المحاضرة الثالثة

Sylow Groups

Proposition 1: If G is a nontrivial P-group, then Z(G) is nontrivial and o(Z(G)) = p^k for $k \geq 1$.

Proof: H.W

Corollary 2. Every P-group of ordered p^2 is abelian.

Proof: Let G be an abelian group with o(G) = p^2 and Z(G) be the centre of G. Since G is not trivial, then Z(G) $\neq$ {e}. Then by Lagrange's theorem o(Z(G)) = p or p^2. If o(Z(G))= p2, then G = Z(G) and so G is an abelian group.

Corollary 3. If G is P-group with order p3 and G is not abelian, then o(Z(G)) = p.
Proof: Since G is not abelian group, then G $\neq$ Z(G) and hence o(Z(G)) $\neq$ p3. Now, by Lagrange's theorem, O(Z(G)) divides O(G). Hence either O(Z(G)) =1 or o(Z(G)) = p or o(Z(G)) = p^2.
If o(Z(G)) =1, then Z(G) = {e} C! since Z(G) is not a trivial subgroup.
If o(Z(G)) = p^2, then O(G/Z) = p. Hence is a cyclic group and so G is an abelian group C! (with hypothesis). Therefore, o(Z(G)) = p.

SYLOW GROUPS

Definition 4: Let G be a finite group with o(G) = p^m. such that p and the integer number $m \geq 1$. Then a subgroup H of G will be called **Sylow P-subgroup** of order p^m.

Remark 5:

- The integer m is the largest positive integer such that pm divides the order of G.
- H is the maximize p-subgroup of G.

Definition 6: Let H and K be two subgroups of G such that H is **conjugate** to K if and only if there is a G such that K = aHa^{-1}.

Remark 7:

1. If H and K are conjugate, then O(H) = O(K).

Proof: Define f: H → K by : $f(h) = aha^{-1}$. Then f is 1-1 and onto. If H is (or K) is P-group with order p^n, then so is K (or H).

2. If H and K are conjugate and H is Sylow P-subgroup, then K is Sylow P-subgroup.

Definition 8: Let H be a subgroup of G. The set NG(H) = N(H) = {a G| aHa^{-1} = H}is said to be **normalize**r of H in G.

Remark 9: If H is a normal subgroup of G, then N(H) = G and if N(H) = H, then H is normalizer subgroup of G.

Theorem 10:

- **i.** N(H) is a subgroup of G and contain H.
- **ii.** H is a normal subgroup of N(H).
- **iii.** N(H) is the largest normal subgroup of G containing H.

Proposition 11: Let G be a group and H be a subgroup of G. Then, there is a 1-1 and onto map between the set of all subgroups conjugate of H and the set of all right coset (left) of N(H) in G.

Corollary 12: If G is a finite group and H subgroup of G, then the number of subgroups of G which conjugate to H are divided o(G).
Proposition. If G is a finite group, then the number of elements of the set [H]K = [K:NK(H)], then the number of elements in [H]K divides o(K) and hence the number of elements in [H]K divides o(G).

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