

Chapter TwoImage Statistics

(2-1) Image Statistics

- Image mean & local mean
- Image variance and local variance
- Image histogram and histogram equalization.
- Image contrast and local contrast

✚ (2-2) The Statistical Image Properties.

The statistical properties of an image provide useful information, such as the total, mean, standard deviation, and variance of the pixel values. The analysis of the statistical properties of images is dictated by the concern of adapting secondary treatments such as filtering, restoring, coding, and shape recognition to the image signal. Several image properties can be calculated from image data, the most important properties (mean μ , standard deviation σ or variance σ^2 , contrast ct , and image histogram) of the image or image local regions.

(2-2-1) Image Mean (μ)

Image means brightness is known as the mean brightness for the image elements (or sub-image or local image region), and it is determined from the following relationship:

$$\text{Mean} = \frac{1}{M \times N} \sum_{x=1}^M \sum_{y=1}^N f(x, y) \dots (2-1)$$

Where M and N denote the height and the width of the image (or sub-image or local image region), the product of them equals the number of image or sub-image elements.

(2-2-2) Standard Deviation (STD or σ)

The standard deviation σ represents the mean of variations of the image (or local or sub-image region) element values from its mean, and it is determined from the following relationship:

$$\text{STD} = \sqrt{\frac{1}{M \times N} \sum_{x=1}^M \sum_{y=1}^N (I(x, y) - \text{Mean})^2} \quad \dots (2-2)$$

The variance σ^2 represents the square value of the standard deviation.

(2-2-3) Image histogram and histogram equalization

Histograms plot how many times (frequency or occurrence) each intensity value in the image occurs. An image histogram is a graph that shows the number of pixels in each image, pixel intensity level (pixel value) or in each index of the indexed color image. The image histogram contains the information needed for image equalization, as image pixels are extended to give a reasonable contrast.

- **Graph and grayscale:** We can get the image histogram by drawing the pixel value distribution across the full grayscale range.

Hist. of pixel(n) = (number of pixels of value=n) / total number of pixels

Example: Produce a histogram given the following image (a matrix filled with integers) with the grayscale value ranging from 0 to 7, that is, with each pixel encoded into 3 bits.

$$I = [0, 1, 2, 2, 6, 2, 1, 1, 2, 1, 1, 3, 4, 3, 3, 0, 2, 5, 1, 1]$$

Or given by :-

$$I = \begin{bmatrix} 0 & 1 & 2 & 2 & 6 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 3 & 4 & 3 & 3 \\ 0 & 2 & 5 & 1 & 1 \end{bmatrix}$$

Solution:

Since the image (I) of many pixels ($N*M=20$) is encoded using 3 bits for each pixel, we have the pixel value ranging from 0 to 7. The count for each grayscale is listed in the Table below.

Image pixel level value (v)	Number of pixels (occurrence) $O_c(v)$	Probability of v $P(v)=O_c(v)/N*M$	Cumulative Probability $C_p(v)$
0	2	0.1	0.1
1	7	0.35	0.45
2	5	0.25	0.70
3	3	0.15	0.85
4	1	0.05	0.90
5	1	0.05	0.95
6	1	0.05	1.00
7	0	0.00	1.00

Based on the grayscale distribution counts, the histogram is created as shown below:

