Chapter-3-
Dynamic of a Particle.
General Motion

① grad $(\phi + \psi)$

$$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$$

② div $(\vec{A} + \vec{B})$

$$\nabla \cdot (\vec{A} + \vec{B}) = \nabla \cdot \vec{A} + \nabla \cdot \vec{B}$$

③ curl $(\vec{A} + \vec{B})$

$$\nabla \times (\vec{A} + \vec{B}) = \nabla \times \vec{A} + \nabla \times \vec{B}$$

④ $\nabla \cdot (\phi \vec{A}) = (\nabla\phi) \cdot \vec{A} + \phi(\nabla \cdot \vec{A})$

⑤ $\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A}(\nabla \times \vec{B})$

⑥ div grad ϕ

$$\nabla \cdot (\nabla\phi) = \nabla^2\phi = \frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dy^2} + \frac{d^2\phi}{dz^2}$$

⑦ curl grad ϕ

$$\nabla \times (\nabla\phi) = \text{zero}$$

⑧ div curl $\vec{A}$

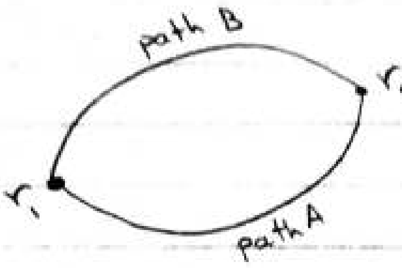
$$\nabla \cdot (\nabla \times \vec{A}) = \text{zero}$$

⑨ $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

" Conservation Forces " القوى المحافظة

The condition of conservation force

① if the work is not depended for the path the Force is conservation and its field called conservation field.



* W_A الشغل اللازم لنقل جسم من r_1 الى r_0 على المسار A

$$W_A = \int_{r_1}^{r_0} \vec{F} \cdot d\vec{r}$$

* W_B = = = على المسار B

$$W_B = \int_{r_1}^{r_0} \vec{F} \cdot d\vec{r}$$

$$W_{path\,A} = W_{path\,B}$$

$$\int_{r_1}^{r_0} \vec{F} \cdot d\vec{r} = \int_{r_1}^{r_0} \vec{F} \cdot dr$$

$$\int_{r_1}^{r_0} \vec{F} \cdot d\vec{r} - \int_{r_1}^{r_0} \vec{F} \cdot dr = 0$$

$$\int_{r_1}^{r_0} \vec{F} \cdot dr + \int_{r_0}^{r_1} \vec{F} \cdot d\vec{r} = 0$$

$$\boxed{\oint \vec{F} \cdot d\vec{r} = 0}$$

شرط ثاني من شروط القوة المحافظة

اذا كان التكامل المغلق للقوة حول مسار مغلق يساوي صفر تكون القوة محافظة.

(3) if $\vec{F} = -\nabla V$

اذا كانت القوة تساوي سالب انحدار الجهد تكون القوة محافظة

(4) if $\nabla \times \vec{F} = 0$ then the force is Conservation

(5) if $E = K.E + P.E = \text{constant}$
then the force is conservation

ex)

$$F = \hat{i}(y^2 z^3 - 6xz^2) + \hat{j}(2xyz^3) + \hat{k}(3xy^2z^2 - 6x^2 z)$$

show that if the force is conservation or no?

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\nabla \times \vec{F} = \hat{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \hat{j}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$$= \hat{i}(6xyz^2 - 6xyz^2) + \hat{j}\left[(3y^2z^2 - 12xz) - (3y^2z^2 - 12xz)\right] + \hat{k}(2yz^3 - 2yz^3)$$

$= \text{Zero}$

∴ القوة محافظة

if

ex) $V = x^2 + yx + xz^1$ Find the force field?

$$\vec{F} = -\nabla V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$$

$$\vec{F} = -\hat{i}(2x + y + z) - \hat{j}x - \hat{k}x$$

ex) show that if the Force is conservation or no?

