University of Baghdad

College of Engineering

Department of
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Experiment of Heat Transfer by Forced Convection

The Reynolds Analogy

for 4th Class
Department of Chemical Engineering

by

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1. Introduction

It is true that the forced convection method of heat transfer, which involves moving a fluid, such as air or water to carry heat from one place to another is a popular and commonly utilized technique. This technique is used in a variety of everyday and technical applications to regulate temperatures, cool or heat systems, and speed up heat exchange procedures. In order to compel the fluid to travel over a surface or through a system and improve heat transmission, forced convection uses mechanical devices like fans, blowers, or pumps. Analogies are frequently used in forced convection, where they can be a useful tool for making abstract or difficult-to-understand ideas more understandable to a wider audience. This is especially true when attempting to explain these concepts to students, co-workers, or people who may not have a strong technical background. It is simpler for people to understand the fundamental concepts behind forced convection heat transfer when analogies are used to create comparisons between a less recognizable subject and one that is more accessible. One of these analogies is Reynolds's analogy.

The Reynolds analogy is a technical procedure for heat transfer and fluid dynamics that connects the momentum transfer in fluid flow to the heat transfer in convection. For the analysis of heat transfer issues involving fluid flow, it was initially presented by Osborne Reynolds a British scientist and engineer, in 1883 (Figure.1) and is a crucial tool in chemical engineering and other engineering fields. Osborne Reynolds, made a substantial addition to our understanding of fluid movement and heat transmission. His research laid the groundwork for our current knowledge of fluid dynamics and heat transport in physics and engineering. The Reynolds number idea, one of his most well-known contributions, is crucial for understanding fluid motion-related problems with fluid flow and heat transfer.

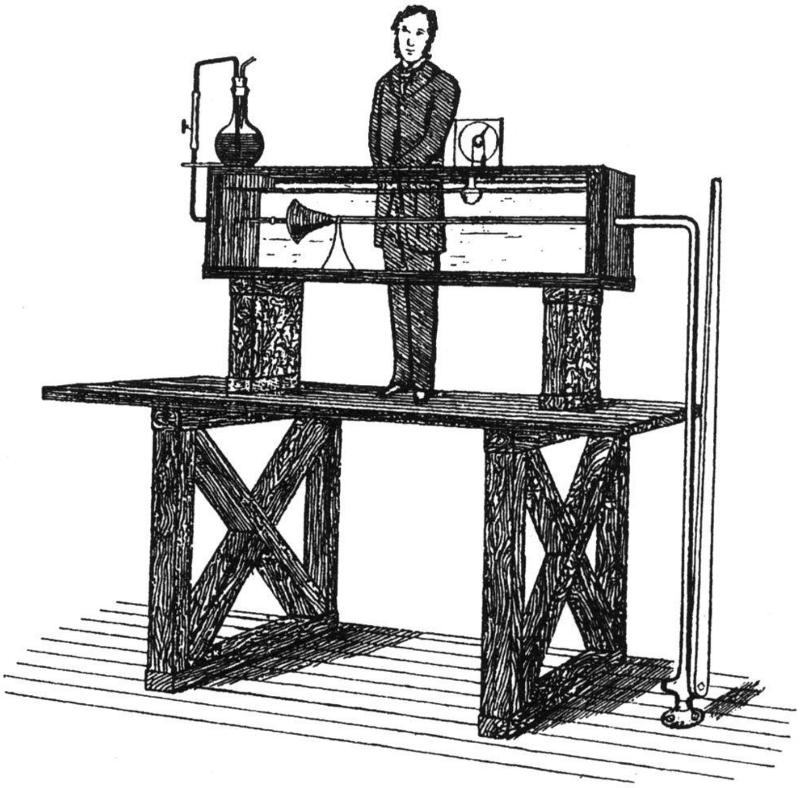

Figure.1: The Experiment of Osborne Reynolds on Fluid Dynamics in Pipes

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2. The Principles of Reynolds Analogy

The Reynolds analogy is based on the observation that, in many turbulent flows, whether in a pipe or a flat plate (Figure.2), the momentum and heat transfer are closely connected because they share a common source in turbulent eddies and have profiles that are comparable. The Reynolds analogy is applicable for a variety of turbulent flows due in large part to this underlying similarity in the behavior of velocity and temperature profiles.

