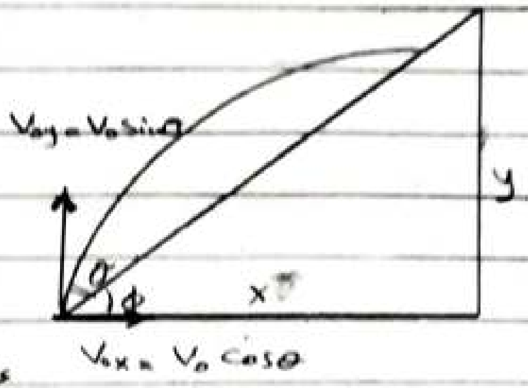
بندقية — أسفل

eq.(14) A gun is located at the bottom of a hill of constant slope (ϕ). show that that the range of the gun measured up the slope of the hill is :

زاوية ميل القذيفة (θ)

$$\tan\phi = \frac{y}{x}$$



مركبة x

$$y = x\tan\phi$$

$$\therefore y = (V_{ox} t)\tan\phi$$

$$\therefore y = t\, v_o \cos\theta \frac{\sin\phi}{\cos\phi} \text{ ---- } (1)$$

على فرض اهمال مقاومة الهواء

$$y = V_{oy} t - \frac{1}{2} g t^2$$

$$y = (v_o \sin\theta) t - \frac{1}{2} g t^2 \text{ ---- } (2)$$

الطرفين متساويين

$$t v_o \cos\theta \frac{\sin\phi}{\cos\phi} = t v_o \sin\theta - \frac{1}{2} g t^2$$

$$\frac{1}{2} g t^{\cancel{2}} = \cancel{t} v_o \sin\theta - \cancel{t} v_o \cos\theta \frac{\sin\phi}{\cos\phi}$$

$$t = \frac{2v_o}{g}\left(\sin\theta - \cos\theta \frac{\sin\phi}{\cos\phi}\right) \quad \frac{\cos\phi}{\cos\phi}$$ نضرب بالمقدار

$$t = \frac{2v_o}{g\cos\phi}(\sin\theta\cos\phi - \cos\theta\sin\phi)$$

$$t = \frac{2V_o}{g\cos\phi}\sin(\theta - \phi) \quad , \cos\phi = \frac{X}{R} \rightarrow R = \frac{X}{\cos\phi}$$

$$\therefore R = \frac{(v_o\cos\theta)t}{\cos\phi}$$

سنعوض قيمة (t)

$$\therefore R = \frac{2v_o^2 \cos\theta \sin(\theta - \phi)}{g\cos^2\phi}$$

eq. 11) Show that the maximum value of the slope in the above problem is:

$$\frac{v_0^2}{g(1+\sin\phi)}$$

ملاحظة

$$R = \frac{2v_0^2 \cos\theta \sin(\theta-\phi)}{g\cos^2\phi}$$

$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

نفرض

$A = \theta \ , \ B = \theta - \phi$

$$\cos\theta \sin(\theta-\phi) = \frac{1}{2}\left[\sin(2\theta-\phi) - \sin\phi\right]$$

$$R = \frac{v_0^2\left[\sin(2\theta-\phi) - \sin\phi\right]}{g\cos^2\phi}$$

لكي يكون R_{max}

$\sin(2\theta-\phi) = 1$

$$R_{max} = \frac{v_0^2\left[1-\sin\phi\right]}{g(1-\sin^2\phi)}$$

$$R_{max} = \frac{v_0^2\left[\cancel{1-\sin\phi}\right]}{g(1+\sin\phi)\cancel{(1-\sin\phi)}}$$

$$R_{max} = \frac{v_0^2}{g(1+\sin\phi)}$$

* The Harmonic Oscillator in two and three Dimensions

(رجاع)

the linear restoring force which is always directed toward the origin.

$$\therefore \vec{F} = -k\vec{r}$$

$$m\frac{d^2\vec{r}}{dt^2} = -k\vec{r}$$

$$m(\hat{i}\ddot{x} + \hat{j}\ddot{y} + \hat{k}\ddot{z}) = -k(\hat{i}x + \hat{j}y + \hat{k}z)$$

$$m\ddot{x} = -kx$$

$$m\ddot{y} = -ky$$

$$m\ddot{z} = -kz$$

في حالة الحركة في مستوى

$$m\ddot{x} = -kx \Rightarrow m\ddot{x} + kx = 0 \quad \text{——} \quad (1)$$

$$m\ddot{y} = -ky \Rightarrow m\ddot{y} + ky = 0 \quad \text{——} \quad (2)$$

the solution of eq (1)

$$x = A_1 \sin(\omega_0 t + \phi_1) \quad \text{——} \quad (3)$$

the solution of eq (2)

