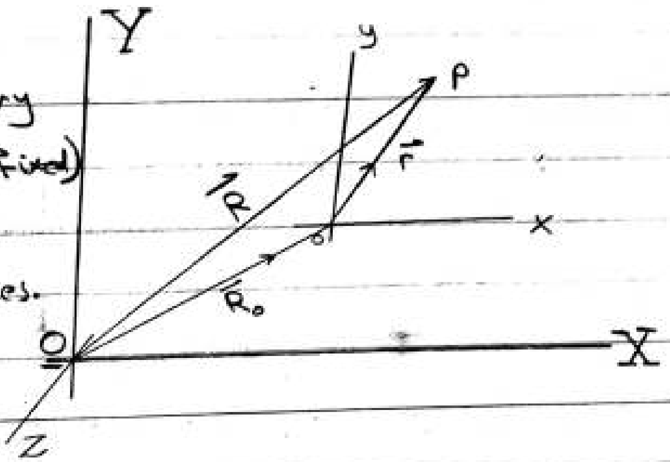
* Translation Motion of the Coordinate system.

The simplest type of motion of the coordinate system is that of pure translation.



(OXYZ) are the primary coordinate axes (assumed fixed)

(Oxyz) are the moving axes.

In the case of pure translation, that respective axes OX and Ox, and so on, remain paraller.

$\vec{R}$:- the position vector of aparticle (p) in fixed system.

$\vec{r}$:- the position Vector of apardicle (p) in moving system.

R_o :- the displacement of a moving origin. ازاحة

thus:-

$$\vec{R} = \vec{r} + R_o$$

by taking the first and second derivatives with respect to time:-

$$\therefore \ \vec{V} = \vec{v} + \vec{V}_o$$

$$\vec{A} = \vec{a} + \vec{A}_o$$

$\vec{V}_o$, $\vec{A}_o$ are the velocity and acceleration, respectively, of moving origin.

v, a, are the velocity and acceleration, respectively, of (P) in the moving system.

In particular: حالة خاصة

. If the moving system is not accelerating, so that $\vec{A}_o = \text{zero}$

$$\therefore \vec{A} = \vec{a}$$

that is, the acceleration is the same with respect to either system.

This statement is true only for the case in which there is no rotation of the moving system.

General Motion of the Coordinate System

الحركة العامة للمحاور

$$\vec{R} = \vec{R}_o + \vec{r}$$

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$

$$\therefore \vec{R} = \vec{R}_o + \hat{i}x + \hat{j}y + \hat{k}z$$

$$\frac{d\vec{R}}{dt} = \vec{V}_o + \underbrace{\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}}_{\text{سرعة الجسم بالنسبة للمحاور المتحركة}} + \underbrace{x\frac{d\hat{i}}{dt} + y\frac{d\hat{j}}{dt} + z\frac{d\hat{k}}{dt}}_{\text{بسبب اتجاهات دوران المحاور}}$$

Oxyz

$$\text{but} \quad \frac{d\hat{i}}{dt} = \vec{\omega} \times \hat{i}$$

$$\frac{d\hat{j}}{dt} = \vec{\omega} \times \hat{j}$$

$$\frac{d\hat{k}}{dt} = \vec{\omega} \times \hat{k}.$$

سرعة الجسيم متحرك في نظام محاور
لأحدها ثابت والثاني يتحرك حركة انتقالية
ودورانية

$$\therefore \frac{d\vec{R}}{dt} = \vec{\dot{r}} + \vec{\omega} \times \vec{r} + V_0$$

* ظهر الحد (V_0) بسبب الحركة الانتقالية للمحاور المتحركة فقط.

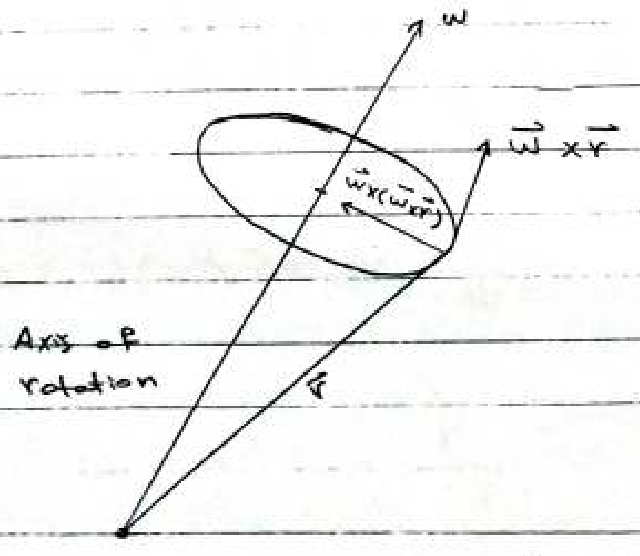
منه

$$\left\{ \frac{d^2\vec{R}}{dt^2} = \vec{\ddot{r}} + \underbrace{2\vec{\omega} \times \dot{r}} + \underbrace{\dot{\omega} \times r} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r})} + A_0 \right\}$$

