المحاضرة الأولى

THE CONVERSE OF LAGRANGE THEORY

Introduction

In these lectures, we study some applications of the group theory such as we study conjugate elements and conjugate groups, the converse of Lagrange theorem, P-group, Sylow P-group, Sylow theorem, Normalizer of subgroup, Nilpotent Group and solvable group. Moreover, some applications of Sylow theorems are considered.

Given a group G of order n, and integer m dividing n. One cannot be certain that G possesses a subgroup of order m. in fact, that is one of considerable difficulty. So in general, that is not true, for example, the subgroup A_4 of the permutation group S_4 is of order 12 and the number 6 divides 12 but A_4 hasn't subgroup of order 6. However, in this lecture, we discuss the case under which that thing is true. We begin the first case:

Theorem 1: If G is a finite abelian group of order n with a prime number p divided n, then G contains a subgroup of ordered p.

Theorem 2: Let G be a finite group with ordered n. for all prime numbers p divided n, there is a subgroup with order p.

CONJUGATE ELEMENTS

Definition 3: Let G be a group and x, y G. Then x and y is said to be **conjugate** if there is a G such that $y = axa-1$.

Remarks 4:

1. The conjugate relation is equivalent and hence it is made a partition on G.

2. We will refer to class of the element x by $[x] = \{y\ G|\ x = aya-1\}$

3. The center of G is denoted by Z(G) where:

$$Z(G) = \{x\ G \mid ax = xa \text{ for all } a\ G\}$$

$$= \{x \in G \mid xa = ax \text{ for all } a \in G\}$$

4. $x \in Z(G)$ if and only if [x] contains only one element.

5. The set $C(y) = \{a \in G \mid ay = ya\}$ is said to be the **center** of the element y.

6. C(y) is a subgroup of G.

7. There is an isomorphism between the right coset of the subgroup C(y) and the conjugate elements of y.

Remark 5: If G is a finite group, then:

$$|G| = |\ Z(G)| + \Sigma \text{ (where } y\ Z(G)$$

is said to be a **conjugate class equation**.

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