Probability Theory

Def: A set is be countable if its elements can be arranged in the form of sequence or if it is finite, it is said to be countably infinite otherwise the set is uncountable.

ex: ① the natural number $N=\{1,2,3,4,---\}$. This set is countably infinite.

② The interval of real number $I=[a, b]$. This set is noncountably infinite.

③ The negative odd integer number $Y=\{-\infty, ---, -5, -3, -1\}$. The set Y is countably infinite

④ The unit interval of real number $I=\{x: 0 \leq x \leq 1\}$. The set I is uncountably infinite.

Def: Any set which elements are sets also is called the class of set or family of set.

ex: let $A=\{\{2\}, \{3,5\}, \{6,2,9\}\}$

The members of set A are also sets

A is family of set or class of set

Def: The class of all subset of any set is called the Power set and denoted by $\beta(\cdot)$,
If A is finite and has n elements, then $\beta(A)$ will have 2^n.

ex: Let $A = \{1, 2, 3\}$ then $\beta(A)$ is $2^3 = 8$ set

$\beta(A) = \{\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, A, \phi\}$

Theorem: Given class of set $\{A_i\}$, $i = 1, 2, 3, \dots, n$, the sequance sets $A_1, A_2, A_3, \dots, A_n$ are disjoint sets, such that

$$\bigcup_{i=1}^{n} A_i = \sum_{i=1}^{n} A_i$$

σ-Algebra and their Generators:

Let Ω be an arbitrary set and $\beta(\Omega)$ its power set, that is, the system of all subsets of Ω.

1- Eevery family $(A_i)_{i \in I}$ of elements of $\beta(\Omega)$.

2- $\bigcup_{i=I} A_i$ and $\bigcap_{i=I} A_i$ are also in $\beta(\Omega)$

3- For every set A, $\beta(\Omega)$ contains A^c

4- The system of sets $\phi \in \beta(\Omega)$

Def: A system Φ of subset of a set Ω is called σ-algebra (in Ω) if it has the following properties:

1- $\Omega \in \Phi$
2- if $A \in \Phi \Rightarrow A^c \in \Phi$
3- for every sequence A_n of sets of Φ. $\bigcup_{n=1}^{\infty} A_n$ lies in Φ

Remark
1- The power set $\beta(\Omega)$ is allways σ-algebra
2- For every set Ω, the system of all sets $A \subset \Omega$ for which either A or A^c is countable (means countable finite or countable infinite) is σ-algebra
3- If all sets A_n are countable, then $\bigcup_{n=1}^{\infty} A_n$ is also countable
4- If Φ is σ-algebra in a set Ω and $\acute{\Omega}$ is subset of Ω, then $\acute{\Omega} \cap \Phi = \{\acute{\Omega} \cap A ; A \in \Phi\}$, $\acute{\Omega} = \phi$ is σ-algebra $\acute{\Phi}$ in $\acute{\Omega}$ the $\acute{\Phi}$ consists of all subsets of $\acute{\Omega}$ belonging to Φ.

Theorem: Eevery intersection (finitely many or infinitely many) σ-algebra in a set Ω is it self a σ-algebra in a set Ω

ex:
Let the natural number set, where the set A is even number which less than seven.

1- the space Ω is : $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, ----\}$
2- the set A is : $A = \{2, 4, 6\}$
3- the power set of A is ; The power set $= 2^n = 2^3 = 8$
$\beta(A) = \{\{2\}, \{4\}, \{6\}, \{2,4\}, \{2,6\}, \{4,6\}, \{2,4,6\}, \phi$

4- The sequence of sets A_n is:

$A_1 = \{2\}$, $A_2 = \{4\}$, $A_3 = \{6\}$, $A_4 = \{2,4\}$, $A_5 = \{2,6\}$, $A_6 = \{4,6\}$

$A_7 = \{2,4,6\}$, $A_8 = \{\phi\}$

5- The σ-algebra φ is:

$\varphi = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}$ or

$\varphi = \{\{2\}, \{4\}, \{6\}, \{2,4\}, \{2,6\}, \{4,6\}, \{2,4,6\}, \{\phi\}\}$

Def: (σ-field)

A nonempty class of subset of Ω which is closed under the countable unions and complement which contains ϕ is known as σ-field (or σ-algebra).

Remark:

1- σ-field $=$ σ-algebra

2- If φ is σ-field, it is closed under the countable intersections and contains the set Ω.

3- σ-field of great intreast in the study of probability is the Borel σ-field of subsets of the real line R.

Theorem: Every countable set of real numbers is Borel set

Theorem: If A is any set (countable or uncountable) and φ_i is σ-field of subset of Ω for each $i \in A$, then

$\bigcap_{i \in A} \varphi_i$ is also σ-field.

Remark: If the set $A \subset \Omega$ and not contains ϕ, then A is algebra but not σ-algebra

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Def: If φ is σ-field of set subset of Ω, a set function $\mu : \varphi \to R^+$ is said to be measure on (Ω, φ) if:

1. $\mu(\phi) = $ zero
2. Given any denumberable (finite or countable) collection $A_1, A_2, A_3, \ldots$ of disjoint set $\mu(\bigcup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} \mu(A_n)$.

Def: If φ is σ-algebra of subsets of Ω and if μ is measure on (Ω, φ), then (Ω, φ, μ) is called a measure space

Def
A measure space (Ω, φ, μ) is probability space if $\mu(\Omega) = 1$.

Remarks:

1- we call the pair (Ω, φ) a measurable space.

2- we called the triple (Ω, φ, μ) a measure space.

3- for every finite number of pairwise disjoint sets $A_1, A_2, A_3, \ldots \in R$ then

$$\mu\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \mu(A_i)$$

Permutation: التباديل

A permutation is an arrangement of objects (things) in a define order.

