Definition :- The Disjoin set المجموعة المنفصلة

Let A, B are two sets, then A, B is called disjoin set if $A \cap B = \phi$.

for example, E & O are disjoin set since $E \cap O = \phi$

Definition :- The Difference الفرق

The difference between the sets A, B is the set

$$A - B = \{x ; x \in A \wedge x \notin B\}$$

for example, Let $A = \{a, b, c, d\}$, $B = \{f, b, d, g\}$

then $A - B = \{a, c\}$

Definition :- Complement المتممة

The complement of the set A is the set which contains elements not belong to A, denoted by A^c i.e

$$A^c = \{x ; x \notin A\} = U - A$$

for example, Let E = even numbers & $U = \mathbb{N}$

then $E^c = U - E = O \leftarrow$ is odd numbers

Proposition :- Let A, B are subsets of U, then

1- $A - A = \phi$, $A - \phi = A$, $\phi^c = U$, $U^c = \phi$

2- $A \cup A^c = U$, $A \cap A^c = \phi$

3- $(A^c)^c = A$

4- If $A \subseteq B$ then $B^c \subseteq A^c$

5- $(A-B) \cap (B-A) = \phi$

6- $A - B = A - (A \cap B)$

7- $A - B = A \cap B^c$

8- $A - B = B^c - A^c$

proof :- 3- Let $x \in (A^c)^c \longrightarrow x \notin A^c \longrightarrow x \in A$

$\therefore (A^c)^c \subseteq A$ --- ①

Let $x \in A \longrightarrow x \notin A^c \longrightarrow x \in (A^c)^c$

$\therefore A \subseteq (A^c)^c$ --- ②

from ① & ② we get $A = (A^c)^c$

4- Let $x \in B^c \longrightarrow x \notin B \xrightarrow[A \subseteq B]{\text{Since}} x \notin A$

$\rightarrow x \in A^c$

So that, $B^c \subseteq A^c$

6- Suppose that $(A-B)\cap(B-A)\neq\phi$

$\rightarrow$ there exists $x\in(A-B)\cap(B-A)$

$\rightarrow x\in(A-B)$ and $x\in(B-A)$

$\rightarrow (x\in A$ and $x\notin B)$ and $(x\in B$ and $x\notin A)$ C!

hence $(A-B)\cap(B-A)=\phi$

7- $A-B=A\cap B^c$

Let $x\in A-B \leftrightarrow x\in A \wedge x\notin B$

$\leftrightarrow x\in A \wedge x\in B^c$

$\leftrightarrow x\in A\cap B^c$

8- $A-B=B^c-A^c$

Let $x\in A-B \longleftrightarrow x\in A \wedge x\notin B$

$\longleftrightarrow x\notin A^c \wedge x\in B^c$

$\leftrightarrow x\in B^c \wedge x\notin A^c$

$\leftrightarrow x\in B^c-A^c$

Problem :- Let A, B are two sets proof that

1- $A \subseteq B$ iff $p(A) \subseteq p(B)$

2- $p(A \cap B) = p(A) \cap p(B)$

3- $p(A) \cup p(B) \subseteq p(A \cup B)$. Is the inverse is true ? why?

Proof :-

1- $\Rightarrow$) Let $c \in p(A)$, for every set c

$\rightarrow c \subseteq A \rightarrow c \subseteq B \rightarrow c \in p(B)$

so that , $p(A) \subseteq p(B)$

$\Leftarrow$) Let $x \in A$

$\rightarrow \{x\} \subseteq A \rightarrow \{x\} \in p(A) \rightarrow \{x\} \in p(B)$

$\rightarrow \{x\} \subseteq B \rightarrow x \in B$

$\therefore A \subseteq B$

2- Let $E \in p(A \cap B) \leftrightarrow E \subseteq A \cap B$

$\leftrightarrow E \subseteq A \wedge E \subseteq B$

$\leftrightarrow E \in p(A) \wedge E \in p(B)$

$\leftrightarrow E \in p(A) \cap p(B)$

Distributive Laws قوانين التوزيع

Let A, B and C are sets, then

1- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

2- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

proof:-

1- Let $x \in A \cap (B \cup C)$

$\rightarrow x \in A \wedge x \in (B \cup C)$

$\rightarrow x \in A \wedge (x \in B \vee x \in C)$

$\rightarrow (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$

$\rightarrow x \in (A \cap B) \vee x \in (A \cap C)$

$\rightarrow x \in (A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ --- (1)

Now,

Let $y \in (A \cap B) \cup (A \cap C)$

$\rightarrow y \in (A \cap B) \vee y \in (A \cap C)$

$\rightarrow (y \in A \wedge y \in B) \vee (y \in A \wedge y \in C)$

$\rightarrow y \in A \wedge (y \in B \vee y \in C)$

$\rightarrow y \in A \wedge y \in (B \cup C)$

$\rightarrow y \in A \cap (B \cup C)$

$\therefore (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ --- (2)

from (1) & (2) we get $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De Morgan's Laws

Let A, B be any two sets, then

1- $(A \cup B)^c = A^c \cap B^c$

2- $(A \cap B)^c = A^c \cup B^c$.

