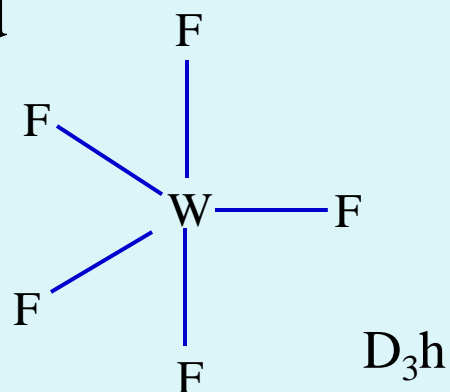
$$C_{2v}$$

(v) WFs

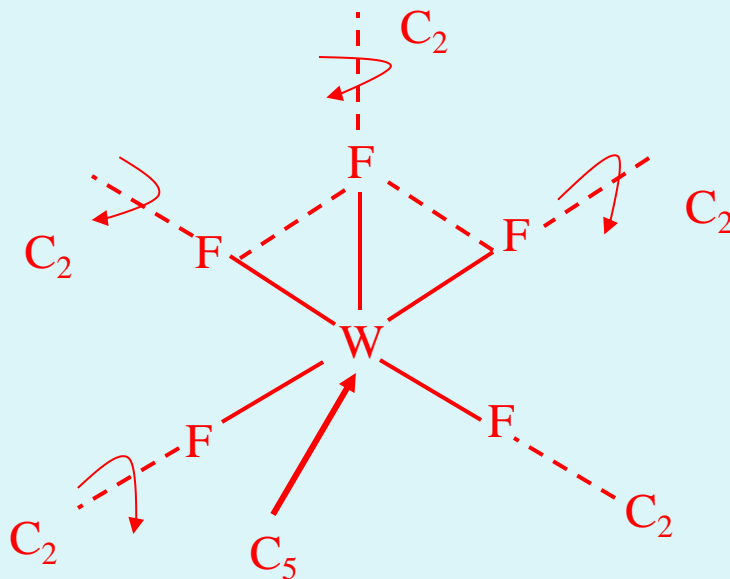
(a) Trigonal bipyramid

$$3\sigma_v + C_3 \perp 3C_2 + \sigma_h$$

$$S_3$$



(B) Pentagon



$$C_5 \perp 5C_2 + S_5$$

$$5\sigma_v, \sigma_h$$

$$D_{5h}$$