$$\vec{F} = \hat{i}xy + \hat{j}xz + \hat{k}yz$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & xz & yz \end{vmatrix}$$

$$= \hat{i}(z - x) - \hat{j}(0) + \hat{k}(z - x).$$

∴ النتيجة لا تساوي صفر ، فالقوة غير محافظة

ex) when the force field is conservation?

$$\vec{F} = \hat{i}(ax + by^2) + \hat{j}cxy$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ax+by^2 & cxy & 0 \end{vmatrix} = \hat{k}(c - 2b)y$$

if the force is conservation then $\nabla \times \vec{F} = 0$

$$\therefore c - 2b = 0 \Rightarrow c = 2b$$

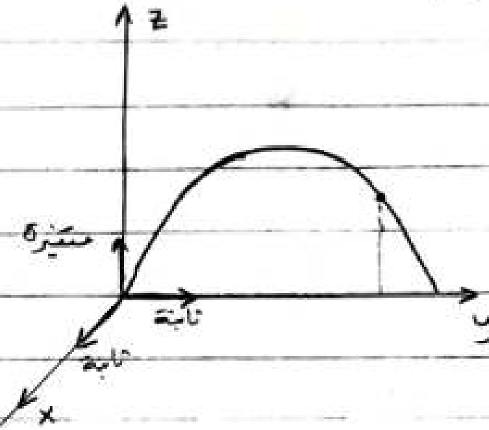
* Motion of a projectile in a Uniform Gravitational Field

① No Air Resistance

For a projectile there is only one force acting namely the force of gravity and equal the weight.

اذا كانت القوة الوحيدة المؤثرة على الجسم
هي قوة الجاذبية والتي تساوي وزن الجسم فيقال
ان الجسم يسقط سقوطاً حراً.

the differential equation of motion:.



$$\cancel{m}\frac{d^2\vec{r}}{dt^2} = -\cancel{m}g\,\hat{k}$$

$$\frac{d^2\vec{r}}{dt^2} = -g\,\hat{k}$$

مادام المعادلة الأخيرة لا تحتوي على الكتلة (m) فان حركة الجسم الساقط سقوط حر لا تعتمد
على الكتلة.

the force function is clearly of the separable type and is also conservative.

From Force Conservation Law?

$$\frac{1}{2}\cancel{m}v_0^2 = \frac{1}{2}\cancel{m}v^2 + \cancel{m}gz \quad \Big] \times \frac{2}{m}$$

$$v^2 = v_0^2 - 2gz \quad \text{— (1)}$$ سرعة القذيفة في أي لحظة
وهي نفس معادلة الحركة الخطية

من المعادلة التفاضلية

$$\frac{d^2\vec{r}}{dt^2} = -g\hat{k} \Rightarrow \frac{d\vec{v}}{dt} = -g\hat{k}$$

$$d\vec{v} = -g\vec{k}\int dt \Rightarrow \vec{v} = -gt\hat{k} + \text{const}$$

if $t=0$ $\therefore v = v_0$, $\therefore \text{const} = v_0$

$$\boxed{\begin{aligned} \vec{v} &= v_0 - gt\hat{k} \\ v &= v_0 - gt \end{aligned}} \Rightarrow (2)$$

تشبه معادلة الحركة الخطية

$$\vec{v} = \frac{d\vec{r}}{dt} = v_0 - gt\hat{k}$$

$$\int d\vec{r} = \int \vec{v}_0 dt - g\int t\,dt\,\hat{k}$$

$$r = v_0 t - \frac{1}{2}gt^2\hat{k} \quad \text{------} \quad (3)$$

وهي تشبه معادلة الحركة الخطية

$$\therefore \quad x = \dot{x}_0 t$$

$$y = \dot{y}_0 t$$

$$z = \dot{z}_0 t - \frac{1}{2}gt^2$$

Linear Air Resistances.

