* Derivative , Integration of a Vectors and Kinematics of a particle

if you have $\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$

① $\frac{d\vec{A}}{dt} = \hat{i}\frac{dA_x}{dt} + \hat{j}\frac{dA_y}{dt} + \hat{k}\frac{dA_z}{dt}$

② $\frac{d}{dt}(\vec{A} + \vec{B}) = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$

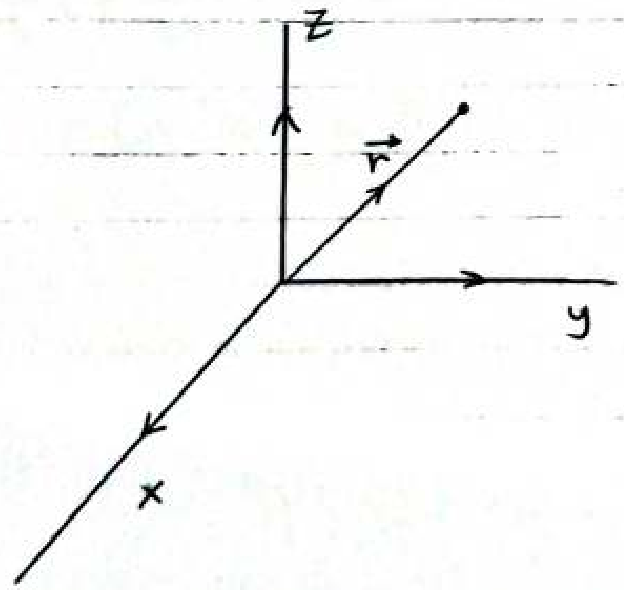
③ $\frac{d}{dt}(n\vec{A}) = \frac{dn}{dt}\vec{A} + n\frac{d\vec{A}}{dt}$

④ $\frac{d}{dt}(\vec{A}\cdot\vec{B}) = \frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt}$

⑤ $\frac{d}{dt}(\vec{A}\times\vec{B}) = \frac{d\vec{A}}{dt}\times\vec{B} + \vec{A}\times\frac{d\vec{B}}{dt}$

* Position Vector of a particle

$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$



* The Velocity Vector :.

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(\hat{i}x + \hat{j}y + \hat{k}z)$$

$$\vec{V} = \hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt} + \hat{k}\frac{dz}{dt}$$

$$\vec{V} = \hat{i}\dot{x} + \hat{j}\dot{y} + k\dot{z}$$

$$\dot{x} = \frac{dx}{dt}, \quad \dot{y} = \frac{dy}{dt}, \quad \dot{z} = \frac{dz}{dt}$$

the magnitude of the Velocity is called "Speed"

$$V = |\vec{V}| = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

* Acceleration Vector :.

The time derivative of the Velocity is called the acceleration

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z})$$

$$\vec{a} = \hat{i}\frac{d\dot{x}}{dt} + \hat{j}\frac{d\dot{y}}{dt} + \hat{k}\frac{d\dot{z}}{dt}$$

$$\vec{a} = \hat{i}\ddot{x} + \hat{j}\ddot{y} + k\ddot{z}$$

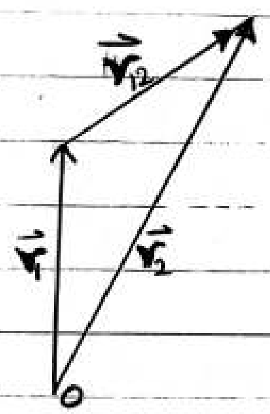
* Vector Integration:

the Integration of $\vec{a}$ is $\Rightarrow$ Velocity ($\vec{V}$)

The Integration of $\vec{V}$ is $\Rightarrow$ position Vector ($\vec{r}$)

* Relative Velocity:

If you have two particles



r_1 : position Vector for first particle

r_2 : position Vector for second particle

$$\vec{r}_1 + \vec{r}_{12} = \vec{r}_2$$

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$\vec{V}_{12} = \frac{d\vec{r}_{12}}{dt} = \frac{d\vec{r}_2}{dt} - \frac{d\vec{r}_1}{dt}$$

$$\boxed{\vec{V}_{12} = \vec{V}_2 - \vec{V}_1}$$

$$\boxed{V_2 = V_{12} + V_1}$$ السرعة الحقيقية للجسم الثاني

سرعة الجسم المقاسة بالنسبة لشيء ثابت هي سرعة حقيقية

" " " " لشيء متحرك هي سرعة نسبية

* Tangential and Normal Components of Acceleration

The Velocity vector of a moving particle $(\vec{V})$ can be written as the product of the particle's speed (v) and Unit Vector (τ) that gives the direction of the particle's motion. Thus

$$\vec{V} = v\,\tau$$

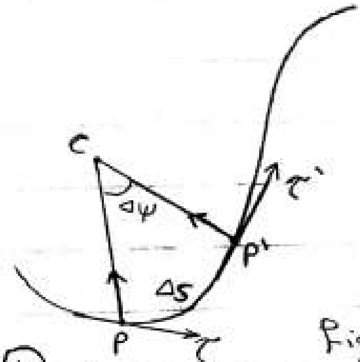


Fig (1)

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(v\,\tau)$$

$$\vec{a} = \dot{v}\,\hat{\tau} + v\left(\frac{d\hat{\tau}}{dt}\right) \quad \text{---} (1)$$

the direction of $\Delta\tau$ become perpendicular to the direction of (τ) in the limit as $\Delta\psi$ and Δs approach zero.
and $\Delta\tau$ approch from $\Delta\psi$, then

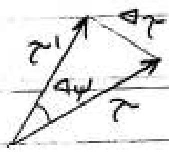


Fig (2)

$$\left|\frac{d\tau}{d\psi}\right| = 1 \quad , \quad \frac{d\hat{\tau}}{d\psi} = \hat{n}$$

$\hat{n} \Rightarrow$ unit normal vector

to Find $\left(\frac{d\hat{\tau}}{dt}\right)$

$$\frac{d\hat{\tau}}{dt} = \frac{d\tau}{d\psi}\cdot\frac{d\psi}{dt}$$

$$\frac{d\hat{\tau}}{dt} = \hat{n}\,\frac{d\psi}{dt}$$

$* \ \frac{d\psi}{d\psi}$

$* \ \frac{ds}{ds}$

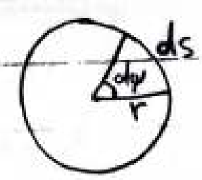


Fig (3)

الزاوية × نق = طول القوس

$$ds = r\,d\psi$$

$$\boxed{\frac{d\psi}{ds} = \frac{1}{r}}$$

$$\frac{d\hat{\tau}}{dt} = \hat{n}\,\frac{d\psi}{ds}\cdot\frac{ds}{dt}$$

$$\frac{d\hat{\tau}}{dt} = \hat{n}\left(\frac{1}{r}\right)v = \frac{v}{r}\,\hat{n}$$

the above value for $\frac{d\hat{\tau}}{dt}$ is now inserted into eq (1) to yield the final result

$$a = \underbrace{\dot{v}\,\tau}_{\text{Tangential Component}} + \underbrace{\left(\frac{v^2}{r}\right)\hat{n}}_{\text{Normal Component}}$$

Normal Component is always directed toward the center of curvature on the Concave side of the path of motion.