The rule of permutation is:

$$P_r^n = \frac{n!}{(n-r)!}$$

Remark: $P_n^n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$

Ex: ① Let $n=7, r=2$ then $P_2^7 = \frac{7!}{(7-2)!} = \frac{7\times6\times\cancel{5!}}{\cancel{5!}} = 42$

② let $n=7, r=7$ the $P_7^7 = \frac{7!}{(7-7)!} = \frac{7!}{0!} = 7! = 5040$

let $n=7, r=1$ the $P_1^7 = \frac{7!}{(7-1)!} = \frac{7\times\cancel{6!}}{\cancel{6!}} = 7$

⊛ If we arrange (n) objects in a raw, then the permutation became $n!$ which come from the following:

$$P_n^n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

⊛ If we arrange (K) objects from (n) objects, then the permutation is as follows: $P_K^n = \frac{n!}{(n-K)!}$

⊛ If we arrange (n) objects in a circlue, then the permutation become $(n-1)!$ which come from the following:

$$1 * P_{n-1}^{n-1} = \frac{(n-1)!}{(n-1-n+1)!} = \frac{(n-1)!}{0!} = (n-1)!$$

Ex: Consider taking three letters from English language. In how many ways be arranged these letters ① in a row, ② in a circle, ③ from all letters?

Sol: ① $n! = 3! = 3\times2\times1 = 6$

$A = \{abc, acb, bca, bac, cab, cba\}$

② $(n-1)! = (3-1)! = 2! = 2$

$B = \{abc, acb\}$

③ $P^n_r = P^{26}_3 = \frac{26!}{(26-3)!} = \frac{26!}{23!} = \frac{26\times25\times24\times\cancel{23!}}{\cancel{23!}} = 15600$

ex: Consider three energy states (A, B, C) ① In how many ways be arranged these energy stats. ② In how many ways be arrange two of these energy states?

Sol: ① $P^3_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 3! = 3\times2\times1 = 6$

$A_1 = \{ABC, ACB, BAC, BCA, CAB, CBA\}$

② $P^3_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = 3! = 3\times2\times1 = 6$

$B = \{AB, AC, BA, CA, BC, CB\}$

ex: In how many ways be arrange two digits number from five numbers which are 6, 7, 3, 2, 4?

$P^5_2 = \frac{5!}{(5-2)!} = \frac{5\times4\times\cancel{3!}}{\cancel{3!}} = 5\times4 = 20$

ex:
In how many ways be arrange three digits number from four number which are as 4, 2, 0, 1 ?

Sol:

$$P_1^3 \times P_2^3 = \frac{3!}{(3-1)!} \times \frac{3!}{(3-2)!} = 3 \times 6 = 18$$

A = {420, 421, 401, 410, 402, 412, 241, 240, 201, 214, 204, 210, 142, 124, 104, 140, 102, 120}

Combination :

A combination is a selection of objects (thing) without regard to order.

The rule of combination is:

$$C_r^n = \frac{n!}{r!(n-r)!} \quad , \quad C_n^n = \frac{n!}{n!(n-n)!} = \frac{\cancel{n!}}{\cancel{n!}\,0!} = 1$$

If r = 1, then $C_1^n = \frac{n!}{1!(n-1)!} = \frac{n\cancel{(n-1)!}}{1!\cancel{(n-1)!}} = n$

ex: In how many ways be selected two person to make Committe from three person?

Sol:

$$C_2^3 = \frac{3!}{2!(3-2)!} = \frac{3!}{2!\times 1!} = \frac{3\times \cancel{2!}}{\cancel{2!}} = 3$$

Committes = {AB, AC, BC}

ex:
A chemist has (10) samples of water taken from the wastwater of paper factory Three samples of water are randomly selected and test the ocidity.

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Sol:
$$C_3^{10} = \frac{10!}{3!(10-3)!} = \frac{10!}{3!\times 7!} = \frac{10\times 9\times 8\times \cancel{7!}}{3!\times \cancel{7!}} = \frac{720}{6}$$
$$= 120$$

ex: A biological has six defferent cages of white rats in the animal room, he want to select three cages to use it in experiment.

Sol:
$$C_3^{6} = \frac{6!}{3!(6-3)!} = \frac{6!}{3!\times 3!} = \frac{6\times 5\times 4\times \cancel{3!}}{3!\times \cancel{3!}} = \frac{120}{6} = 20 \text{ ways}$$

he can select (3) cages from six cages.

ex: Consider four energy states (A, B, C, D).
① If we want to select three energy states.
② If we want to select two energy states.

Sol ①
$$C_3^{4} = \frac{4!}{3!(4-3)!} = \frac{4\times 3!}{3!\times 1!} = \frac{4}{1} = 4$$

$A_1 = \{ABC, ABD, ACD, BCD\}$

②
$$C_2^{4} = \frac{4!}{2!(4-2)!} = \frac{4\times 3\times \cancel{2!}}{2!\times \cancel{2!}} = \frac{12}{2} = 6$$

$A_2 = \{AB, AC, AD, BC, BD, CD\}$

ex: How many committees of two chemists and one physicist can be formed from 4 chemists and 3 physicist?

Sol:
$$C_2^{4}\cdot C_1^{3} = \frac{4!}{2!(4-2)!}\cdot\frac{3!}{1!(3-1)!} = \frac{4\times 3\times \cancel{2!}}{2!\times \cancel{2!}}\cdot\frac{3\times \cancel{2!}}{1!\times \cancel{2!}} = 6\times 3 = 18$$

Thus 18 different committees can be formed.

Binomial Theorem

An algebraic expression containing two termes is called a binomial expression, The general type of a binomial is $a+b$, $x-2$, $3x+4$ etc.

Binomial Theorem:

The rule or formula for expension of $(a+b)^n$, where n is any positive integr power, is called binomial theorem. Such that:

$$(a+b)^n = C_0^n a^n b^0 + C_1^n a^{n-1} b^1 + C_2^n a^{n-2} b^2 + \cdots + C_r^n a^{n-r} b^r + \cdots + C_n^n a^{n-n} b^n$$

or briefly $(a+b)^n = \sum_{r=0}^{n} C_r^n a^{n-r} b^r$

for any positive integr power n

Remarks: The Coefficients of the successive terms are

$C_0^n, C_1^n, C_2^n, \ldots, C_r^n, \ldots, C_n^n$

are called Binomial Coefficients

Note ① Sum of binomial coefficients is 2^n

② we can represent the binomial ~~coef~~ theorem as the form

$$(a+b)^n = a^n + \frac{n}{1!} a^{n-1} b + \frac{n(n-1)}{2!} a^{n-2} b^2 + \frac{n(n-1)(n-2)}{3!} a^{n-3} b^3 + \cdots$$

$$\cdots + \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} a^{n-r} b^r + \cdots + b^n$$