The differential equation is:

$$m\frac{d^2\vec{r}}{dt^2} = -mg\hat{k} - c\vec{v}$$

$$m(\hat{i}\ddot{x} + \hat{j}\ddot{y} + \hat{k}\ddot{z}) = -mg\hat{k} - c(\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z})$$

$\therefore \quad m\ddot{x} = -c\dot{x} \Rightarrow \ddot{x} = \frac{-c}{m}\dot{x}$

$(\ddot{x} = -\gamma\dot{x})$ ---- (1) $\qquad \frac{c}{m} = \gamma$

$\therefore \quad m\ddot{y} = -c\dot{y} \Rightarrow \ddot{y} = \frac{-c}{m}\dot{y}$

$(\ddot{y} = -\gamma\dot{y})$ ---- (2)

$\therefore \quad m\ddot{z} = -mg - c\dot{z}$

$(\ddot{z} = -\gamma\dot{z} - g)$ ---- (3)

to find the velocity in (x) direction

$\ddot{x} = -\gamma\dot{x}$ ---- (1)

$$\frac{d\dot{x}}{dt} = -\gamma\dot{x}$$

$$\int_{\dot{x}_0}^{\dot{x}} \frac{d\dot{x}}{\dot{x}} = -\gamma\int_0^t dt$$

$$\ln\frac{\dot{x}}{\dot{x}_0} = -\gamma t$$

$(\dot{x} = \dot{x}_0 e^{-\gamma t})$ ---- (1')

From eq. (2)

$$\ddot{y} = -\gamma \dot{y} \Rightarrow \frac{d\dot{y}}{dt} = -\gamma \dot{y} \Rightarrow \int_{\dot{y}_0}^{\dot{y}} \frac{d\dot{y}}{\dot{y}} = -\gamma \int_0^t dt$$

$$\ln \frac{\dot{y}}{\dot{y}_0} = -\gamma t$$

$$\left(\dot{y} = \dot{y}_0 e^{-\gamma t} \right) \quad \text{----} (2')$$

From eq. (3)

$$\frac{d\dot{z}}{dt} = -\gamma \dot{z} - g \Rightarrow \int_{\dot{z}_0}^{\dot{z}} \frac{d\dot{z}}{-\gamma \dot{z} - g} = \int_0^t dt$$

$$\frac{1}{-\gamma} \int_{\dot{z}_0}^{\dot{z}} \frac{-\gamma \, d\dot{z}}{-\gamma \dot{z} - g} = \int_0^t dt$$

$$\ln \frac{(-\gamma \dot{z} - g)}{(-\gamma \dot{z}_0 - g)} = -\gamma t$$

$$\ln \frac{\gamma \dot{z} + g}{\gamma \dot{z}_0 + g} = -\gamma t$$

$$\gamma \dot{z} + g = (\gamma \dot{z}_0 + g) \, e^{-\gamma t}$$

$$\left(\dot{z} = \frac{-g}{\gamma} + \left(\dot{z}_0 + \frac{g}{\gamma} \right) e^{-\gamma t} \right) \quad \text{----} (3')$$

* to Find the position Components From eq. (1')

$$\dot{x} = \dot{x}_0 \, e^{-\gamma t} \Rightarrow \frac{dx}{dt} = \dot{x}_0 \, e^{-\gamma t}$$

$$\int_{x_0}^{x} dx = \int_0^t \dot{x}_0 e^{-\gamma t} \, dt \quad \Big] \frac{-\gamma}{-\gamma}$$

$$x\Big]_{x_0}^{x} = \frac{\dot{x}_0}{-\gamma} \int_0^t e^{-\gamma t} (-\gamma) \, dt \Rightarrow x - x_0 = \frac{-\dot{x}_0}{\gamma} \left[e^{-\gamma t} \right]_0^t$$

اذا كان الموضع الابتدائي في نقطة الاصل

$$x = \frac{-\dot{x}_0}{\gamma} \left[e^{-\gamma t} - 1 \right]$$

$$x = \frac{\dot{x}_0}{\gamma} \left[1 - e^{-\gamma t} \right] \quad \text{-----} (1'')$$

وبنفس الطريقة نجد (y)

$$y = \frac{\dot{y}_0}{\gamma} \left(1 - e^{-\gamma t} \right) \quad \text{-----} (2'')$$