$$y = A_2 \sin(\omega_0 t + \phi_2) \quad \text{——} \quad (4)$$

- $\omega_{01} = \omega_{02} = \omega_0$ — افترضنا حالة الجهاز موحد الخواص

$$k_1 = k_2 = k_3$$

$\phi = \phi_2 - \phi_1$ — اختلاف

$\phi_1 = 0$ على فرض أن $\Rightarrow \phi = \phi_2$

$x = A_1 \sin\omega_0 t$ — نعوض في معادلة (3)

$$\frac{x}{A_1} = \sin\omega_0 t$$

$$\omega_0 t = \sin^{-1}\frac{x}{A_1}$$

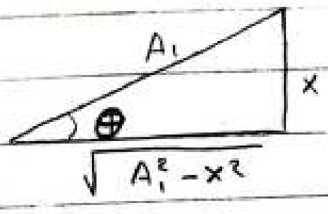
$$t = \frac{1}{\omega_0}\sin^{-1}\frac{x}{A_1}$$

نعوض في معادلة (4)

$$y = A_2 \sin\left[\cancel{\omega_0}\left(\frac{1}{\cancel{\omega_0}}\sin^{-1}\frac{x}{A_1}\right) + \phi\right]$$

$$y = A_2\left[\sin\left(\sin^{-1}\frac{x}{A_1}\right)\cos\phi + \cos\left(\sin^{-1}\frac{x}{A_1}\right)\sin\phi\right]$$

$$y = A_2\left[\frac{x}{A_1}\cos\phi + \frac{\sqrt{A_1^2 - x^2}}{A_1}\sin\phi\right]$$



معادلة المسار

$$y = \frac{A_2}{A_1}\left(x\cos\phi + \sqrt{A_1^2 - x^2}\,\sin\phi\right)$$

شكل المسار يعتمد على زاوية الطور (ϕ)

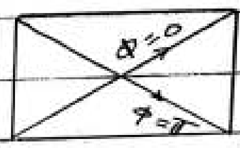
لنأخذ مجموعة من الحالات

(1) اذا كانت $\phi = 0, \pi$

يمثل معادلة خط مستقيم

معادلة المسار $y = \frac{\mp A_2}{A_1}x$

اذا كانت $\phi = 0, \pi$



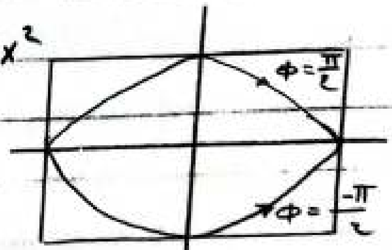
(2) اذا كانت $\phi = \frac{\pi}{2}, \frac{3\pi}{2}$

$$y = \frac{\mp A_2}{A_1}\sqrt{A_1^2 - x^2}$$

$$y^2 = \frac{A_2^2}{A_1^2}\left(A_1^2 - x^2\right) \rightarrow A_1^2 y^2 = A_2^2 A_1^2 - A_2^2 x^2$$

بالقسمة على $A_2^2 A_1^2$

معادلة المسار $\left(\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1\right)$ معادلة قطع ناقص



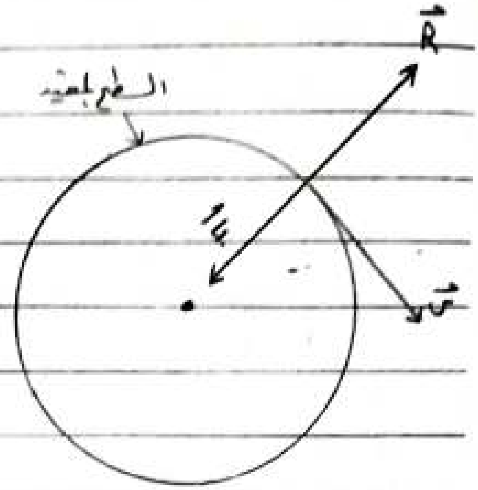
(3) اذا كان $A = A_1 = A_2$

المسار دائري

$$x^2 + y^2 = A^2$$

* The Energ Equation For Smooth Constraint

المدات



The total Force acting on a particle moving under Constraint can be written by:- $(F + R)$