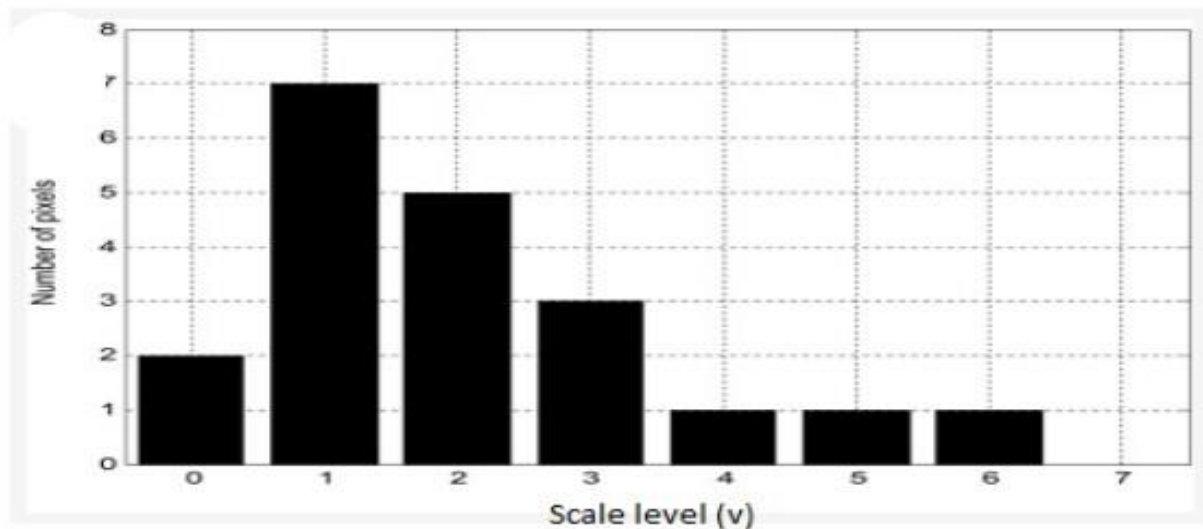


Fig. (2.1) The histogram of Number of pixels (occurrence) $O_c(v)$

As we can see, the image has pixels whose levels are more concentrated in the dark scale in this example

H.w: - plot $C_p(v)$?

Example:

The following Fig (2.2) shows the data (i) and its histogram $h(i)$.

Histograms

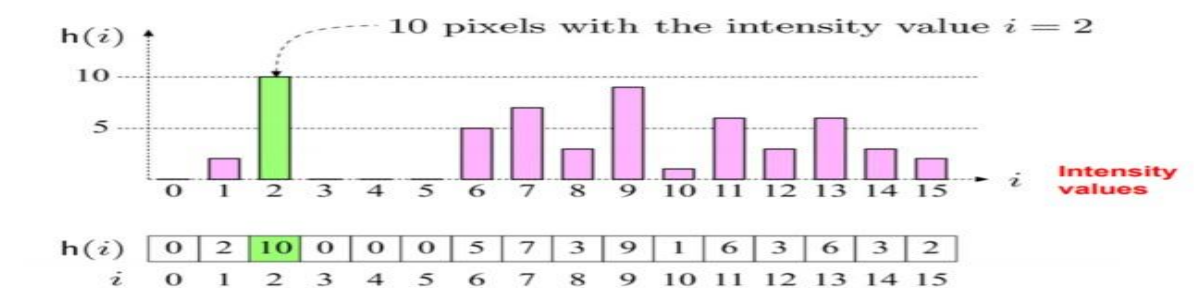


Fig. (2.2) Example of Histograms ($h(i)$)



Fig. (2.3) Real example of the left image and its histogram.

Different images can have the same histogram; the three images

Fig. (2-4) has the same histogram:



Fig (2-4)

Half of the pixels are gray, half are white. The Same histogram means the same statistics. The Distribution of intensities could be different in the image plane.

Q- Can we reconstruct an image from a histogram? No

Histograms help detect image acquisition issues. *Problems with the image can be identified on a histogram (over- and under-exposure, brightness, contrast, & dynamic range).*

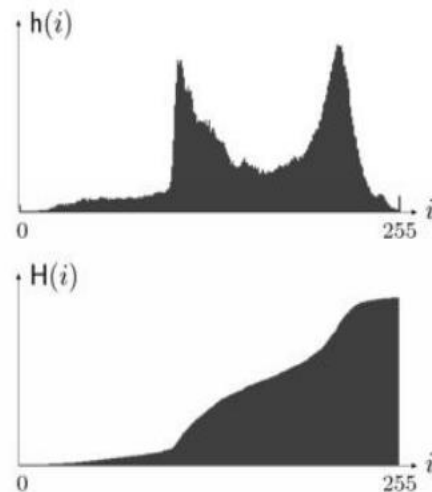
With the histogram, the equalization technique can be developed. Equalization stretches the scale range of the pixel levels to the full range to give an improved contrast for the given image. To do so, the equalized new pixel value is redefined as:

$$\mathbf{J = Round\ integer\ (C_p\ (I) * L) \text{ -----} \quad (2-3)}$$

Where (J) represents the new contrast enhancement image and (L) is the maximum scale pixel value in the image (I). Since the accumulated counts can range from 0 up to 1, and the equalized pixel value can vary from 0 to the maximum scale level (L), i.e., integer value (0 to L). It is due to An accumulation procedure in which the pixel values are spread over the whole range from 0 to the maximum scale level (L).

