eq.6) show that the magnitude of tangential Component of the acceleration is given by the expression

$$a_T = \frac{\vec{a}\cdot\vec{v}}{|\vec{v}|}$$, and that of the normal Component is:. $$a_n = (a^2 - a_T^2)^{1/2}$$

$$\vec{v} = v\hat{\tau}$$
$$\vec{a} = \dot{v}\hat{\tau} + \frac{v^2}{r}\hat{n}$$

$$\vec{a}\cdot\vec{v} = (\dot{v}\hat{\tau} + \frac{v^2}{r}\hat{n})\cdot v\hat{\tau}$$

$$= \dot{v}\hat{\tau}\cdot v\hat{\tau} + \frac{v^2}{r}\underbrace{\hat{n}\cdot v\hat{\tau}}_{\text{صفر}}$$

$$\vec{a}\cdot\vec{v} = \dot{v}\,v = a_T v$$ حيث $\dot{v} = a_T$

$$\therefore a_T = \frac{\vec{a}\cdot\vec{v}}{v}$$

(2) $$\vec{a} = \vec{a}_T + \vec{a}_n$$
$$\vec{a} = a_T\hat{\tau} + a_n\hat{n}$$

مقدار $a = (a_T^2 + a_n^2)^{1/2}$

$$a^2 = a_T^2 + a_n^2$$

$$a_n = (a^2 - a_T^2)^{1/2}$$

eq. 8) A particle moves on a circle of constant radius (b) if the speed of the particle varies with the time (t) according to ($v = At^2$),
For what Value, or values, of (t) does the acceleration Vector make an angle of (45°) with the Velocity vector?

$$\vec{v} \cdot \vec{a} = va\cos\theta \Rightarrow \cos\theta = \frac{\vec{v} \cdot \vec{a}}{va} \quad ---- (1)$$

$$v = At^2 \Rightarrow \dot{v} = 2At$$

$$\vec{a} = \dot{v}\,\hat{\tau} + \frac{v^2}{b}\hat{n}$$

$$\vec{a} = 2At\,\hat{\tau} + \frac{A^2t^4}{b}\hat{n}$$

$$a = |\vec{a}| = \sqrt{4A^2t^2 + \frac{A^4t^8}{b^2}}$$

$$\vec{v} \cdot \vec{a} = At^2\,\hat{\tau} \cdot \left(2At\,\hat{\tau} + \frac{A^2t^4}{b}\hat{n}\right)$$

$$\vec{v} \cdot \vec{a} = 2A^2t^3\,\hat{\tau}\cdot\hat{\tau} + \frac{A^3t^6}{b}\,\underbrace{\hat{\tau}\cdot\hat{n}}_{zero}$$

$$\vec{v} \cdot \vec{a} = 2A^2t^3$$

$$\therefore \cos\theta = \frac{2A^2t^3}{(At^2)\sqrt{4A^2t^2 + \frac{A^4t^8}{b^2}}} = \frac{2\cancel{A^2t^2}}{\cancel{At^2} \cdot \frac{At}{b}(4b^2 + A^2t^4)^{1/2}}$$

$$\cos 45 = \frac{2b}{(4b^2 + A^2t^4)^{1/2}} = \frac{1}{\sqrt{2}}$$

$$4b^2 + A^2t^4 = 8b^2 \Rightarrow 4b^2 = A^2t^4$$

$$t^4 = \frac{4b^2}{A^2} \Rightarrow t^2 = \frac{2b}{A}$$

$$\therefore t = \left(\frac{2b}{A}\right)^{1/2}$$

eq.10) A particle moves on a لولبي helical path with cylindrical coordinates varying with time as

$$R = A \quad , \phi = Bt^2 \quad , \quad z = Ct^2$$

where A, B, C are Constant. Find the velocity and acceleration vectors as a functions of t.