ولإيجاد (z) نأخذ المعادلة (3')

$$\dot{z} = \frac{-g}{\gamma} + \left(\frac{g}{\gamma} + \dot{z}_0 \right) e^{-\gamma t} = \frac{dz}{dt}$$

$$\int_{z_0}^{z} dz = \frac{-g}{\gamma} \int_0^t dt + \left(\frac{g}{\gamma} + \dot{z}_0 \right) \int_0^t e^{-\gamma t} \, dt$$ نضرب ونقسم $(-\gamma)$

$$z - z_0 = \frac{-g}{\gamma} t + \left(\frac{1}{-\gamma} \right) \left(\frac{g}{\gamma} + \dot{z}_0 \right) \int_0^t e^{-\gamma t} (-\gamma) \, dt$$

عند نقطة الاصل $z_0 = \text{zero}$

$$z = \frac{-g}{\gamma} t - \frac{1}{\gamma} \left(\frac{g}{\gamma} + \dot{z}_0 \right) e^{-\gamma t} \Big]_0^t$$

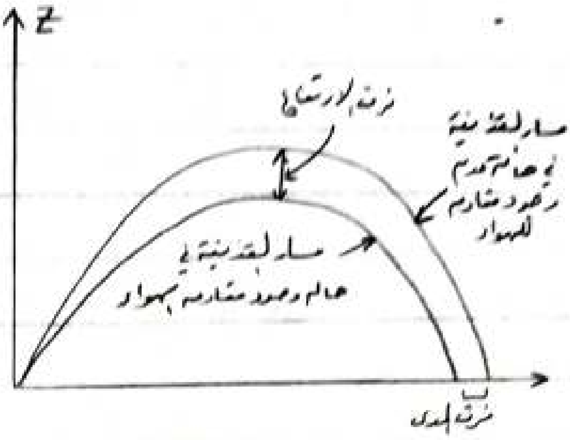
$$z = \frac{-g}{\gamma} t - \left(\frac{g}{\gamma^2} - \frac{\dot{z}_0}{\gamma} \right) \left[e^{-\gamma t} - 1 \right]$$

$$z = \frac{-g}{\gamma} t + \left(\frac{g}{\gamma^2} + \frac{\dot{z}_0}{\gamma} \right) \left(1 - e^{-\gamma t} \right) \quad \text{-----} (3'')$$

وبعد دمج المركبات x ، y ، z نحصل على

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$

$$\vec{r} = \left(\frac{\vec{v}_0}{\gamma} + \frac{\hat{k}g}{\gamma^2}\right)(1 - e^{-\gamma t}) - \hat{k}\frac{gt}{\gamma}$$



في حالة وجود مقاومة هواء صغيرة $(\gamma t \ll 1)$
نستفيد من المعادلة

$$e^{u} = 1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \frac{u^4}{4!} + \cdots$$

فيكون الناتج

$$(1 - e^{-\gamma t}) = \gamma t - \frac{\gamma^2 t^2}{2!} + \frac{\gamma^3 t^3}{3!}$$

وبعد الاختصار يكون الناتج بالتعويض في معادلة (٣)

$$\vec{r} = \vec{v}_0 t - \frac{1}{2}gt^2\hat{k} - \Delta\vec{r}$$

حيث (Δr) تمثل جميع الحدود المتبقية وهو عامل التغيير في مسار القذيفة

$$\Delta\vec{r} = \gamma\left[v_0\left(\frac{t^2}{2!} - \frac{\gamma t^3}{3!} - \cdots\right) + kg\left(\frac{t^3}{3!} - \frac{\gamma t^4}{4!} - \cdots\right)\right]$$

ملاحظة / قارن المعادلة الاخيرة التي تمثل $(\vec{r})$ بوجود مقاومة الهواء بالمعادلة السابقة (بدون وجود مقاومة الهواء)

$\vec{r} = \vec{v}_0 t - \frac{1}{2}gt^2\hat{k}$ عند عدم وجود مقاومة هواء