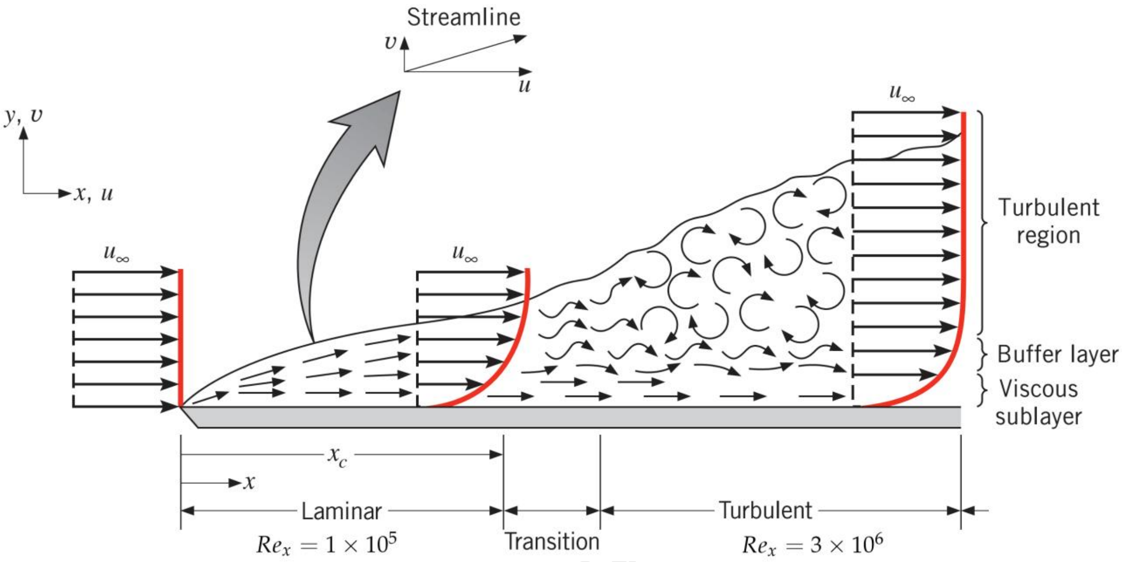

Figure.2: Laminar and turbulent boundary layer generated over a flat plate

The fluid is mixed in turbulent flows by eddies of different sizes and intensities, which improves momentum and heat transmission. Both momentum and thermal energy are transferred through these eddies' variations in velocity and temperature. Because momentum transfer and heat transmission may be compared, the Reynolds analogy offers a helpful framework for calculating heat transfer in turbulent flows. This idea is especially helpful in engineering applications when calculations for heat transmission are required in circumstances where precise heat transfer measurements may be difficult. Engineers may more effectively build and analyze heat exchangers, cooling systems, and other related systems by employing the Reynolds analogy to estimate heat transfer coefficients by utilizing information and relationships from fluid mechanics and heat transfer.

The Reynolds analogy's applicability is frequently debated in the context of the pressure gradient. The Reynolds analogy has traditionally been seen to be more accurate when the flow is in equilibrium, which often indicates a zero pressure gradient. When the velocity and temperature profiles are comparable, the comparison becomes more true. However, the Reynolds analogy may still be used when there is a nonzero pressure gradient, such as in turbulent flows. There may be pressure changes throughout the flow route in many real engineering applications, such as flows via pipes, heat exchangers, or boundary layers, and these flows are frequently turbulent. While the analogy may not be as exact in non-equilibrium settings, it can still yield reasonable heat transfer coefficient estimations.