التعجيل الكوريولي
Coriolis acceleration

التعجيل المستعرض
transverse acceleration

تعجيل الجذب المركزي
centripetal acceleration



الشكل يمثل التعجيل
المركزي

Dynamics of a Particle in a Rotating Coordinate System

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{R}}{dt^2}$$

$$\frac{d^2\vec{R}}{dt^2} = \ddot{\vec{r}} + 2\vec{\omega}\times\dot{\vec{r}} + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + A_o \Big\} \times m$$

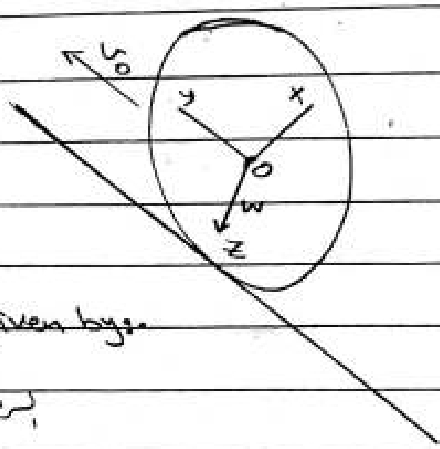
$$m\ddot{\vec{r}} = \vec{F} \underbrace{- 2m\vec{\omega}\times\dot{\vec{r}}} \underbrace{- m\dot{\vec{\omega}}\times\vec{r}} \underbrace{- m\vec{\omega}\times(\vec{\omega}\times\vec{r})} \underbrace{- m\vec{A}_o}$$

القوة الكوريولية	$-2m\vec{\omega}\times\dot{\vec{r}}$	Coriolis force
القوة المستعرضة	$-m\dot{\vec{\omega}}\times\vec{r}$	transverse force
القوة المركزية (الطاردة)	$-m\vec{\omega}\times(\vec{\omega}\times\vec{r})$	centrifugal force
القوة المتبقية	$-mA_o$	remaining force

in which the total "force" is given by:

$$\left[\, \text{"}F\text{"} = \vec{F} + \vec{F}_{cor} + \vec{F}_{trans} + \vec{F}_{cent} - mA_o \,\right]$$

ex) A wheel (عجلة) of radius (b) rolls along the ground with constant forward (امامية) speed (انطلاق) (v), Find the acceleration, relative to the ground, of any point on the rim.



$\vec{r} = \hat{i}\, b$

$\dot{\vec{r}} = 0$

$\ddot{\vec{r}} = 0$

the angular velocity vector is given by:

$\vec{\omega} = \hat{k}\omega = \hat{k}\dfrac{v}{b}$ السرعة الزاوية = $\dfrac{\text{السرعة}}{\text{نق}}$

(٢) لأن ω ثابتة
في العجلة المنتظمة

all terms in the expression for acceleration vanish except the centripetal term.

$$\vec{A} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \hat{k}\omega \times (\hat{k}\omega \times \hat{i} b)$$

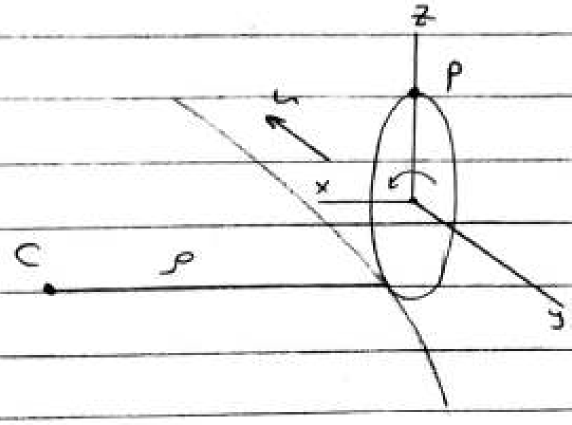
$$= \frac{v^2}{b}\, \hat{k} \times (\hat{k} \times \hat{i})$$

$$= \frac{v^2}{b}\, \hat{k} \times \hat{j}$$

$$= \frac{v^2}{b}\, (-\hat{i})$$

∴ مقدار ($\vec{A}$) يساوي ($\dfrac{v^2}{b}$) ويتجه دائماً نحو مركز العجلة

ex2) A bicycle travels with constant speed around a track (مسار) of radius (ρ). What is the acceleration of the highest point on one of its wheels?



$$\vec{\omega} = \hat{k}\frac{v}{\rho}$$

$$A_o = \hat{i}\frac{v^2}{\rho}$$

the acceleration in the moving system oxyz

$$\ddot{\vec{r}} = -k\frac{v^2}{b}$$ عكس اتجاه

the velocity of this point in the moving system is given by:

$$\dot{r} = -\hat{j}v$$

so, the Coriolis acceleration is:

$$2\vec{\omega}\times\dot{r} = 2\left(\frac{v}{\rho}\hat{k}\right)\times(-\hat{j}v) = 2\frac{v^2}{\rho}\hat{i}$$