The following **Figure (2-5)** shows the image and its histogram and cumulative histogram

Image histogram



Cumulative image histogram

Fig (2-5) Image histogram and Cumulative image histogram

The cumulative histogram is a variation of the histogram in which the vertical axis represents not just the counts for a single gray level but denotes the counts for the intensity in consideration, plus all values less than that intensity.

Image Histogram Equalization

Histogram equalization is a method to process images to adjust the contrast of an image by modifying the intensity distribution of the histogram see **Fig. (2-6)**. The objective of this technique is to give a linear trend to the cumulative probability function associated with the image.

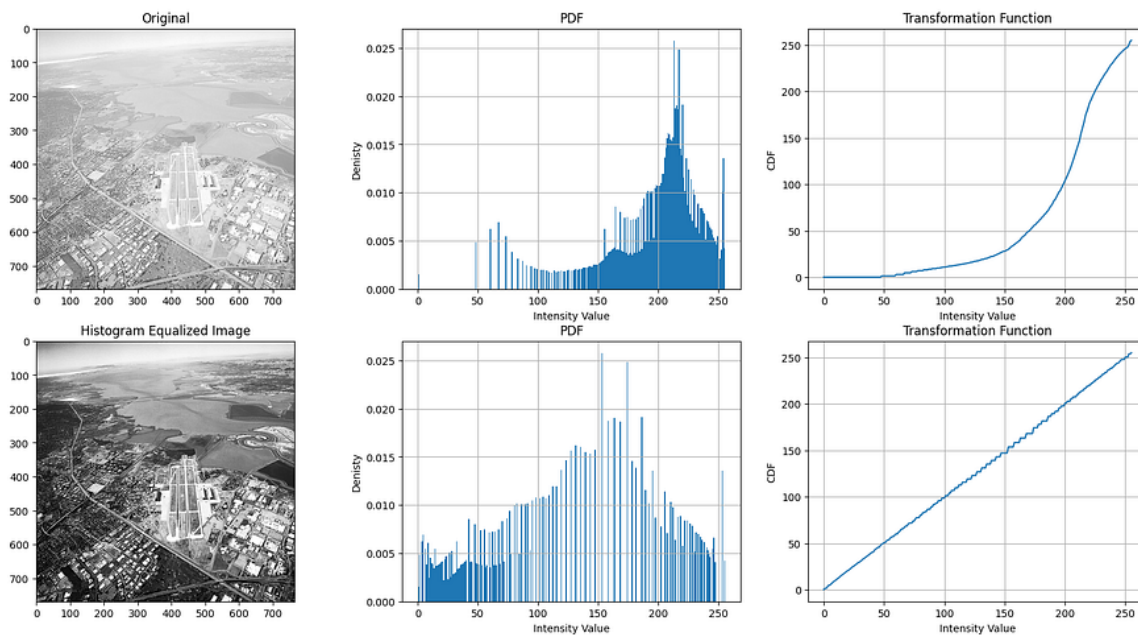


Fig. (2-6) Effect of Histogram Equalization

Example:

Given the following image (matrix filled with integers) with a grayscale value ranging from 0 to 7, that is, with each pixel encoded in 3 bits,

$$I = \begin{bmatrix} 0 & 1 & 2 & 2 & 6 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 3 & 4 & 3 & 3 \\ 0 & 2 & 5 & 1 & 1 \end{bmatrix}$$

Perform equalization using the histogram, and plot the histogram for the equalized image.

Solution:

Based on the histogram result in the Table below, we can compute an accumulative probability $C_p(v)$ for each grayscale level (v), as shown in the Table below.

The maximum scale value is $L=7$ and the total number of image elements is $N*M=20$. The equalized pixel level ($n \ v$) using Eq. (2-1) is given in the last column.