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3. Calculation of Reynolds Analogy

The Reynolds analogy is based on the idea that the transfer of momentum and heat in a fluid can be related using dimensionless numbers. In particular, it relates the Nusselt number (Nu) and Stanton number (St), which describe heat transfer, to the Reynolds number (Re), which characterizes fluid flow, and Prandtl number (Pr) which represents the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity in a fluid. These dimensionless numbers can be linked by mathematical equations with skin friction coefficient or friction coefficient (C_f or f) as follows:

$$R_e = \frac{\rho \times U \times L}{\mu} = \frac{\rho \times U \times d}{\mu} = \frac{U \times L \times v}{\mu} \qquad (1)$$

Where:
ρ: fluid density.
U: velocity of the fluid based on the actual cross section area of the pipe.
L: characteristic length of the object or conduit through which the fluid is flowing.
d: inside diameter of the pipe.
v: kinematic viscosity of the fluid.
μ: dynamic viscosity of the fluid.

$$C_f \frac{Re}{2} = Nu \qquad (pr = 1) \qquad (2)$$

This equation is known as the **Reynolds analogy**, which is a substantial analogy because it allows us to calculate the heat transfer coefficient for fluids with Pr<1 by using only the friction coefficient, which is easier to measure. Reynolds analogy can also be written in terms of a Stanton number:

$$\frac{C_f}{2} = St \qquad (3)$$

$$St = \frac{h}{\rho \times U \times C_p} \qquad (4)$$

Where:
C_p: specific heat capacity of the fluid at constant pressure.
h: heat transfer coefficient.

$$and, St = \frac{Nu}{Re \times Pr} \qquad (5)$$

$$and, Nu = \frac{h\times L}{k} \tag{6}$$

Where:
L: characteristic length, which represents the length of the surface in contact with the fluid.
k: thermal conductivity of the fluid.

$$C_f = \frac{\tau_w}{0.5\times\rho\times U^2} \tag{7}$$

Where:
τ_w: wall shear stress.
U: free stream velocity of the fluid far away from the wall.

$$Pr = \frac{v}{\alpha} = \frac{\mu\times C_p}{k} \tag{8}$$

Where:
v: momentum diffusivity (Kinematic viscosity).
α = thermal diffusivity.

A modified Reynolds analogy has been developed to account for the fact that the Prandtl number is typically not equal to one:

$$\frac{C_f}{2} = St\times Pr^{2/3} \qquad 0.6 < Pr < 60 \tag{9}$$

4. Forced Convection Apparatus

A forced convection apparatus shown in Figure.3 and manufactured by Tecquipment limited Nottingham, is used to perform the Reynolds analogy experiment, and through the results obtained, it is possible to derive the Nusselt number, determine the Stanton number, and determine the validity of the Reynolds analogy for the fluid used, which is air. The forced convection heat transfer apparatus consists of the following parts:

Part 1: Fan or Blower: The fan operates at a constant speed. The function of this fan is to supply the unit with fluid, represented by air.

Part 2: Stream or Flow Pipe: Its function is to transport the air pumped by the fan to the rest of the unit.

Part 3: Orifice Meter: It is a plate that is attached to the flow pipe and has a diameter less than it. The plate aims to cause a change in the flow pressure of the fluid (pressure drop), through which the flow rate can be measured before and after this plate.

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Part 4: Fluid Tank: This tank is connected on one side to the orifice meter using a plastic tube, and on the other side it is connected to the manometer. When there is a change in pressure, the fluid in the tank will move toward the manometer.

Part 5: Thermometer: It is used to measure the inlet temperature of the fluid transported by the stream pipe before it enters the test part. The temperature is recorded before the heating is turned on.

Parts 6 and 7: Manometer: This gauge is used to measure the fan pressure drop resulting from the orifice plate (Part 3) before the fluid enters the test part. The manometer in Part 6 is inclined at an angle of 30° and is called the Inclined Manometer. Part 7 is the vertical manometer.

Parts 8 and 9: U-Manometer: It is used to measure the pressure drop in the test part resulting from the fluid flow.