- the angular velocity is constant $\Rightarrow \therefore \dot{\omega} = \text{zero}$

$\therefore$ centripetal acceleration is also zero, because

$$\vec{\omega}\times(\vec{\omega}\times\vec{r}) = \frac{v^2}{\rho^2}\hat{k}\times(\hat{k}\times b\hat{k}) = \text{zero}$$

Thus, the total acceleration of the highest point on the wheel is:

$$\vec{A} = 3\frac{v^2}{\rho}\hat{i} - \frac{v^2}{b}\hat{k}$$

ex.3) A bug (بقة) crawls (تزحف) outward (الى الخارج) with constant speed (v) along the spoke of a wheel which is rotating with constant angular velocity (ω) about a vertical axis. Find all the forces acting on the bug.

$$\vec{r} = \hat{i}x = \hat{i}vt$$
$$\dot{\vec{r}} = \hat{i}v$$
$$\ddot{\vec{r}} = 0$$

If we choose the z axis to be vertical, then

$$\vec{\omega} = \hat{k}\omega$$

The various forces are then given by:

Coriolis force:

$$-2m\vec{\omega}\times\dot{\vec{r}} = -2m\omega v(\hat{k}\times\hat{i}) = -2m\omega v\hat{j}$$

transverse force:

$$-m\dot{\vec{\omega}}\times\vec{r} = 0 \qquad (\omega = \text{constant})$$

القوة الطاردة Centrifugal force:

$$-m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$
$$= -m\omega^2[\hat{k}\times(\hat{k}\times\hat{i}x)]$$
$$= -m\omega^2 x(\hat{k}\times\hat{j})$$
$$= m\omega^2 x\hat{i}$$

في الحالة النهائية نضع

$$\vec{F} - 2m\omega v\hat{j} + m\omega^2 x\hat{i} = 0$$

هنا F هي القوة الحقيقية التي يسلطها الدولاب على البقة

« Chapter (4) Central Force and Celestial Mechanics القوى المركزيه والميكانيكا السماوي

A force whose line of action passes through fixed point or (center of force) is called Central force.
For they include such forces as gravity, electrostatic forces.

* The law of Gravity:

Every particle in the universe attracts every other particle with a force that varies directly as the product of the masses of the two particles and inversely as the square of their distance apart.

$$\vec{F}_{12} = G \frac{m_1 m_2}{r_{12}^2} \hat{n}$$

where

$\vec{F}_{12}$: is the force on particle (1), of mas (m_1) exerted by particle (2).

$\vec{F}_{21}$: is the force on particle (2), of mass (m_2) exerted by particle (1)

$\vec{r}_{12}$: is the directed line from particle (1) to particle (2)

the law of action and reaction requires that

$$\vec{F}_{12} = -\vec{F}_{21}.$$

The constant of proportionality (G) is known as the universal constant of gravitation.

$$G = (6.6 * 10^{-11}) \ Nm^2/kg^2$$

"Potential Energy in a Gravitational Field" الطاقة الكامنة في مجال الجاذبية

Let us consider the work (W) required to move a test particle of mass (m) along path in gravitational field of another particle of mass M.

The force $\vec{F}$ on the test particle is given by

$$\vec{F} = -G\frac{Mm}{r^2}\hat{n}$$

التغلب على هذه القوة   قوة خارجية

then, overcome, this force, an external force ($-\vec{F}$) must be applied. The work dW done in moving the test particle through a distance (dr) is thus given by:-

$$dW = -\vec{F}\cdot d\vec{r} = G\frac{Mm}{r^2}\hat{n}\cdot d\vec{r}$$

not:- $(\vec{n}\cdot d\vec{r}) = dr$

$$\therefore W = GMm\int_{r_1}^{r_2}\frac{dr}{r^2} = -GMm\left[r^{-1}\right]_{r_1}^{r_2}$$

$$W = -GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \quad \text{—— (a)}$$

We can define the potential energy of a particle of mass at a given point in the gravitational field of another particle as the work done in moving the test particle from some (arbitrary) reference position to the point in question.

It is convenient to take the reference position at infinity.

putting $r_1 = \infty$ and $r_2 = r$ in equation (a) we have :-

$$V_{(r)} = G\,Mm \int_{\infty}^{r} \frac{dr}{r^2} = -G\frac{Mm}{r}$$

$$\therefore \quad V_{(r)} = \frac{-k}{r} \qquad (\text{where } GMm = k)$$

$$\Phi\,(\text{gravitational potential}) = \frac{\text{gravitational potential energy}}{\text{mass}}$$

$$\Phi = \frac{V}{m}$$

thus the gravitational potential in field of a particle of mass M is given by

$$\boxed{\Phi = \frac{-GM}{r}}$$