بعد مرور وقت كافي حيث $t >>> \frac{m}{c}$ يمكن اهمال المقدار الاسي ليعلن السرعة تقترب من $\left(\frac{-mg}{c}\right)$ ويسمى حينئذ ذلك بسرعة المنتهى.

سرعة المنتهى :- هي تلك السرعة التي تكون فيها قوة المقاومة مساوية تماماً ومعاكسة لوزن الجسم بحيث محصلة القوى الكلية على الجسم تساوي صفر.

terminal velocity:- it is that velocity at which the force of resistance is just equal and opposite to the weight of the body, so that the total force is zero.

Terminal Velocity $\quad v_t = \frac{mg}{c}$

characteristic time $\quad \tau = \frac{m}{c}$
الزمن المميز

$$\therefore \quad v = -v_t \,(v_t + v_0)\, e^{-\frac{t}{\tau}}$$

$$v = \frac{dx}{dt} \Rightarrow dx = v\,dt$$

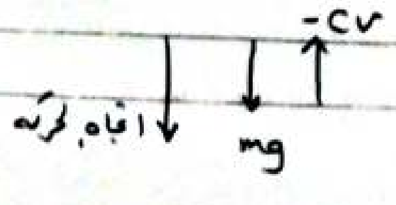
$$\therefore \int_{x_0}^{x} dx = \int_0^t -v_t + (v_t + v_0)\, e^{-\frac{t}{\tau}}\, dt$$

$$x - x_0 = -v_t t + (v_t + v_0)\int_0^t e^{-\frac{t}{\tau}}\, dt$$

$$x = x_0 - v_t t - \tau\,(v_t + v_0)\left[e^{-\frac{t}{\tau}} - 1\right]$$

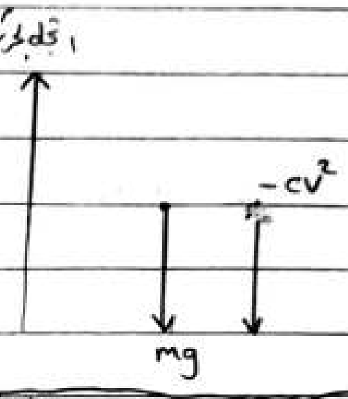
في حالة الهبوط المعادلة التفاضلية

$$mg - cv = m\frac{dv}{dt}$$



(2) if the motion is fast the resistance is proportional to the second power of (v)

ج) في حالة الصعود



$$-mg-cv^2=m\frac{dv}{dt}$$

$$\int_{t_0}^{t} dt=\int_{v_0}^{v}\frac{m\,dv}{-mg-cv^2}$$

$$t-t_0=-\frac{m}{c}\int_{v_0}^{v}\frac{dv}{\frac{mg}{c}+v^2}$$

Not $\int\frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}$

$$t-t_0=\frac{-m}{c}\cdot\frac{1}{\sqrt{\frac{mg}{c}}}\tan^{-1}\frac{v}{\sqrt{\frac{mg}{c}}}$$

$$t-t_0=-\sqrt{\frac{m^2}{c^2}}\sqrt{\frac{c}{mg}}\tan^{-1}\frac{v}{\sqrt{\frac{mg}{c}}}$$

$$t-t_0=-\sqrt{\frac{m}{cg}}\tan^{-1}\frac{v}{\sqrt{\frac{mg}{c}}}$$

in this case the terminal velocity $(v_t)=\sqrt{\frac{mg}{c}}$

and the characteristic time $(\tau)=\sqrt{\frac{m}{cg}}$

$$\therefore t_0-t=\tau\tan^{-1}\frac{v}{v_t}\Rightarrow\tan^{-1}\frac{v}{v_t}=\frac{t_0-t}{\tau}$$

نأخذ tan للطرفين

$$v=v_t\tan\left(\frac{t_0-t}{\tau}\right)$$

في حالة الهبوط $-mg+cv^2=m\frac{dv}{dt}$ المعادلة التفاضلية

ملاحظة/

$$\int\frac{dx}{a^2-x^2}=\frac{1}{a}\tanh^{-1}\frac{x}{a}$$

ex-1m). For a falling body, If we choose the (x) direction to be positive upward Find

1. the equation of total Energy
2. turning point
3. Find (v) from energy equation
4. find (t)

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(1) $E = \frac{1}{2} m v_0^2 + V_0(x) = \frac{1}{2} m v^2 + V(x)$

$$\underbrace{V_0(x)}_{mg\overset{0}{x} = zero}$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + mgx$$