Part 10: Test pipe: Made of copper with a smaller diameter than the flow pipe, it is surrounded by a heater and insulated from the outside with a layer of fiberglass. Thirteen thermocouples are mounted on the test pipe to measure the change in temperature in both the inside pipe, outer wall, and insulator. These thermocouples are arranged as follows: Seven thermocouples (1–7) measure the change in temperature inside the pipe; three thermocouples (8, 10, and 12) measure the temperature change on the outer wall of the pipe; and finally, three thermocouples (9, 11, and 13) measure the temperature of the insulator.

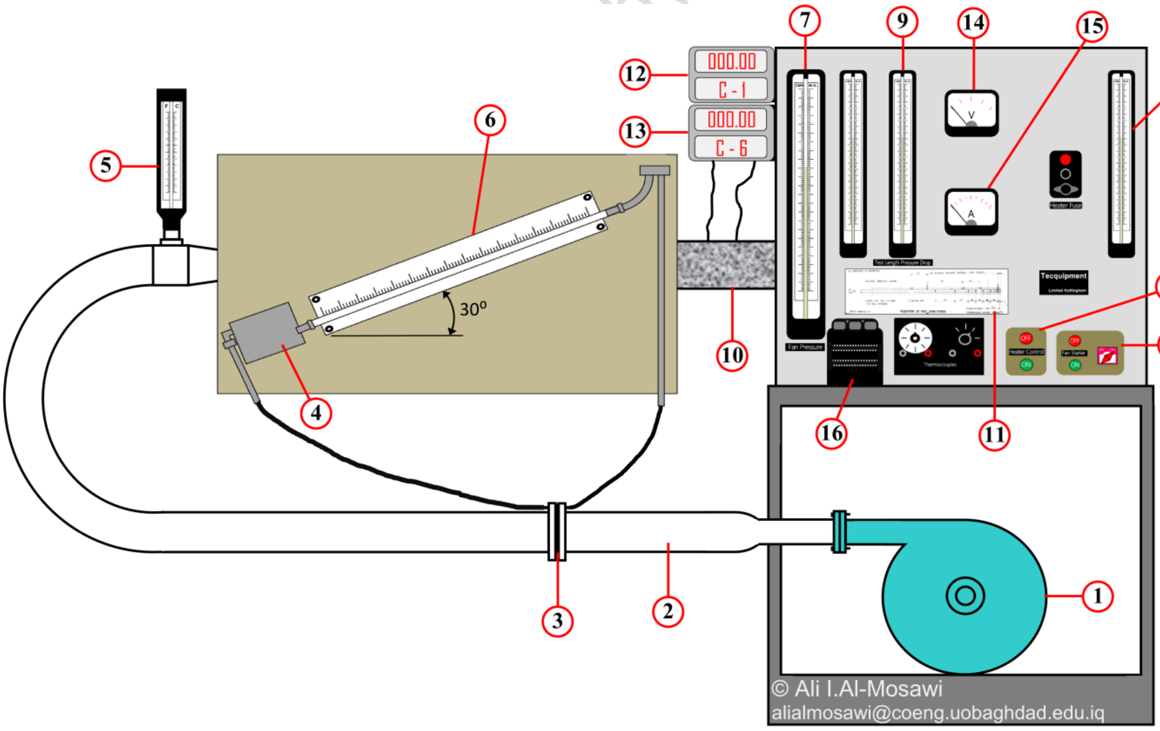

Figure.3: Forced Convection Apparatus

Part 11: Test pipe details plate: Included in this plate are all the details that the student needs for calculations. The diameter of the test pipe is 31.75 mm, its wall thickness is (1.625 mm), and the length of the part on which the thermocouples are mounted is 1720 mm. The test length is 1542 mm, which is located between the pressure tappings placed at the beginning and ending of the test pipe. The end of the pipe is connected to the U- manometer. The plate also lists the distribution of thermocouples in the test pipe and the distances separating them. Thermocouples (8) and (9) are located outside the test length, while the remaining thermocouples are located inside it.