The following points can be observed in the expansion of $(a+b)^n$

1. There are $(n+1)$ terms in the expansion
2. The 1^{st} term is a^n and $(n+1)$ the term or the last term is b^n
3. The exponent of "a" decreases from n to zero
4. The exponent of "b" increases from zero to n
5. The sum of the exponents of a and b in any term is equal to index n
6. The co-efficients of the term equidistant from the beginning and end of the expansion are equal as $C_r^n = C_{n-r}^n$

ex: Expand $(x+y)^4$ by binomial theorem

Sol:

$$(x+y)^4 = C_0^4 x^4 y^0 + C_1^4 x^3 y^1 + C_2^4 x^2 y^2 + C_3^4 x^1 y^3 + C_4^4 x^0 y^4$$

$$= x^4 + 4x^3y + \frac{4\times3}{2\times1} x^2 y^2 + \frac{4\times3\times2}{3\times2\times1} x y^3 + y^4$$

$$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

ex: Expand $\left(\frac{x^2}{2} - \frac{2}{x}\right)^4$ by binomial theorem

Sol:

$$\left(\frac{x^2}{2} - \frac{2}{x}\right)^4 = C_0^4 \left(\frac{x^2}{2}\right)^4 + C_1^4 \left(\frac{x^2}{2}\right)^3 \left(-\frac{2}{x}\right)^1 + C_2^4 \left(\frac{x^2}{2}\right)^2 \left(-\frac{2}{x}\right)^2 + C_3^4 \left(\frac{x^2}{2}\right)\left(-\frac{2}{x}\right)^3$$

$$+ C_4^4 \left(\frac{x^2}{2}\right)^0 \left(-\frac{2}{x}\right)^4$$

$$= \frac{x^8}{16} - x^5 + 6x^2 - \frac{16}{x} + \frac{16}{x^4}$$

ex) Expand $(1.04)^5$ by the binomial formula and find its value to two decimal places

Sol) $(1.04)^5 = (1+0.04)^5$

$$(1+0.04)^5 = (1)^5 + C_1^5 (1)^4 (0.04) + C_2^5 (1)^3 (0.04)^2 + C_3^5 (1)^2 (0.04)^3 + C_4^5 (1)^1 (0.04)^4 + C_5^5 (1)^0 (0.04)^5$$

$$= 1 + (0.04)(5) + (0.04)^2(10) + (0.04)^3(10) + (0.04)^4(5) + (0.04)^5$$

$$= 1 + 0.2 + 0.016 + 0.00064 + 0.000128 + 0.0000001024$$

$$= 1.22$$

General Term:

The term $C_r^n a^{n-r} b^r$ in the expansion of binomial theorem is called the General term or $(r+1)^{th}$ term. It is denoted by T_{r+1}, Hence

$$T_{r+1} = C_r^n a^{n-r} b^r$$

Note) The General term is used to find out the specified term or the required coefficient of the term in the binomial expansion

ex) Find the eight term in the expansion of $\left(2x^2 - \frac{1}{x^2}\right)^{12}$

Sol) The general term is $T_{r+1} = C_r^n a^{n-r} b^r$

$$\therefore T_8 = T_{7+1} = C_7^{12} a^{12-7} b^7$$

$a = 2x^2 \ , b = -\frac{1}{x^2} \ , n = 12 \ , r = 7$

therefore $T_{7+1} = C_7^{12} (2x^2)^{12-7} (-\frac{1}{x^2})^7$

$T_8 = \frac{12 \times 11 \times 10 \times 9 \times 8 \times \cancel{7!}}{\cancel{7!}(12-7)!} (2x^2)^5 (-\frac{1}{x^2})^7$

$= 792 (32x^{10})(\frac{-1}{x^{14}}) \quad = 25344 (-\frac{1}{x^4})$

$\therefore T_8 = -\frac{25344}{x^4}$

ex: find the coefficient of x^{5} in the expension of $(2x+3y)^8$

sol:

$x^5 \Rightarrow x^{8-3} , y^3$

$T_4 = T_{3+1} = C_3^8 (2x)^{8-3} (3y)^3$

$= C_3^8 (2x)^5 (3y)^3 = \frac{8 \times 7 \times 6 \times 5!}{3!(8-3)!} (2x)^5 (3y)^3$

$= \frac{8 \times 7 \times 6 \times \cancel{5!}}{3! \times \cancel{5!}} (2x)^5 (3y)^3$

$= 56 (2x)^5 (3y)^3 = 48384 x^5 y^3$

$\therefore$ the Coeffiecient of the term has x^5 is 48384

and the term contains x^5 is T_4

Theorem:

Let $n \in N$ (the set of natural numbers) and $r = 0, 1, 2, \ldots, n$

then
$$C^n_r = C^n_{n-r}$$

proof:

$$C^n_{n-r} = \frac{n!}{(n-r)!\,(n-(n-r))!} = \frac{n!}{(n-r)!\,(n-n+r)!}$$

$$= \frac{n!}{(n-r)!\,r!} = C^n_r$$

Multinomial Expansion:

For real numbers $x_1, x_2, \ldots, x_m$ and non negative integers $n, r_1, r_2, \ldots, r_m$ the following hold.

$$(x_1 + x_2 + \cdots + x_m)^n = \sum \frac{n!}{r_1!\,r_2!\,r_3!\cdots r_m!} x_1^{r_1} x_2^{r_2} x_3^{r_3} \cdots x_m^{r_m}$$

where Σ denoted the sum of all combinations of $(r_1, r_2, r_3, \ldots, r_m)$ s.t $r_1 + r_2 + r_3 + \cdots + r_m = n$

$$(x_1 + x_2 + x_3 + \cdots + x_m)^n = \sum \frac{n!}{(n-r_1-\cdots-r_m)!\,r_1!\cdots r_{m-1}!} x_1^{n-r_1-\cdots-r_{m-1}} x_2^{r_1} \cdots x_m^{r_{m-1}}$$

The coefficients of Multinomial represented by

$$\sum \frac{n!}{n-r_1-r_2-\cdots-r_{m-1}} = C^n_{r_1} C^n_{r_2} C^n_{r_3} \cdots C^n_{r_{m-1}}$$

Probability Measure

Def: (Random experiment)

A random experiment is an experiment whose outcomes cannot be predicted with certainty.