Image pixel level value (v)	Number of pixels التكرار (occurrence) $O_c(v)$	Probability of v $P(v)=O_c(v)/(N*M)$	Cumulative Probability $C_p(v)$	New Image pixel level value (Equalized pixel Level) (nv) $= C_p(v) * 7$
0	2	0.1	0.1	1
1	7	0.35	0.45	3
2	5	0.25	0.70	5
3	3	0.15	0.85	6
4	1	0.05	0.90	6
5	1	0.05	0.95	7
6	1	0.05	1.00	7
7	0	0.00	1.00	7

To see how the old pixel-level

$I(x, y) = 4$ is equalized to the new pixel level $J(x, y) = 6$, which we apply above Eq(2-1). Where $C_p(4) = 0.90$

then: The new value:

$n_v = \text{Round integer}(C_p(4) * 7) = \text{Round integer}(0.90 * 7) = \text{Round integer}(6.3) = 6.$

So, the equalized image using the above Table is finally obtained by replacing each old pixel value(v) in the old image (I) with its corresponding equalized new pixel value (n v) in the enhanced image (J) that given by

$$J = \begin{bmatrix} 1 & 3 & 5 & 5 & 7 \\ 5 & 3 & 3 & 5 & 3 \\ 3 & 6 & 6 & 6 & 6 \\ 1 & 5 & 7 & 3 & 3 \end{bmatrix}$$

To see how the histogram is changed, see Fig. 2-7. We compute the pixel-level counts according to the equalized image. The result is given in the Table below, and the new histogram for the equalized image is below.

Image pixel level value (v)	Number of pixels (occurrence) Oc(nv)	Probability of v $P(v)=Oc(nv)/N * M$	Cumulative Probability $Cp(nv)$
0	0	0.0	0.0
1	2	0.1	0.1
2	0	0.0	0.1
3	7	0.35	0.45
4	0	0.0	0.45
5	5	0.25	0.70
6	4	0.2	0.90
7	2	0.1	1.00

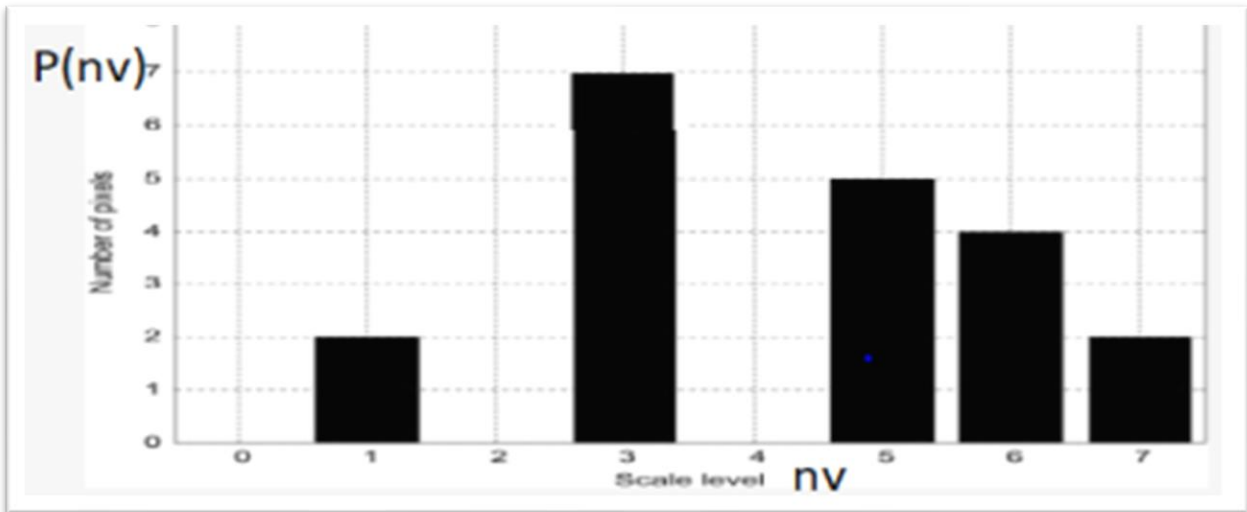


Fig. (2-7)

Next, we apply the image histogram equalization to enhance an image and its histogram in Fig. (2-8) below. We see that there are many pixels counts residing at the lower scales in the histogram. Hence, the image looks rather dark and may be underexposed.