Part 12: Upper temperature gauge: It is responsible for reading the temperatures of the thermocouples located inside the test pipe (1–7).

Part 13: Bottom temperature gauge: This gauge reads the temperatures of the thermocouples located on the wall of the test pipe and on the insulator (8–13).

Part 14: Voltmeter.

Part 15: Ammeter.

Part 16: Power Supply: It is figuratively called a Variac, and its function is to control voltages and currents.

Part 17: Heater start and stop switch.

Part 18: Fan start and stop switch.

5. Measurement Procedure

The measurement procedure consists of the following steps:

1. Before starting the unit, a reading of the U-Manometer (Part 8) is taken, and this reading represents the value (ΔH_o).
2. The fan is initially turned on (Part 1) to supply the system with only flowing air without heating. The flow of air through the narrow plate (Part 3) attached to the stream pipe will cause a pressure drop that is read via the inclined manometer (Part 6). The reading is taken after reaching a steady state and represents (ΔH_1).
3. After the fluid flows into the pipe, the reading of the U-Manometer (Part 8) is recorded again, which represents (ΔH_2), from which the value of (ΔH_o) is subtracted: (ΔH_2-ΔH_o).
4. The temperature of the fluid is measured before entering the test part (T_{inlet}) using a thermometer (Part 5).
5. After obtaining the appropriate air flow in the unit, the heating is turned on (Part 17). Turning on the heating leads to a change in voltage and current, which are controlled by the power supply (Part 16). The heating does not start at the same time as the fan in order to avoid excessive temperature rise in the unit.
6. After reaching a steady state, the voltage value is read (Part 14), and the current value is also taken (Part 15).

7. The temperatures in the test pipe (Part 10) are recorded using thermocouples (Parts 12 and 13). The reading is repeated every two minutes until we reach a steady-state condition, at which point the temperature readings begin to converge in value or the same values are repeated. Then the last reading is taken, which will be included in the calculations.
8. When the measurement is complete and all readings are recorded, the heater is initially closed and waits a little until the unit temperature drops; after that, the unit is completely closed.

6. Results Analysis and Calculations

After the practical results were obtained from the experiment, we will now analyse these results and perform mathematical calculations through which the value of the Reynolds number, Nusselt number, Stanton number, Prandtl number, and the skin friction coefficient are calculated.

The **first step** in data analysis is to collect information about the parameters that went into the experiment, including the test pipe dimensions, thermocouples distribution distances, and orifice meter details, which can be obtained from Part 11. Also, we need the properties of fluid (air), which are listed in Table 1.

Table.1: Properties of Air at 25ºC and 1 atm. Pressure

Density ρ, kg/m^3	Specific Heat, *Cp* J/kg·K	Thermal Conductivity *k*, W/m·K	Thermal Diffusivity α, m^2/s	Dynamic Viscosity μ, kg/m.s	Kinematic Viscosity ν, m^2/s	Prandtl Number Pr
1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296

The **second step** is to organize the results we need for the mathematical calculations in the form of two tables. The first includes the inlet temperature (T_{inelt}), the ΔH obtained from the orifice meter, the ΔH obtained from the U-manometer, voltage, and current. The second table includes the temperatures obtained from the thirteen thermocouples.

Table.2: Template for the Data Recording Table

Parameter	Value
$\Delta H_{orificemeter}$ (ΔH_1)	
$\Delta H_{U\text{-}manometer}$ (ΔH_2-ΔH_o)	
T_{inelt}	
Voltage, V	
Current, I	

Table.3: Template for the Temperatures Recording Table

Inside pipe (Air) Temperature, °C	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Value							
Pipe Wall Temperature, °C	T_8	T_{10}	T_{12}				
Value							
Insulator Temperature, °C	T_9	T_{11}	T_{13}				
Value							

The **third step** is to draw a diagram representing the relationship between temperatures and test length (Figure.4). Test length details are listed in Part 11. The average temperature is taken.