Def: (Sample space)

A sample space of a random experiment is the collection of all possible outcomes.

ex: What is the sample space for an experiment in which we select a rat at random from cage and determine its sex?

$S = \{M, F\}$ where M denotes the male rat and F denotes the female rat.

Note: If the sample space have finit element is called finit sample space.

ex: What is the sample space for an experiment in which we roll a pair of dice, one red and one green?

$$S = \left\{ \begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{matrix} \right\}$$

or $S = \{(x, y) : 1 \leq x \leq 6,\ 1 \leq y \leq 6\}$

where x represents the number rolled on red die and y denotes the number rolled on green die

Def: (σ-field)

A subset A of the sample space S is said to be an event if it belongs to a collection $\mathcal{F}$ of subsets of S satisfying the following three conditions

1. $S \in \mathcal{F}$
2. If $A \in \mathcal{F}$ then $A^c \in \mathcal{F}$
3. for every sequence A_n of sets of $\mathcal{F}$ then $\bigcup_{n=1}^{\infty} A_n$ lies in $\mathcal{F}$

The collection $\mathcal{F}$ is called an event space or σ-field

Note: If A is the outcome of an experiment, then we say that the event A occurred.

Def: (Event)

An event of sample space S or Ω is any set which belong to sample space and subset of Ω.

ex: A women is carrier for classical hemolysis (انحلال الدم). This women have four sons which carrier or not carrier the classical homolysis. Describe the sample space, then find the event that no sons with this disease, find the event that two sons with this disease.

sol: If the son have hemolysis denoted by $= h$

If the son donot have hemolysis denoted by $= n$

The no. of elements in $\Omega = 2^4 = 16$

$$\Omega = \left\{ \begin{array}{l} (hhhh), (hhhn), (hhnh), (hnhh), (nhhh), \\ (hhnn), (nnhh), (hnhn), (nhnh), (hnnh) \\ (nhhn), (nnnh), (nnhn), (nhnn), (hnnn) \\ (nnnn) \end{array} \right\}$$

Let A = event that no sons with this disease

then $A = \{(nnnn)\} = 1$

Let B = event ~~two~~ two sons with this disease

$B = \{(hhnn), (hnnh), (nnhh), (nhnh), (hnhn), (nhhn)\}$

$= 6$

Find the event that at least two of them with disease

$$C = \left\{ \begin{array}{l} (hhnn), (nnhh), (nhnh), (hnhn), (hnnh) \\ (nhhn), (hhhn), (hhnh), (hnhh), (nhhh) \\ (hhhh) \end{array} \right\} = 11$$

ex: If a random experiment represent three teams of football A, B, C. Find the sample space, then find the probability that team A is win?

sol: The outcomes of football = win, loss, tie

The no. of elements in $\Omega = 3^2 = 9$

$$\Omega = \left\{ \begin{array}{l} (\text{win}A, \text{loss}B), (\text{win}A, \text{loss}C), (\text{tie}A, \text{tie}B) \\ (\text{tie}A, \text{tie}C), (\text{loss}A, \text{win}B), (\text{loss}A, \text{win}C) \\ (\text{win}B, \text{loss}C), (\text{tie}B, \text{tie}C), (\text{win}C, \text{loss}B) \end{array} \right\} = 9$$

$$A = \{ (\text{win}A, \text{loss}B), (\text{win}A, \text{loss}C) \} = 2$$

$$Pr(A) = \frac{n_A}{n} = \frac{2}{9}$$

Ω is finit and countable.

ex → The experiment consists of rolling die until (6) is obtai

Find the sample space?

$$\Omega = \{ 6, N6, NN6, NNN6, \ldots \}$$

where N is any number except six

Ω is infinite and countable.

ex → Two balls are placed at random into four cells. Describe the sample space?

sol: The no. of elements in $\Omega = 4^n = 4^2 = 16$

$$\Omega = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4) \\ (2,1), (2,2), (2,3), (2,4) \\ (3,1), (3,2), (3,3), (3,4) \\ (4,1), (4,2), (4,3), (4,4) \end{array} \right\}$$

Ω is finite and countable.

ex: First describe the sample space of rolling a pair of dice then describe the event A that the sum of numbers is 7

Sol:

$$\Omega = \{(x,y) : 1 \leq x \leq 6,\ 1 \leq y \leq 6\}$$

and the event A is $A = \{(1,6),(6,1),(2,5),(5,2),(4,3),(3,4)\}$

Def: (countable space)
If the sample space contains of a countable number of sample points, then the sample space is said to be countable sample space.

Def: (continuous sample space)
If a sample space contains uncountable number of sample points, then it is called a continuous sample space.

Note: An events A, B in sample space Ω, then $A \cup B$, A^c, $A \cap B$, B^c are also entitled to be events.

Def: (probability measure)
Let Ω be the sample space of a random experiment.
A probability measure $P : \mathcal{F} \to [0,1]$ is a set function which assigns real numbers to the various events of Ω satisfying

1. $P(A) \geq 0$ for all event $A \in \mathcal{F}$
2. $P(\Omega) = 1$
3. $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

if $A_1, A_2, A_3, \ldots, A_i, \ldots$ are mutually disjoint events of Ω

Classical probability

If a random experiment whose sample space is Ω can results in (n) mutually exclusive, then if (n_A) one of these outcomes has an attribute from event A. The probability of event A is

$$P(A) = \frac{\text{number of outcomes in event A}}{\text{number of outcomes in sample space } \Omega} = \frac{n_A}{n}$$

ex, Two electrons are placed at random into three levels

① Find the probability that one electron in second level.

② Find the probability that at least one electron in 2^{nd} level.