F :- external force

R :- the Force of Constraint

the equation of motion Can be written as:-

$$m \frac{d\vec{v}}{dt} = \vec{F} + \vec{R}$$

$$m \frac{d\vec{v}}{dt} \cdot \vec{v} = \vec{F} \cdot \vec{v} + \underset{}{\overset{\text{صفر}}{\vec{R} \cdot \vec{v}}} \qquad (\cos 90 = \text{zero})$$

$$m \frac{d\vec{v}}{dt} \cdot \vec{v} = \vec{F} \cdot \vec{v}$$

$$m \int \vec{v} \cdot d\vec{v} = F \cdot \int (\vec{v}\, dt)$$

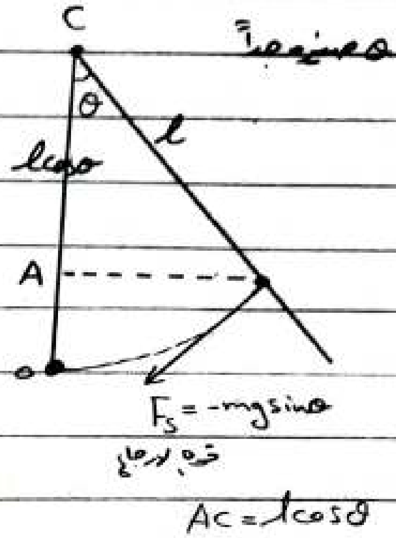
$$m \frac{v^2}{2} = \underbrace{\vec{F} \cdot \vec{r}}_{\text{الشغل}} + \text{constant}$$

$$\frac{1}{2} m v^2 - W = \text{constant}$$

$$W = - V(x) \qquad \text{الشغل = - الطاقة الكامنة}$$

$$\therefore \frac{1}{2} m v^2 + V(x) = \text{const.} = E$$

*(The Simple Pendulum) البندول البسيط



$$-mg\sin\theta = m\ddot{s}$$

$$m\ddot{s} + mg\sin\theta = 0$$

نق × زاوية = طول القوس

$$s = \theta l \Rightarrow \dot{s} = \dot{\theta} l \Rightarrow \ddot{s} = \ddot{\theta} l$$

$$m\ddot{\theta} l + mg\sin\theta = 0$$

$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0 \quad \Rightarrow \sin\theta \simeq \theta$$

$$\left[\ddot{\theta} + \frac{g}{l}\theta = 0\right]$$ المعادلة التفاضلية للبندول

$$\ddot{x} + \omega_0^2 x = 0$$ وهذه المعادلة تشبه

$$\omega_0^2 = \frac{g}{l} \Rightarrow \omega_0 = \sqrt{\frac{g}{l}}$$

زمن الذبذبة $T = \frac{2\pi}{\omega_0} \Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$

Finding The restoring Force F_s

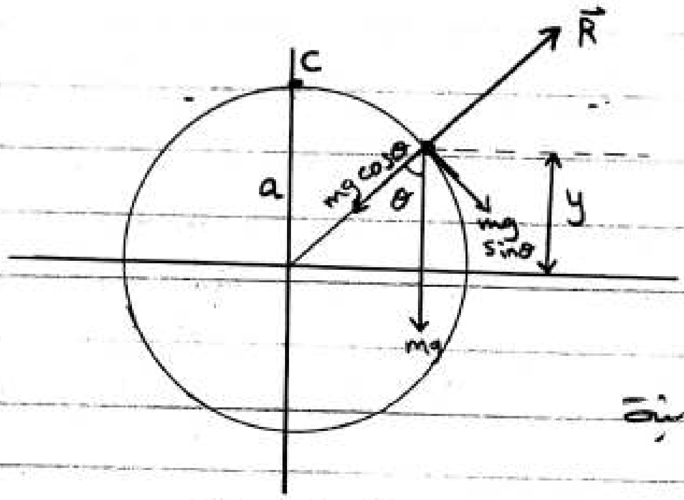
$$F_s = \frac{-dV}{dx}, \quad V = mgh \Rightarrow V = mg(l - l\cos\theta)$$

$$V = mgl(1-\cos\theta) = mgl\left(1 - \cos\frac{s}{l}\right)$$

$$\frac{dV}{ds} = mgl(\sin\theta)\frac{1}{l} = mg\sin\theta$$

$$F_s = \frac{-dV}{ds} = -mg\sin\theta$$

eq) A particle is placed on the up of smooth (c) sphere of radius (a), As the particle slides down Find the Vertical distance from the point will it leave to the x-axis?