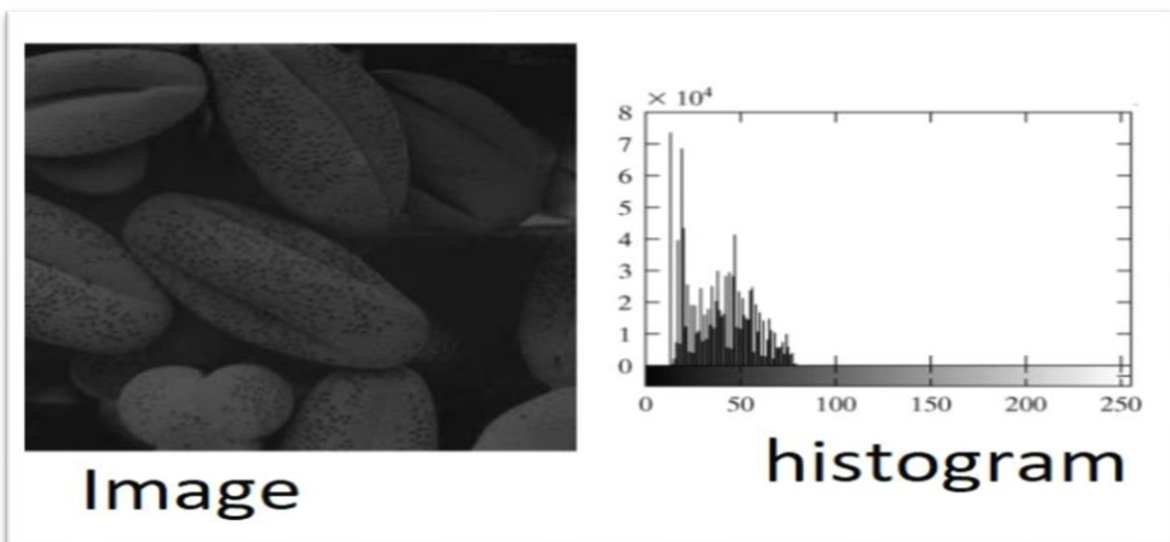


Fig. (2-8) Original grayscale image, & its Histogram.

Figure (2-9) below shows the equalized grayscale image for an image in Figure (2-8) above using the histogram equalization method and its histogram. As shown in the histogram, the equalized pixels reside more on the larger scale, and hence the equalized image has improved contrast.

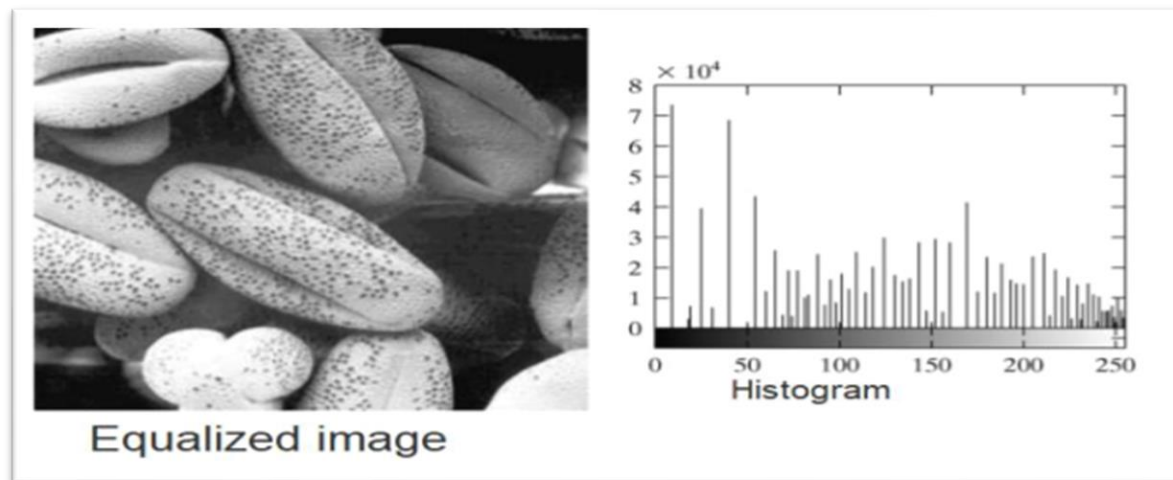


Fig. (2-9) Equalized grayscale image, & its Histogram.

(2-2-4) Image Contrast

The contrast of a grayscale image indicates how easily objects in the image can be distinguished. High-contrast images have many distinct intensity values. The low-contrast image uses a few intensity values.

Histograms and Contrast: Low contrast, Normal contrast, and High contrast images and their histograms are shown in **Fig. 2-10**.

