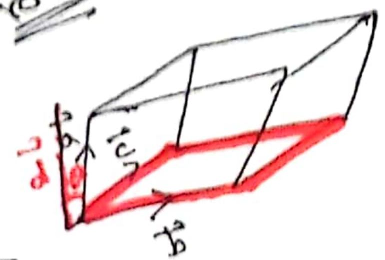
(1)

dot, cross product على التطبيقات الهندسية

Parallelepiped Volume

prove



$\cos\theta = \frac{d}{|\vec{a}|}$

$\boxed{d = |\vec{a}|\cos\theta}$

$V = \text{Area of base} * \text{height}$

$= |\vec{b}\times\vec{c}| * d$

$= |\vec{b}\times\vec{c}|\,|\vec{a}|\cos\theta$

$= |\vec{a}|\,|\vec{b}\times\vec{c}|\cos\theta$

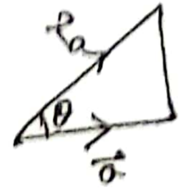
$$\boxed{V = |a\cdot(b\times c)|}$$

ex $\vec{A} = (0,1,-3)$, $\vec{b} = (1,2,3)$, $c(-1,0,1)$

$$\vec{A} = \begin{vmatrix} 0 & 1 & -3 \\ 1 & 2 & 3 \\ -1 & 0 & 1 \end{vmatrix} = -10$$

$\boxed{V = |-10| = 10}$

Triangle Area

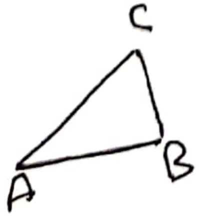


$A = \frac{1}{2}|a||b|\sin\theta$

$$\boxed{A = \frac{1}{2}|\vec{a}\times\vec{b}|}$$

ex : $A(0,1,2)$, $B(-1,0,2)$, $C(1,2,0)$

S/o



$AB = \langle -1,-1,0\rangle$

$AC = \langle 1,1,-2\rangle$

$$AB\times AC = \begin{vmatrix} i & j & k \\ -1 & -1 & 0 \\ 1 & 1 & -2 \end{vmatrix}$$

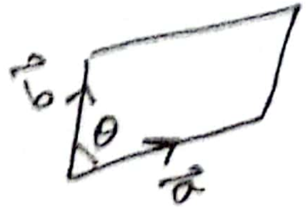
$= \langle 2,-2,4\rangle$

$|AB\times AC| = \sqrt{4+4+16} = \sqrt{24} = 2\sqrt{6}$

$A = \frac{1}{2}|AB\times AC|$

$= \frac{1}{2}\cdot 2\sqrt{6} = \sqrt{6}$

Parallelogram Area



$A = |\vec{a}||\vec{b}|\sin\theta$

$$\boxed{A = |\vec{a}\times\vec{b}|}$$

ex $\vec{a} = \langle 1,-1,0\rangle$

$\vec{b} = \langle 0,1,2\rangle$

Area of parallelogram

S/o. $A = |\vec{a}\times\vec{b}|$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{vmatrix}$$

$= \langle -2,-2,1\rangle$

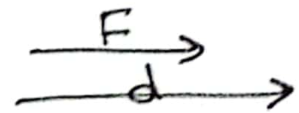
$|a\times b| = \sqrt{4+4+1}$

$= \sqrt{9}$

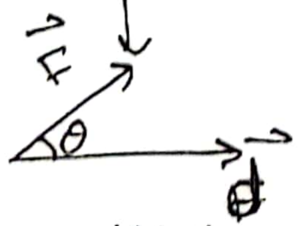
$\boxed{A = 3}$

التطبيقات الفيزيائية

(الشغل) work



$$w = |F||d|$$



$$w = |d||F|\cos\theta$$

$$w = F \cdot d$$

عزم الدوران (Torque)

هو يسبب في دوران الجسم حول محور

$$\vec{\tau} = \vec{r} \times \vec{F} = Nm$$

Ex1 A crate is hauled 8m up the ramp by a constant force of 20N applied at an angle of 30° to the ramp. Calculate the work done by the force.

$$W = \vec{F} \cdot \vec{d}$$
$$= |F||d| \cos\theta$$
$$= (20)(8)\cos 30$$
$$= 160 \frac{\sqrt{3}}{\cancel{2}}$$
$$= 80\sqrt{3} \text{ J}$$

$|d| = |8|$
$|F| = 20 \text{ N}$
$\theta = 30$

$\langle 1, 1, 0 \rangle$

Ex2 A particle located at the position vector $\vec{r} = \hat{i} + \hat{j}$ (m) and a force $\vec{F} = 2i + 3\hat{j}$ (N), determine Torque about the origin

$\langle 2, 3, 0 \rangle$

Sol:

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 2 & 3 & 0 \end{vmatrix}$$
$$= +\hat{i}(0) - \hat{j}(0) + \hat{k}(3-2)$$
$$\tau = 1\hat{k}$$
$$\boxed{\tau = \langle 0, 0, 1 \rangle} \text{ N·m}$$