$$R = A \qquad \phi = Bt^2 \qquad z = ct^2$$
$$\dot{R} = 0 \qquad \dot{\phi} = 2Bt \qquad \dot{z} = 2ct$$
$$\ddot{R} = 0 \qquad \ddot{\phi} = 2B \qquad \ddot{z} = 2c$$

$$\vec{v} = \dot{R}\,\hat{i}_R + R\dot{\phi}\,\hat{j}_\phi + \dot{z}\,\hat{k}$$
$$\vec{v} = 0 + 2ABt\,\hat{j}_\phi + 2ct\,\hat{k}$$

$$\vec{a} = (\ddot{R} - R\dot{\phi}^2)\,\hat{i}_R + (2\dot{R}\dot{\phi} + R\ddot{\phi})\,\hat{j}_\phi + \ddot{z}\,\hat{k}$$

$$= (0 - 4AB^2t^2)\,\hat{i}_R + (0 + 2AB)\,\hat{j}_\phi + 2c\,\hat{k}$$

$$\vec{a}\cdot\vec{v} = av\cos\theta \Rightarrow \cos\theta = \frac{\vec{a}\cdot\vec{v}}{av}$$

نطبقها ثم نستخرج الزاوية

eq.12) prove that $\vec{A} \cdot \frac{d\vec{A}}{dt} = A \frac{dA}{dt}$

$$\vec{A} \cdot \vec{A} = A^2$$

نأخذ المشتقة للطرفين

$$\frac{d}{dt}(\vec{A} \cdot \vec{A}) = \frac{d}{dt}(A^2)$$

نشتق الطرف الأيسر

$$\frac{d\vec{A}}{dt} \cdot \vec{A} + \vec{A} \cdot \frac{d\vec{A}}{dt} = 2\vec{A} \cdot \frac{d\vec{A}}{dt}$$

نشتق الطرف الأيمن

$$2A\frac{dA}{dt}$$

$$\therefore \cancel{2}\vec{A} \cdot \frac{d\vec{A}}{dt} = \cancel{2}A\frac{dA}{dt} \Rightarrow \therefore \vec{A} \cdot \frac{d\vec{A}}{dt} = A\frac{dA}{dt}$$

و.هـ.م

eq.14) prove that $\frac{d}{dt}(\vec{r} \cdot [\vec{v} \times \vec{a}]) = \vec{r} \cdot (\vec{v} \times \dot{\vec{a}})$

$$\frac{d}{dt}(\vec{r} \cdot [\vec{v} \times \vec{a}]) = \vec{r} \cdot \frac{d}{dt}[\vec{v} \times \vec{a}] + [\vec{v} \times \vec{a}] \cdot \dot{r}$$

$$= \vec{r} \cdot [\vec{v} \times \dot{\vec{a}} + \dot{v} \times \vec{a}] + [\vec{v} \times \vec{a}] \cdot \vec{v}$$

$$= \vec{r} \cdot [\vec{v} \times \dot{\vec{a}} + \underbrace{\vec{a} \times \vec{a}}_{\text{صفر}}] + \underbrace{[\vec{v} \times \vec{v}]}_{\text{الصفر}} \cdot \vec{a}$$

$$= \vec{r} \cdot [\vec{v} \times \dot{\vec{a}}]$$

ex) two vectors

$$\vec{A} = 5t^2\,\hat{i} + t\,\hat{j} + t^3\,\hat{k}$$
$$\vec{B} = (\sin t)\,\hat{i} - (\cos t)\,\hat{j}$$

Find ① $\frac{d}{dt}(\vec{A}\cdot\vec{B})$, ② $\frac{d}{dt}(\vec{A}\times\vec{B})$

① $\vec{A}\cdot\vec{B} = 5t^2\sin t - t\cos t + 0$

$$\frac{d}{dt}(\vec{A}\cdot\vec{B}) = 5t^2\cos t + 10t\sin t - [-t\sin t + \cos t]$$
$$= 5t^2\cos t + 10t\sin t + t\sin t - \cos t$$
$$= 5t^2\cos t + 11t\sin t - \cos t$$
$$= \cos t\,(5t^2 - 1) + 11t\sin t$$

② $$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 5t^2 & t & t^3 \\ \sin t & -\cos t & 0 \end{vmatrix} =$$

$$\hat{i}(0 + t^3\cos t) - \hat{j}(t^3\sin t) + \hat{k}(-5t^2\cos t - t\sin t)$$

تم الاشتقاق

ex) Suppose the position vector of a particle is given by

$$\vec{r} = \hat{i}\, b \sin\omega t + \hat{j}\, b \cos\omega t + \hat{k}\, c \quad , \text{ Find}$$

1. The distance from this position to origin
2. Velocity vector and the speed
3. acceleration Vector 4. The figure of path