③ Find the probability that at most one electron in 2^{nd} level.

sol,

The number of elements in $\Omega = 3^2 = 9$

the first level $= L_1$, the second level $= L_2$, the third level $= L_3$

$$\Omega = \begin{Bmatrix} (L_1,L_1), (L_1,L_2), (L_1,L_3) \\ (L_2,L_1), (L_2,L_2), (L_2,L_3) \\ (L_3,L_1), (L_3,L_2), (L_3,L_3) \end{Bmatrix} = 9$$

① the event A represent one electron in the second level.

$$A = \{(L_2,L_1), (L_2,L_3), (L_1,L_2), (L_3,L_2)\} = 4$$

$$\therefore P(A) = \frac{n_A}{n} = \frac{4}{9}$$

② the event B represent at least on electron in 2^{nd} level.

$$B = \{(L_2,L_1), (L_2,L_3), (L_2,L_2), (L_1,L_2), (L_3,L_2)\} = 5$$

$$P(B) = \frac{n_B}{n} = \frac{5}{9}$$

③ the event C represent at most one electron in the 2^{nd} level.

$$C = \{(L_1,L_2), (L_2,L_1), (L_2,L_3), (L_3,L_2), (L_1,L_1), (L_3,L_3), (L_3,L_1), (L_1,L_3)\} = 8$$

$$P(C) = \frac{n_C}{n} = \frac{8}{9}$$

The theorems of classical probability:

① If A is any event in the sample space, then $0 \leq P(A) \leq 1$,
since $0 \leq \frac{n_A}{n} \leq 1$ for every n.

proof:

Since $\phi \subset A \subset \Omega$, then $P(\phi) \subset P(A) \subset P(\Omega)$

But $P(\phi) = \text{Zero}$, $P(\Omega) = 1$

Then $0 \leq P(A) \leq 1$

② The probability of sample space is $P(\Omega) = 1$

proof:
Let $A = \Omega$ so that $A^c = \Omega^c = \phi$

then $P(\Omega) = 1 - P(\phi) = 1 - 0 = 1$

③ ~~If~~ A and B are two mutually exclusive events, then
$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

④ If $A_i, (i = 1, 2, 3, \dots, n)$ are mutually exclusive events, then
$P(A_1 \text{ or } A_2 \text{ or } \dots, \text{or } A_n) = P(A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n)$
$= P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$

⑤ The null event denoted by ϕ then $P(\phi) = \text{Zero}$. because $n_\phi = \text{Zero}$

proof: Let $A = \phi$ then $A^c = \Omega$
$P(\phi) = 1 - P(\Omega) = 1 - 1 = 0$

⑥ If A and B are two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

proof: The event $A \cup B$ be represent as a union of nonintersecting sets as follows:

$A \cup B = A \cup (A^c \cap B)$ then
$P(A \cup B) = P(A) + P(A^c \cap B)$ ---- ①

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The event B can be represent as

$B=(A\cap B)\cup(A^c\cap B)$. then $P(B)=P(A\cap B)+P(A^c\cap B)$

$P(B)-P(A\cap B)=P(A^c\cap B)$ ---- ②

substituted equ ② in equ ①

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

⑦ If A is any event in sample space Ω. then

$$P(A^c)=1-P(A)$$

proof let $\Omega=A\cup A^c$ and $\phi=A\cap A^c$

then $P(A\cap A^c)=P(\phi)$

$\therefore P(A\cap A^c)=0$

$P(\Omega)=P(A\cup A^c)$

$=P(A)+P(A^c)-P(A\cap A^c)$

$1=P(A)+P(A^c)-0$

$\therefore P(A)=1-P(A^c)$

⑧ If A and B are two independent events then

$$P(A\cap B)=P_r(A)*P(B)$$

⑨ if $A\subset B$ then $P(A)\leq P(B)$

Def: Two events A and B are independent if and only if

$$P(A\cap B)=P(A)P(B)$$

⑩ De Morgan's laws state that if $\{A_n\}$, $n\geq 1$, is any sequence of sets, then

$$\left(\bigcup_{n=1}^{\infty}A_n\right)^c=\left(\bigcap_{n=1}^{\infty}A_n^c\right)$$

$\because P(A^c)=1-P(A)$ then $P(\bigcup_n A_n)=1-P(\bigcap_n A_n^c)$

ex consider four particles placed at random in one energy state. Two particles are selected at random from this state energy.

1. what is the probability that three particles in state
2. what is the probability that at least one particles in state
3. what is the probability that at most one particles in state
4. what is the probability that the first particle select and the second particle will be arrange.

sol

$$\Omega = C_2^4 = \frac{4!}{2!(4-2)!} = \frac{4!}{2!\times 2!} = \frac{4\times3\times\cancel{2!}}{2!\times\cancel{2!}} = \frac{12}{2} = 6$$

$$\Omega = \{AB, AC, AD, BC, BD, CD\}$$

sol 1:

$$C_2^3 = \frac{3!}{2!(3-2)!} = \frac{3\times\cancel{2!}}{\cancel{2!}\times 1!} = 3$$

if the particles in energy state are $\{A, B, C\}$ then the event $A = \{AB, AC, BC\}$

$$\therefore P(A) = \frac{n_A}{n} = \frac{3}{6} = \frac{1}{2}$$

sol 2:

$$C_1^2 + C_2^2 = \frac{2!}{1!(2-1)!} + \frac{2!}{2!(2-2)!} = 2+1 = 3$$

let the event H represent at least one particle

$$P(H) = \frac{n_H}{n}$$

$$\therefore P(H) = \frac{3}{6} = \frac{1}{2}$$

sol ③

$$C_1^2 + C_0^2 = \frac{2!}{1!(2-1)!} + \frac{2!}{0! \times 2!} = 2+1=3$$

let the event G represent at most one particles in state

$$\therefore P(G) = \frac{n_G}{n} = \frac{3}{6} = \frac{1}{2}$$

sol ④

$$C_1^2 + P_1^2 = \frac{2!}{1!(2-1)!} + \frac{2!}{1!} = 4$$

let the event E represent that first particle select and 2^{nd} arrange

$$P(E) = \frac{n_E}{n} = \frac{4}{6} = \frac{2}{3}$$

ex, The weather bureaus air pollution index classified each days, where the weather are [extremely good, good, fair, poor, extremely poor]. From past experience indicate that 50% of days are extremely good, 22% as good, 18% as fair, 8% as poor and 2% as extremely poor.