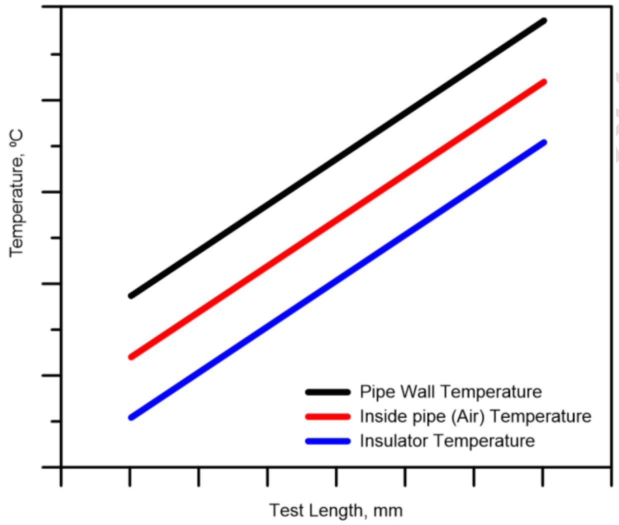

Figure.4: Template for the Temperatures Recording Diagram

Finally, the **fourth step** is to perform mathematical calculations to determine the dimensionless numbers.

From the equation below we will calculate the value of the friction coefficient (C_f):

$$\Delta p = \frac{1}{\rho \times g} \times \left(\frac{m}{A_{TL}}\right)^2 \times \left(\frac{4C_f \times L}{2d_{TL}} + \frac{T_1 - T_2}{T_{av}}\right) \tag{10}$$

Where:
Δp: pressure drop.
ρ: fluid density.
g: acceleration of gravity (9.81 m/s²).
m: mass flow rate.
A_{TL}: cross-sectional area of test pipe.
L: test length.
d_{TL}: Diameter of test pipe.
T_1 and T_2: Temperatures of thermocouples which reached to steady state condition.
T_{av}: the sum of the temperatures of the thermocouples inside the test pipe (1-7) divided by their number.

Now we calculate the height of the fluid above the center of the orifice (OR) and test length (TL):

$$\Delta h_{TL} = (\Delta h_2 - \Delta h_{\circ}) \times \left(\frac{1000 - \rho}{\rho}\right) \qquad (11)$$

$$\Delta h_{OR} = \Delta h_1 \times sin30 \qquad (12)$$

Then we convert the height (Δh) into pressure (p):

$$p = \rho \times g \times \Delta h \qquad (13)$$

The next step is to calculate the mass flow rate value

$$m = \rho \times Cd \times A_{OR} \times \sqrt{2g \times h} \qquad (14)$$

Where:
Cd: discharge coefficient, its value ranges from 0.6 to 0.8.
A_{OR}: cross-sectional area of the orifice.

$$\mathrm{A_{OR}} = \frac{\pi}{4}\,\mathrm{d}^2 \qquad (15)$$

Where:
d: diameter of orifice.

$$T_{av} = \frac{T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7}{7} \qquad (16)$$

After we calculate the value of the friction coefficient, we will now calculate the value of the heat transfer coefficient, which is involved in calculating dimensionless numbers:

$$Q = m \times C_p \times \Delta T \qquad (17)$$

Q: heat transfer rate.
ΔT: The difference in air temperature represents the difference between the temperature of the first thermocouple and the seventh thermocouple.

We substitute the value of the heat transfer rate (Q) calculated from Equation 17 into the equation below:

$$Q = h \times A \times \Delta T \qquad (18)$$

Where:
h: heat transfer coefficient.
A: surface area of test pipe.
ΔT: temperature difference between wall and fluid.

$$\Delta T = T_{wall} - T_{air} \qquad (19)$$

$$T_{wall} = \frac{T_8 + T_{10} + T_{12}}{3} \qquad (20)$$

$$T_{air} = \frac{T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7}{7} \qquad (21)$$

$$A = \pi \times d \times L \qquad (22)$$

Where:
d: test pipe diameter.
L: the test length.

After all the data is available, we can now calculate Reynolds number, Nusselt number, Stanton number, and Prandtl number.

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