---

① $r = |\vec{r}| = \sqrt{b^2 \sin^2\omega t + b^2 \cos^2\omega t + c^2}$

$$= \sqrt{b^2(\sin^2\omega t + \cos^2\omega t) + c^2} = \sqrt{b^2 + c^2}$$

② $\vec{V} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(\hat{i}\, b \sin\omega t + \hat{j}\, b \cos\omega t + \hat{k}\, c)$

$$= \hat{i}\, b\omega \cos\omega t - \hat{j}\, b\omega \sin\omega t + k(0)$$

$$V = |\vec{V}| = \sqrt{\omega^2 b^2 \cos^2\omega t + b^2\omega^2 \sin^2\omega t} = \sqrt{b^2\omega^2(\sin^2\omega t + \cos^2\omega t)}$$
$$= b\omega$$

نستنتج أن متجه بسرعة دائرية لـ (xy) لأن مركبة k تساوي صفر والانطلاقة ثابتة مقدارها (bw)

③ $\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(\hat{i}\, b\omega \cos\omega t - \hat{j}\, b\omega \sin\omega t)$

$$\vec{a} = -\hat{i}\, b\omega^2 \sin\omega t - \hat{j}\, b\omega^2 \cos\omega t)$$

$$a = |\vec{a}| = \sqrt{b^2\omega^4 \sin^2\omega t + b^2\omega^4 \cos^2\omega t}$$

$$= \sqrt{b^2\omega^4(\sin^2\omega t + \cos^2\omega t)} = b\omega^2$$

④ $\Rightarrow$

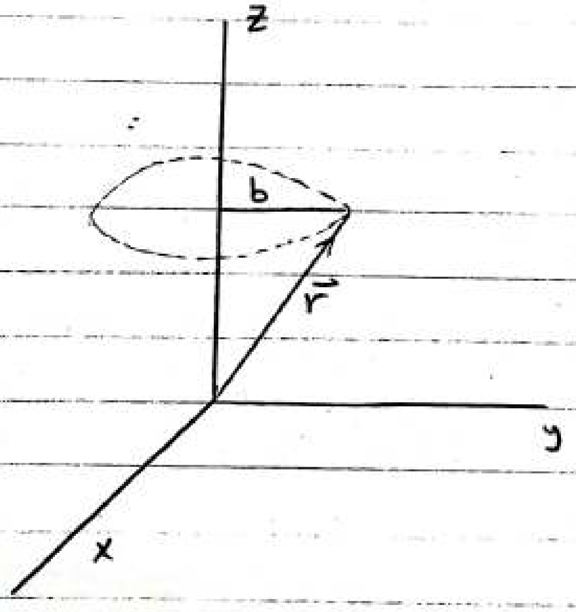
4 لايجاد شكل المسار نجد $\vec{a} \cdot \vec{v}$

$$\vec{a} \cdot \vec{v} = (-\hat{i}\, b\omega^2 \sin\omega t - \hat{j}\, b\omega^2 \cos\omega t) \cdot (\hat{i}\, b\omega \cos\omega t - \hat{j}\, b\omega \sin\omega t + k(0))$$

$$- b\omega^2 \sin\omega t \cdot b\omega \cos\omega t + b\omega^2 \cos\omega t \cdot b\omega \sin\omega t$$

$$= \text{zero}$$

بما ان $\vec{a} \cdot \vec{v} = 0$ نستنتج ان التعجيل عمودي على السرعة



ex) A particle moves on spiral path such that the position in polar Coordinats is given by :.

حلزوني

$r = bt^2$, $\theta = ct$, c,b Constant

Find the velocity and acceleration as a function of (t)?

$\theta = ct$ $\qquad$ $r = bt^2$

$\dot{\theta} = c$ $\qquad$ $\dot{r} = 2bt$

$\ddot{\theta} = zero$ $\qquad$ $\ddot{r} = 2b$

$\vec{v} = \dot{r}\,\hat{i}_r + r\dot{\theta}\,\hat{j}_\theta$

$\vec{v} = 2bt\,\hat{i}_r + cbt^2\,\hat{j}_\theta$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{i}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\hat{j}_\theta$

$= (2b - 2bt^2c^2)\,\hat{i}_r + 4bct\,\hat{j}_\theta$