① what is the probability that the day classify as poor or extremely poor

② what is the probability that the day classify as poor or fair or good

③ what is the probability that the day classify poor and good

④ what is the probability complement that day classify extremely good

⑤ what is the probability complement that day classify poor or fair

sol, let A = extremely good, B = good, C = fair, D = poor, E = extremely poor

sol ①: $P(\text{Poor or extremely poor}) = P(D \cup E) = 8\% + 2\% = 10\%$

sol ②: $P(\text{poor or fair or good}) = P(D \cup C \cup B) = P(D) + P(C) + P(B)$

$$= 8\% + 18\% + 22\% = 48\%$$

Sol (3):

$$P(\text{Poor and good}) = P(D \cap B) = P(D) * P(B)$$
$$= 8\% * 22\% = \frac{176}{10000}$$

Sol (4)

$$P(\text{extremely poor})^c = P(A)^c = 1 - P(A)$$
$$= 1 - 50\% = 50\%$$

Sol (5):

$$P(\text{poor or fair})^c = P(D \cup C)^c = 1 - P(D \cup C)$$
$$= 1 - [P(D) + P(C)]$$
$$= 1 - [8\% + 18\%] = 74\%$$

Relative frequency probability,

If the number of independent trials is infinite then

$$P(A) = \lim_{n \to \infty} \frac{h}{n}$$

where h is the number of occureness of event A
n is the number of independent trials.

ex, Find the probability of getting an even number, if you draw one number from natural number set.

Sol,

$$N = \{1, 2, 3, 4, ------, \infty\}$$

n is even integer numbers $= \{2, 4, 6, 8, ---, \infty\}$

n is odd integer numbers $= \{3, 5, 7, 9, ---, \infty\}$

$2n$ is even and odd integer numbers, then $P(A = \text{even}) = \lim_{n \to \infty} \frac{n}{2n} = \frac{1}{2}$

Equally likely probability:

From a certain random experiment, there are finite number of outcomes, say N, if the probability is equally likely.

ex ① The probability of sex human or sex children

$P(\text{male}) = \frac{1}{2}$, $P(\text{female}) = \frac{1}{2}$

② The probability of tossing coin $S = \{H, T\} \Rightarrow n = 2$

$\therefore P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$

③ The probability of tossing die then $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n = 6$

$P(1 \text{ appear}) = \frac{1}{6}$, $P(2 \text{ appear}) = \frac{1}{6}$, ----, $P(6 \text{ appear}) = \frac{1}{6}$

Def: (probability space)

Is define as triple $(\Omega, \mathcal{F}, P(\cdot))$ where Ω is sample space, $\mathcal{F}$ is a collection (σ-algebra) of events (each subset of Ω) and $P(\cdot)$ is probability function with domain Ω i.e

$P: \Omega \to [0, 1]$

then

ex: suppose three perfectly balanced and identical coins are tossed. Find the probability that at least one of them lands heads.

sol:

$$S = \{(HHH), (HHT), (HTH), (HTT), (THT), (THH), (TTH), (TTT)\}$$

the probability of each element in the sample space $= \frac{1}{8}$

let A_1 be the event that the first coin lands heads
A_2 " " " " " 2^{nd} " lands heads
A_3 " " " " " 3^{rd} " " "
we must compute $P(A_1 \cup A_2 \cup A_3)$

But $(A_1 \cup A_2 \cup A_3)^c = (A_1^c \cap A_2^c \cap A_3^c)$ De Morgan's laws

$\therefore P(A_1 \cup A_2 \cup A_3)^c = P(A_1^c \cap A_2^c \cap A_3^c)$

But $A_1^c \cap A_2^c \cap A_3^c = \{TTT\}$

then $P(A_1 \cup A_2 \cup A_3)^c = P(A_1^c \cap A_2^c \cap A_3^c) = \frac{1}{8}$

then
$$\begin{aligned} P(A_1 \cup A_2 \cup A_3) &= 1 - P(A_1 \cup A_2 \cup A_3)^c \\ &= 1 - P(A_1^c \cap A_2^c \cap A_3^c) \\ &= 1 - \frac{1}{8} = 7/8 \end{aligned}$$

Theorem: Let A_n, $n \geq 1$, be events

① If $A_1 \subset A_2 \subset A_3 \subset ----$ and $A = \bigcup_{i=1}^{\infty} A_n$ then

$$\lim_{n \to \infty} P(A_n) = P(A)$$

② If $A_1 \supset A_2 \supset A_3 \supset ---$ and $A = \bigcap_{i=1}^{\infty} A_n$ then

$$\lim_{n \to \infty} P(A_n) = P(A)$$

Proof: ① suppose $A_1 \subset A_2 \subset ---$ and $A = \bigcup_{n=1}^{\infty} A_n$.

set $B_1 = A_1$ and foreach $n \geq 2$, let B_n denoted those point which are in A_n but not in A_{n-1}, i.e $B_n = A_n \cap A_{n-1}^c$.

Apoint ω is in B_n if and only if ω is in A and A_n is the first set in the sequence $A_1, A_2, ---$ Containing ω.

B_n are disjoint

$$A_n = \bigcup_{i=1}^{n} B_i$$

and

$$A_n = \bigcup_{i=1}^{\infty} B_i$$

$$P(A_n) = P\left(\bigcup_{i=1}^{\infty} B_i\right)$$

$$P(A_n) = \sum_{i=1}^{\infty} P(B_i)$$

Now

$$\lim_{n \to \infty} \sum_{i=1}^{\infty} P(B_i) = \sum_{i=1}^{\infty} P(B_i)$$

$$\lim_{n \to \infty} P(A_n) = \lim_{n \to \infty} \sum_{i=1}^{n} P(B_i)$$

$$= \sum_{i=1}^{\infty} P(B_i) = P(A)$$

Proof ②:

Suppose $A_1 \supset A_2 \supset \cdots$ and $A = \bigcap_{n=1}^{\infty} A_n$ then

$A_1^c \subset A_2^c \subset \cdots \subset A_n^c \subset \cdots$

then $A^c = \bigcup_{n=1}^{\infty} A_n^c$

then by the proof of part ① erliare

$$\lim_{n\to\infty} P(A_n^c) = P(A^c)$$

since $P(A_n^c) = 1 - P(A_n)$ and $P(A^c) = 1 - P(A)$

$$\therefore \lim_{n\to\infty} P(A_n) = \lim_{n\to\infty} (1 - P(A_n^c))$$

$$= 1 - \lim_{n\to\infty} P(A_n^c)$$

$$= 1 - P(A^c) = P(A)$$

ex: A box contain (20) similar balls on each ball is written a number from (1) to (20). One ball was taken at random from the box. What is the probability that the ball