(2) بنقطة الرجوع

E طاقة $= V(x)$

من معادلة الطاقة الكلية

$$\frac{1}{2} m v_0^2 = 0 + mg X_{max}$$

نقطة الرجوع $\left[X_{max} = \frac{v_0^2}{2g} \right]$

(3) من معادلة الطاقة الكلية

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + mgx$$

نضرب المعادلة بـ $\left(\frac{2}{m}\right)$

$$v_0^2 = v^2 + 2gx$$

$$\boxed{v^2 = v_0^2 - 2gx}$$

ويعتبر هنا اشتقاق لقيمة v^2 للمعادلة الخطية

3 $v^2 = v_0^2 - 2gx$

$$v = (v_0^2 - 2gx)^{1/2}$$

$$\frac{dx}{dt} = (v_0^2 - 2gx)^{1/2}$$

$$\int_0^t dt = \int_0^x \frac{dx}{(v_0^2 - 2gx)^{1/2}}$$

$$t = \frac{1}{2g}\left[(v_0^2 - 2gx)^{1/2}\right]_0^x$$

$$t = \frac{1}{2g}\left[(v_0^2 - 2gx)^{1/2} - (v_0^2)^{1/2}\right]$$

$$t = \frac{1}{2g}\left[(v_0^2 - 2gx)^{1/2} - v_0\right]$$

ex-2) suppose a block is projected with initial velocity (v_0) on a smooth horizontal plane, but that there is air resistance proportional to (v), $F_{(v)} = -cv$ where c is a constant of proportionality. Find

1. velocity as a function of (t)
2. position as a function of (t)
3. the enough time to move distance (x)

① $F_{(v)} = mv \dfrac{dv}{dx} = m\dfrac{dv}{dt}$

$$\int_0^t dt = \int_{v_0}^{v} \frac{m\,dv}{F_{(v)}} = \int_{v_0}^{v} \frac{m\,dv}{-cv}$$

$$t = \frac{-m}{c} \cdot \int_{v_0}^{v} \frac{dv}{v}$$

$$t = \frac{-m}{c} \ln v \Big]_{v_0}^{v} = \frac{-m}{c}\left(\ln v - \ln v_0\right) = \frac{-m}{c} \ln \frac{v}{v_0}$$

$$\ln \frac{v}{v_0} = -\frac{ct}{m}$$

$$v_{(t)} = v_0\, e^{-\frac{ct}{m}}$$

نستنتج أن السرعة تتناقص أسياً مع الزمن

② $v = \dfrac{dx}{dt} \Rightarrow dx = v_{(t)}\,dt$

$$\int dx = \int v_0\, e^{\frac{-ct}{m}}\, dt$$

نضرب ونقسم $\left(-\frac{c}{m}\right)$

$$x = \frac{-m}{c} \int v_0\, e^{-\frac{ct}{m}} \left(\frac{-c}{m}\right) dt$$

$$x = \frac{-mv_0}{c}\, e^{\frac{-ct}{m}} + \text{const}$$

من شروط الابتدائية

$t=0\ ,\ v=v_0\ ,\ x=0$

$$\text{Constant} = \frac{mv_0}{c}$$

$$x = \frac{-mv_0}{c} e^{-ct/m} + \frac{mv_0}{c}$$

$$x = \frac{mv_0}{c}\left(1 - e^{-ct/m}\right) = x(t)$$

$(t=\infty)$ اقصى مسافة تقطعها الجسم

$$x = \frac{mv_0}{c}$$

(3) $$m\frac{dv}{dt} = -cv$$

$$m\frac{dv}{dx}\frac{dx}{dt} = -cv \qquad \frac{dx}{dx}$$

$$m\cancel{v}\frac{dv}{dx} = -c\cancel{v}$$

$$m\int_{v_0}^{v} dv = -c\int_0^x dx \Rightarrow \int_{v_0}^{v} dv = \frac{-c}{m}x$$

$$v - v_0 = \frac{-c}{m}x$$

$$v = \frac{-c}{m}x + v_0 = \frac{dx}{dt}$$

$$dt = \frac{dx}{v_0 - \frac{c}{m}x} \Rightarrow t = \int_0^x \frac{dx}{v_0 - \frac{c}{m}x}$$