<u>proof:-</u> 1- Let $x \in (A \cup B)^c$

$\rightarrow x \notin A \cup B \rightarrow x \notin A \wedge x \notin B$

$\rightarrow x \in A^c \wedge x \in B^c \rightarrow x \in A^c \cap B^c$

$\therefore (A \cup B)^c \subseteq A^c \cap B^c$

Now, Let $y \in A^c \cap B^c$

$\rightarrow y \in A^c \wedge y \in B^c \longrightarrow y \notin A \wedge y \notin B$

$\rightarrow \cancel{y \notin A \cup B}$

$\rightarrow y \notin A \cup B \longrightarrow y \in (A \cup B)^c$

$\therefore A^c \cap B^c \subseteq (A \cup B)^c$

So that $(A \cup B)^c = A^c \cap B^c$

2- $x \in (A \cap B)^c$

$\leftrightarrow x \notin A \cap B$

$\leftrightarrow x \notin A \vee x \notin B$

$\leftrightarrow x \in A^c \vee x \in B^c$

$\leftrightarrow x \in A^c \cup B^c$

Generalization for union and the intersection of sets

It can be define the union and the intersection of sets for more than two sets finite or infinite.

Let $A_1, A_2, \cdots, A_n$ be sets then

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n = \{x ; x \in A_i \text{ for some } 1 \leq i \leq n\}$$

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n = \{x ; x \in A_i \text{ for all } 1 \leq i \leq n\}$$

for example,

$A_1 = \{2,3\}$, $A_2 = \{3,5,7\}$, $A_3 = \{1,2,3\}$.

$$\bigcup_{i=1}^{3} A_i = A_1 \cup A_2 \cup A_3 = \{1,2,3,5,7\}$$

$$\bigcap_{i=1}^{3} A_i = A_1 \cap A_2 \cap A_3 = \{3\}$$

More general if we have a family of sets $\{A_i\}_{i \in J}$ then

$$\bigcup_{i \in J} A_i = \{x ; x \in A_i \text{ for some } i \in J\}$$

$$\bigcap_{i \in J} A_i = \{x ; x \in A_i \text{ for all } i \in J\}$$

Generalization for De Morgan's Laws

Let $\{A_i\}_{i\in J}$ be a family of sets then

1- $\left(\bigcup_{i\in J} A_i\right)^c = \bigcap_{i\in J} A_i^c$

2- $\left(\bigcap_{i\in J} A_i\right)^c = \bigcup_{i\in J} A_i^c$

Proof:- 1. $x \in \left(\bigcup_{i\in J} A_i\right)^c \longleftrightarrow x \notin \bigcup_{i\in J} A_i$

$\longleftrightarrow x \notin A_i$, for all $i \in J$

$\longleftrightarrow x \in A_i^c$, for all $i \in J$

$\longleftrightarrow x \in \bigcap_{i\in J} A_i^c$

$\therefore \left(\bigcup_{i\in J} A_i\right)^c = \bigcap_{i\in J} A_i^c$

2- $x \in \left(\bigcap_{i\in J} A_i\right)^c \longleftrightarrow x \notin \bigcap_{i\in J} A_i$ for some $i \in J$

$\longleftrightarrow x \notin A_i$ for some $i \in J$

$\longleftrightarrow x \in A_i^c$ for some $i \in J$

$\longleftrightarrow x \in \bigcup_{i\in J} A_i^c$

$\therefore \left(\bigcap_{i\in J} A_i\right)^c = \bigcup_{i\in J} A_i^c$

Example:-

① prove that $(A\cup B)\cap(A\cup B^c) = A$.

Solution:- By Distributive Law (2)

$$(A\cup B)\cap(A\cup B^c) = A\cup(B\cap B^c)$$
$$= A\cup \phi$$
$$= A$$

② prove that $A\cup(A\cup B^c)^c = A\cup B$.

Solution:- By De Morgan's Law

$$A\cup(A\cup B^c)^c = A\cup(A^c\cap(B^c)^c)$$
$$= A\cup(A^c\cap B) \quad \text{by Distributive Law}$$
$$= (A\cup A^c)\cap(A\cup B)$$
$$= U\cap(A\cup B)$$
$$= A\cup B$$

③ prove that $B-(B-A) = A\cap B$

$$B-(B-A) = B\cap(B-A)^c \quad \text{by } A-B = A\cap B^c$$
$$= B\cap(B\cap A^c)^c$$
$$= B\cap(B^c\cup(A^c)^c) \quad \text{De Morgan's Law}$$
$$= B\cap(A\cup B^c)$$
$$= (B\cap A)\cup(B\cap B^c) \quad \text{Distributive Law}$$
$$= (B\cap A)\cup\phi$$
$$= B\cap A$$
$$= A\cap B$$

prove that

1- $(A \cap B) \cup (A - B) = A$

2- $A \cap (B - C) = (A \cap B) - (A \cap C)$

3- $A \cap (A^c \cup B) = A \cap B$

4- $A \cup (A \cup B^c)^c = A \cup B$

5- $A - (B \cap C) = (A - B) \cup (A - C)$

6- $(A \cup B) - C = (A - C) \cup (B - C)$

7- $A \cap (B - C) = (A \cap B) - C$

8- $A \subseteq B$ iff $A = B - (B - A)$

Statements العبارات

statements is a verbal sentence helpful will be denoted by the letters

$p, q, r, \ldots$

The fundamental property of a statement is that it is either true or false, but not both. The truthfulness or falsity of a statement is called its truth value.

Some statements are composite, that is, composed of substatements and various connectives which will be discussed subsequently.

Example:-

1- "Huba is a nice and a clever girl" is a composite statement with substatements "Huba is a nice" and "Huba is a clever girl".

2- "where are you going?" is not a statement since it is neither true nor false

3- "John is sick or old" is a composite statement with substatements "John is sick" or "John is old".

A fundamental property of a composite statement is that its truth value is completely determined by the truth value of each of its substatements and the way they are connected to form the composite statement.