① A: be odd number

② B: be divided by three

Find $P(A \cup B)$, $P(A^c)$. $P(B^c)$

sol: let A be the event that odd number on the ball

let B be the event that the number divided by three

~~Find $P(A\cup B)$~~

$\omega = S = \{1, 2, 3, 4, \cdots, 19, 20\} = 20$

$A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} = 10$

$B = \{3, 6, 9, 12, 15, 18\} = 6$

$$P(A) = \frac{n_A}{n} = \frac{10}{20} = \frac{1}{2}$$

$$P(B) = \frac{n_B}{n} = \frac{6}{20} = \frac{3}{10}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad , \quad A \cap B = \{3, 9, 15\}$$

$$P(A \cap B) = \frac{n_{A \cap B}}{n} = \frac{3}{20}$$

$$\therefore P(A \cup B) = \frac{1}{2} + \frac{3}{10} - \frac{3}{20} = \frac{13}{20}$$

$$P(A^c) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(B^c) = 1 - P(B) = 1 - \frac{3}{10} = \frac{7}{10}$$

Conditional probability:

Def: Let (A) and (B) be two events, then the conditional probability of B given A written $P(B|A)$ is defined to be

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad \text{where } P(A) > 0$$

and $$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{where } P(B) > 0$$

Note: where $P(A) = 0$ the conditional probability of B given A is undefined.

ex: suppose cards numberd (1) to (10) are placed in a box, mixedup and then one of the cards is drawn. If we are told that the number on the drawn card is at least five, then what is the conditional probability that is number 10

Sol:

$$\Omega = S = \{1, 2, 3, 4, \dots, 9, 10\} = 10$$

let E denote the event that the number of the drawn card is 10 and let F be the event that at least five.

$$\therefore P(E|F) = \frac{P(E\cap F)}{P(F)}$$

$E = \{10\} = 1$, $F = \{5, 6, 7, 8, 9, 10\} = 6$

$E\cap F = \{10\} = E$

$\therefore P(E\cap F) = \frac{1}{10}$

$P(F) = \frac{n_F}{n} = \frac{6}{10}$

$$\therefore P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{\frac{1}{10}}{\frac{6}{10}} = \frac{1}{6}$$

ex) A family has two children. What is the conditional probability that both are boys given that at least one of them is boy?

sol) $S = \{(b,b), (b,g), (g,b), (g,g)\} = 4$

let E denote the event that both children are boys and let F the event that at least one of them is boy.

$$P(E|F) = \frac{P(E\cap F)}{P(F)}$$

$E = \{(b,b)\} = 1$ $\Rightarrow E\cap F = \{(b,b)\} = 1$

$F = \{(b,g), (g,b), (b,b)\} = 3$

$\therefore P(E\cap F) = \frac{1}{4}$ and $P(F) = \frac{3}{4}$

$$\therefore P(E|F) = \frac{\frac{n_{E\cap F}}{n}}{\frac{n_F}{n}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

ex: A school involve 80% boys and 20% girls. 60% of boys are smoking and 40% of girls are smoking too. One student are chosen at random, what is the probability that this student boy and smoking

sol:

let b the event that the student is boy

let g " " " " " is girl

$$P(b) = \frac{80}{100} = \frac{8}{10} = 0.8$$

$$P(g) = \frac{20}{100} = \frac{2}{10} = 0.2$$

$$P(b|S) = \frac{P(b \cap S)}{P(S)}$$

$\because P(b \cap S) = P(b) \cdot P(S)$

$\therefore P(b \cap S) = 0.8 * 0.6 = 0.48$

$P(S \cap g) = P(S) \cdot P(g) = 0.4 * 0.2 = 0.08$

$P(S) = P(S \cap b) + P(S \cap g) = 0.48 + 0.08 = 0.56$

$$\therefore P(b|S) = \frac{P(b \cap S)}{P(S)} = \frac{0.48}{0.56} = \frac{48}{56} = \frac{6}{7}$$

ex: suppose a factory has two machines A and B that make 60% and 40% of the total production respectively. of their output, Machine A produces 9% defective items, while machine B produce 15% defective items. Find the probability that a given defective part was produce by machine B.

sol:

$$P(B|D) = \frac{P(B \cap D)}{P(D)}$$

$P(D \cap A) = P(D) \cdot P(A) = 0.09 * 0.6 = 0.054$

$P(D \cap B) = P(D) \cdot P(B) = 0.15 * 0.40 = 0.060$

$P(D) = P(D \cap A) + P(D \cap B) = 0.054 + 0.060 = 0.114$

$$\therefore P(B|D) = \frac{P(B\cap D)}{P(D)} = \frac{0.060}{0.114} = \frac{30}{57}$$

Def: The probability that two event A and B, both occur is given by the multiplication rule:

$$P(A\cap B) = P(A) * P(B|A)$$

Theorem: If A and B are independent events with non-zero Probabilities then:

$$P(A|B) = P(A) \quad \text{and} \quad P(B|A) = P(B)$$

proof: As A and B are independent and $A\cap B = B\cap A$ we have

$$P(A\cap B) = P(B\cap A) = P(A) * P(B)$$

Hence:

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A) * \cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

$$P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{P(B)\cdot \cancel{P(A)}}{\cancel{P(A)}} = P(B)$$

ex: Flip a coin and then independently cast a die. What is the probability of observing heads on the coin and 2 or 3 on the die?

sol: let A denote the event of observing a head on the coin and let B be the event of observing a 2 or 3 on the die.

$$P(A\cap B) = P(A) * P(B)$$
$$= \left(\tfrac{1}{2}\right)\left(\tfrac{2}{6}\right) = \frac{1}{6}$$

Theorem: If A and B are independent events. Then A^c and B are independent. Similarly A and B^c are independent.

proof: since A and B are independent then

$$P(A \cap B) = P(A) * P(B)$$

we want show that A^c and B are independent that is

$$P(A^c \cap B) = P(A^c)\, P(B)$$

since
$$\begin{aligned} P(A^c \cap B) &= P(A^c \mid B) \cdot P(B) \\ &= [1 - P(A \mid B)]\, P(B) \\ &= P(B) - P(A \mid B)\, P(B) \\ &= P(B) - P(A \cap B) \\ &= P(B) - P(A)\, P(B) \\ &= P(B)[1 - P(A)] \\ &= P(B)\, P(A^c) \end{aligned}$$