نضرب ونقسم على $-\frac{c}{m}$

$$t = \frac{-m}{c}\ln\left(\frac{v_0 - \frac{c}{m}x}{v_0}\right)$$

eq-9) The force acting on a particle varies with the distance x according to the power law

$$F_{(x)} = -K x^n$$

① Find the potential energy function

② If $v = v_0$ at time $t = 0$ and $x = 0$, find v as a function of x

③ Determine the turning points of the motion

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① $F_{(x)} = -\dfrac{dV_{(x)}}{dx}$

$$\int dV_{(x)} = -\int F_{(x)}\, dx = \int K x^n\, dx$$

$$V_{(x)} = \frac{K x^{n+1}}{n+1} + \text{constant}$$

من الشروط الابتدائية يمكن معرفة ثابت التكامل

$t = 0, \; x = 0$ صفر

$\therefore \text{Constant} = V_{(x)}$, $V_{(x)} = mgx$

$\therefore V_{(x)} = \text{zero}$

$\therefore \text{constant} = \text{zero}$

الطاقة الكامنة $\therefore V_{(x)} = \dfrac{K x^{n+1}}{n+1}$

② $\frac{1}{2} m v_0^2 + V_{(x_0)} = \frac{1}{2} m v^2 + V_{(x)} = E$ قانون حفظ الطاقة

$$\frac{1}{2} m v^2 = \frac{1}{2} m v_0^2 - V_{(x)} \quad \Big] \times \frac{2}{m}$$

$$v^2 = v_0^2 - \frac{2}{m} \frac{K x^{n+1}}{n+1}$$

$$v_{(x)} = \left[v_0^2 - \frac{2K x^{n+1}}{m(n+1)} \right]^{1/2}$$

(3) عند نقطة الرجوع الطاقة الكامنة = الطاقة الكلية
والسرعة تساوي (صفر)

$$V_{(x)} = E$$

$$v_{(x)} = 0$$

$$0 = \sqrt{v_0^2 - \frac{2kx^{n+1}}{m(n+1)}}$$

$$v_0^2 = \frac{2k\,x^{n+1}}{m(n+1)}$$

$$x^{n+1} = \frac{m(n+1)v_0^2}{2k}$$

$$x = \left[\frac{m(n+1)v_0^2}{2k}\right]^{\frac{1}{n+1}}$$

eq-15) The force acting on a particle of mass (m) is given by $F = kvx$, k, is a constant.
The particle passes through the origin with speed v_0 at time $t=0$. Find (x) as a function of (t)

حسب قانون نيوتن الثاني

$$k\cancel{v}x = m\cancel{v}\frac{dv}{dx}$$

$$m\,dv = kx\,dx \Rightarrow \int_{v_0}^{v} dv = \frac{k}{m}\int_0^x x\,dx$$

$$v - v_0 = \frac{kx^2}{2m} \Rightarrow v = v_0 + \frac{kx^2}{2m} = \frac{dx}{dt}$$

$$dx = v\,dt \Rightarrow dt = \frac{dx}{v}$$

$$\int_0^t dt = \int_0^x \frac{dx}{v_0 + \frac{kx^2}{2m}}$$

$$t = \frac{2m}{k}\int_0^x \frac{dx}{\frac{2mv_0}{k} + x^2}$$

ملاحظة

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$t = \frac{2m}{k}\frac{1}{\sqrt{\frac{2mv_0}{k}}}\tan^{-1}\frac{x}{\sqrt{\frac{2mv_0}{k}}}$$

$$\tan^{-1}\frac{x}{\sqrt{\frac{2mv_0}{k}}} = \frac{kt\sqrt{\frac{2mv_0}{k}}}{2m}$$

بأخذ tan للطرفين ثم يتم استخراج قيمة (x) eq-6) A block slides on a horizontal surface which has been Lubricated (مزيت) with a heavy oil such that the block suffers a viscous resistance that varies as the square root of the speed:. $F_{(v)} = -cv^{1/2}$

If the initial speed of the block is v_0 at time $t = 0$, find the values of (v) and (x) as a functions of the time (t).

$$m\frac{dv}{dt} = -cv^{1/2}$$

نجري الفصل ونكامل بالنسبة (v)

$$\int_{v_0}^{v} v^{-\frac{1}{2}}\, dv = \frac{-c}{m}\int_{0}^{t} dt \Rightarrow v^{1/2}\Big]_{v_0}^{v} * 2 = \frac{-ct}{m}$$

$$\sqrt{v} - \sqrt{v_0} = \frac{-ct}{2m} \Rightarrow \sqrt{v} = \sqrt{v_0} - \frac{ct}{2m}$$

$$v = \left[\sqrt{v_0} - \frac{ct}{m}\right]^2$$

$$v = \frac{dx}{dt} \Rightarrow dx = v\,dt$$

$$\int_{x_0}^{x} dx = \left(v_0 - \frac{2ct}{2m}\sqrt{v_0} + \frac{c^2t^2}{4m^2}\right)dt$$

ثم نكامل