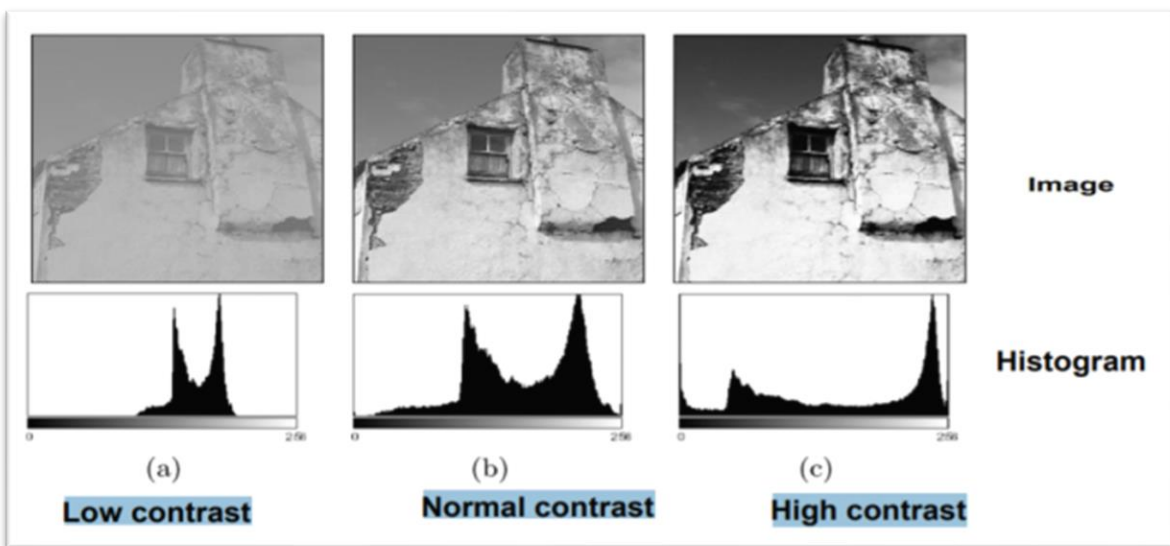


Fig. (2-10) Type of Contrast of an image and Histogram

Good or High Contrast? Widely spread intensity values + large difference between min and max intensity values in the above image.

Image Contrast (Luminance Contrast): is the relationship between the *luminance* of a brighter area of interest and that of an adjacent darker area.

- **Weber Contrast:** - One of the oldest luminance contrast statistics, Weber Contrast, is also often used for these patterns (small, sharp-edged graphic objects like symbols and text characters on larger uniform backgrounds):

Weber Contrast equation given by:

$$C_W = \frac{I_{max} - I_{min}}{I_{min}} \dots\dots\dots (2-4)$$

Where I_{max} and I_{min} represent the maximum and minimum luminance of the image or local image values, respectively.

- **Michelson contrast:**

Michelson contrast measures the relation between the spread and the sum of the two luminances. Also known as *visibility* is commonly used for patterns where both bright and dark features are equivalent and take up similar fractions of the area (e.g. sine-wave gratings). (Its value is between zero and one.) The Michelson contrast is defined as

$$C_M = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \dots\dots\dots (2-5)$$

Where I_{max} and I_{min} represent the maximum and minimum luminance of the image or local image values, respectively.

Example: Find the two contrast values for the center pixel of the following the mask using the adjacent points.

2	2	100
3	16	250
4	10	251

Solution

I_{max} in the 3*3 mask is 251, and **I_{min}** =2, so the contrast values as follow:

1. Weber Contrast: $C_w = (251-2)/2 = 124.5$

2. Michalson contrast: C_M

$$= (251-2) / (251+2) = 0.9841$$

Notes: -

- 1- Contrast is the sharpness in the details. There is good sharpness, but if the camera contains aberrations, the sharpness decreases.
- 2- The cause of blur or lack of clarity in the image is either diffraction or aberration.
- 3- The most important information in the image is located in the edge area, so when the edge is lost in the image, we lose most of the important information.
- 4- If the edges are present in large numbers, the Contrast is large, Conversely, if the number of edges is small, the contrast will be small.
- 5- Another type of contrast is statistical contrast, which depends on the standard deviation and the means.

$$C_s = \frac{\sigma}{\mu} \dots\dots\dots(2-6)$$

SNR or signal-to-noise ratio

is the ratio between the desired information or the power of a signal and the undesired signal or the power of the background noise.

$$SNR = \frac{\mu}{\sigma} \dots\dots\dots(2-7)$$