بما ان السطح املس ∴ الطاقة الكلية ثابتة

نستخدم قانون حفظ الطاقة

the total energy is constant

$$E_C = E_D$$

$$\left(\tfrac{1}{2}mv^2 + V\right)_C = \left(\tfrac{1}{2}mv^2 + V\right)_D$$

$$mga = \tfrac{1}{2}mv^2 + mgy$$

نضرب ب $\left(\frac{2}{m}\right)$

$$v^2 = 2ga - 2gy = 2g(a-y)$$

$$\therefore v = \sqrt{2g(a-y)}$$

بما ان الحركة منحنية فان مجموع القوتين تساوي القوة المركزية

$$-mg\cos\theta + R = \frac{-mv^2}{a}$$

$$R = mg\cos\theta - \frac{mv^2}{a}$$

$$\cos\theta = \frac{y}{a}$$, ونعوض عن قيمة v

$$R = \frac{mgy}{a} - \frac{2mg(a-y)}{a}$$

$$R = \frac{mg}{a}(y - 2a + 2y) \Rightarrow R = \frac{mg}{a}(3y - 2a)$$

. عندما يترك الجسم سطح الكرة (القوة العمودية تساوي صفر)

$$0 = \frac{mg}{a}(3y - 2a)$$

$$3y - 2a = 0$$

$$3y = 2a$$

$$y = \frac{2}{3}a$$

ارتفاع النقطة التي يترك عندها الجسم سطح الكرة .

Eq.3) A particle of mass (m) moves in a Force field whose potential function is given by
$V = ax + by^2 + cz^2$.

If the particle passes through the origin with Speed (v_0), what will its speed be and when it passes through the point (1,1,1)?

في البداية نثبت ان القوة محافظة والتي بالمحافظة تكون دالة للموقع فقط.

$$E = T_{(r)} + V_{(r)}$$

$$F = -\nabla V = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) = -(\hat{i}a + \hat{j}\,2by + \hat{k}\,2cz)$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a & 2by & 2cz \end{vmatrix} = \hat{i}(0-0) + \hat{j}(0-0) + \hat{k}(0-0) = \text{zero}$$

∴ القوة محافظة.

نطبق قانون حفظ الطاقة

$$\left(\tfrac{1}{2}mv^2 + \underset{\text{صفر}}{V}\right)_{(0,0,0)} = \left(\tfrac{1}{2}mv^2 + V\right)_{(1,1,1)}$$

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv^2 + (a*1 + b*1 + c*1)$$

$$\left.\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv^2 + (a+b+c)\right] * \tfrac{2}{m}$$

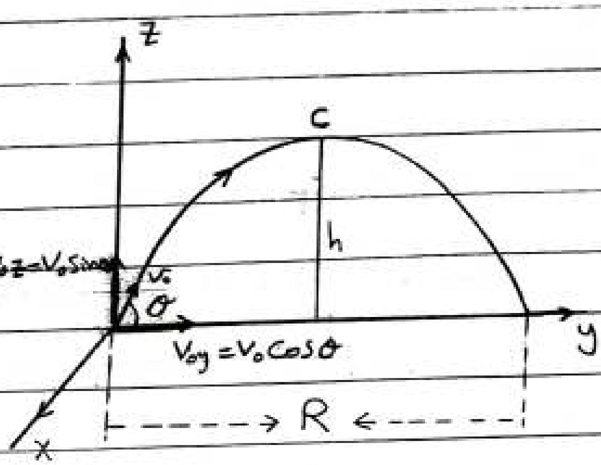
$$v_0^2 = v^2 + \tfrac{2}{m}(a+b+c)$$

$$\boxed{v_0 = \left[v^2 + \tfrac{2}{m}(a+b+c)\right]^{1/2}}$$

سرعة الجسم عند النقطة المطلوبة

eq.7) A projectile is fired from the origin with initial speed (v_0) at an angle of elevation (θ) with the horizontal. If the air resistance is neglected, show that if the ground is level, the projectile hits the ground a distance

$\dfrac{v_0^2 \sin 2\theta}{g}$, This is called the horizontal range (R).



$\vec{V} = V_0 - gt$

$\vec{V}_z = V_{0z} - gt$

عند أقصى ارتفاع السرعة = صفر

$\therefore V_{0z} = gt$

$V_0 \sin\theta = gt$

$t = \dfrac{V_0 \sin\theta}{g}$ ⇒ الزمن اللازم للوصول إلى أقصى ارتفاع (زمن الصعود)

زمن الهبوط + زمن الصعود = T (زمن الطيران)

$$\therefore \boxed{T = \frac{2v_0 \sin\theta}{g}}$$

بما أن $R = V_{0y} T$

$$R = V_0 \cos\theta \, \frac{2v_0 \sin\theta}{g}$$

$$R = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

$\sin 2\theta = 2 \sin\theta \cos\theta$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

eq. 8) In problem (7) Find (h) high vertical distance For a projectile?

$$v^2 = v_0^2 - 2gz$$

$$v_z^2 = v_{0z}^2 - 2gz$$

$z = h$ عند أقصى ارتفاع

$v_z = 0$

$$2gh = v_{0z}^2$$

$$2gh = (v_0 \sin\theta)^2$$

$$h = \frac{v_0^2 \sin^2\theta}{2g}$$ أقصى ارتفاع