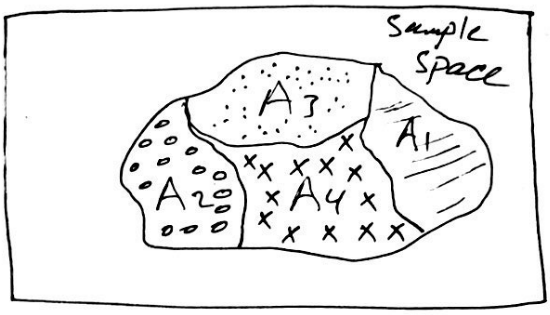
then the events A^c and B are independent
in similar way we can proof that A and B^c are independent.

Bayes Theorem:

Def: Let S be a set and let $P=\{A_i\}_{i=1}^{m}$ be collection of subsets of S. The collection P is called a partition of S if

① $S=\bigcup_{i=1}^{m} A_i$

② $A_i \cap A_j = \phi$ for $i \neq j$



*** Theorem ①** (Total probability) If the events $\{B_i\}_{i=1}^{m}$ constitute a partition of the sample space S and $P(B_i) \neq 0$ for $i=1,2,3,\dots,m$ then for any event A in S

$$P(A)=\sum_{i=1}^{m} P(B_i)P(A|B_i)$$

proof: let S be a sample space and A be an event in S. let $\{B_i\}_{i=1}^{m}$ be any partition of S. Then

$$A=\bigcup_{i=1}^{m}(A\cap B_i)$$

$$P(A)=\sum_{i=1}^{m} P(A\cap B_i)$$

$$=\sum_{i=1}^{m} P(B_i)P(A|B_i)$$

Thearem (Bayes Theorem) If the events $\{B_i\}_{i=1}^{m}$ constitute a partition of the sample space S and $P(B_i) \neq 0$ for $i=1,2,3,\cdots,m$ then for any event A in S such that $P(A) \neq 0$

$$P(B_K|A) = \frac{P(B_K)P(A|B_K)}{\sum_{i=1}^{m} P(B_i)P(A|B_i)} \qquad K=1,2,\cdots,m$$

Proof:

$$P(B_K|A) = \frac{P(A\cap B_K)}{P(A)} \qquad \text{(definition of conditional probability)}$$

by using theorem ⊛ erligar we get.

$$P(B_K|A) = \frac{P(A\cap B_K)}{\sum_{i=1}^{m} P(B_i)P(A|B_i)}$$

this Theorem is called Bayes theorem

The probability $P(B_K)$ is called prior probability.
The probability $P(B_K|A)$ is called posterior probability.

ex: Two boxes containing marbles are placed on a table. The boxes are labeled B_1 and B_2. Box B_1 contains 7 green marbles and 4 white marbles. Box B_2 contains 3 green marbles and 10 yellow marbles. The boxes are arranged so that the probability of selecting box B_1 is $\frac{1}{3}$ and the probability of selecting box B_2 is $\frac{2}{3}$. Kathy is blindfolded and asked to select a marble. She will win a color TV if she selects a green marble

① What is the probability that Kathy will win the TV

② If Kathy wins the color TV, what is the probability that the green marble was selected from the first box?

Sol

Let A be the event of drawing a green marble.

The prior probabilities are $P(B_1) = \frac{1}{3}$ and $P(B_2) = \frac{2}{3}$

① The probability that Kathy will win the TV is

$$P(A) = P(A \cap B_1) + P(A \cap B_2)$$
$$= P(A|B_1)P(B_1) + P(A|B_2)P(B_2)$$
$$= \left(\frac{7}{11}\right)\left(\frac{1}{3}\right) + \left(\frac{3}{13}\right)\left(\frac{2}{3}\right)$$
$$= \frac{7}{33} + \frac{2}{13} = \frac{91}{429} + \frac{66}{429}$$
$$= \frac{157}{429} = 0.365$$

② Given that Kathy won TV, the probability that green marble was selected from B_1

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2)}$$
$$= \frac{\left(\frac{7}{11}\right)\left(\frac{1}{3}\right)}{\left(\frac{7}{11}\right)\left(\frac{1}{3}\right) + \left(\frac{3}{13}\right)\left(\frac{2}{3}\right)} = \frac{91}{157}$$

Note: Note that $P(A|B_1)$ is the probability of selecting a green marble from B_1 whereas $P(B_1|A)$ is the probability that the green marble was selected from box B_1

ex. A company buys microchips from three suppliers A, B, C. Supplier A has a record of providing microchips that contain 10% defectives, supplier B has defective rate of 5% and supplier C has a defective rate of 2%, suppose 20%, 35% and 45% of current supply came from suppliers A, B, C respectively. If a microchip is selected at random from this supply, ① What is the probability that it is defective?
② what is the probability that the defective microchip came from supplier B?

Sol. ①
Let G_i denote the event that a microchip comes from supplier i, $i = A, B$ or C (Notice that G_A, G_B and G_C form partition of the sample space for the experiment of selecting one microchip)

Let D denote the event that the selected microchip is defective

$P(G_A) = 0.20$, $P(G_B) = 0.35$ and $P(G_C) = 0.45$

$P(D|G_A) = 0.10$, $P(D|G_B) = 0.05$ and $P(D|G_C) = 0.02$

by using theorem Total probability we have

$$P(D) = P(G_A)P(D|G_A) + P(G_B)P(D|G_B) + P(G_C)P(D|G_C)$$

$$= (0.20)(0.10) + (0.35)(0.05) + (0.45)(0.02)$$

$$= 0.02 + 0.0175 + 0.009 = 0.0465$$

Sol ②

$$P(G_B|D) = \frac{P(D|G_B)P(G_B)}{P(D)} = \frac{P(D|G_B)P(G_B)}{P(G_A)P(D|G_A) + P(G_B)P(D|G_B) + P(G_C)P(D|G_C)}$$

$$= \frac{0.05(0.35)}{0.0465} = 0.376$$

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