Three-Dim Isotropic Harmonic oscillator

$m\ddot{x} = -kx$, $m\ddot{y} = -ky$, $m\ddot{z} = -kz$

$x = A_1 \cos\omega t + B_1 \sin\omega t$

$y = A_2 \cos\omega t + B_2 \sin\omega t$

$z = A_3 \cos\omega t + B_3 \sin\omega t$

The six constants of Integration are determined From the Intial position and velocity

$\therefore r = A \cos\omega t + B \sin\omega t$

So In the motion of Isotropic oscillation. the restoring force is Independent of direction of the displacement.

NoN Isotropic oscillator

If the magnitudes of components of the restoring force depend on the direction and displacement It's the case of non Isotropic case.

$m\ddot{x} = -k_1 x$, $m\ddot{y} = -k_2 y$,

$m\ddot{z} = -k_3 z$

So we have three different frequency

$w_1 = \sqrt{\frac{k_1}{m}}$, $w_2 = \sqrt{\frac{k_2}{m}}$ $w_3 = \sqrt{\frac{k_3}{m}}$

The equation of motion ②

$$x = A\cos(\omega_1 t + \alpha)$$
$$y = B\cos(\omega_2 t + \beta)$$
$$z = C\cos(\omega_3 t + \gamma)$$

{ The energy consideration }

$$V(x,y,z) = \frac{1}{2}K_1 x^2 + \frac{1}{2}K_2 y^2 + \frac{1}{2}K_3 z^2$$

$$F_x = -\frac{\partial u}{\partial x} = -K_1 x \qquad F_y = -\frac{\partial u}{\partial y} = -K_2 y \, ,$$

$$F_z = -\frac{\partial u}{\partial z} = -K_3 z$$

The General potential energy

$$V(x,y,z) = \frac{1}{2}K(x^2+y^2+z^2) = \frac{1}{2}Kr^2$$

Total energy $E = \frac{1}{2}mv^2 + \frac{1}{2}Kr^2$

<u>Ex</u> The particle of mass (m) moves in two dimension under the following potential energy function

$$V(r) = \frac{1}{2}K(x^2+4y^2)$$

Find the resulting motion, given the Intial condition at $t=0$: $x=a$, $y=0$, $\dot{x}=0$, $\dot{y}=v_0$.

<u>Sol:</u> This oscillator is Isotropic

The force $F = -\nabla u = -iKx - j4Ky$

$\frac{1}{2}(Kx^2+Ky^2)$

$F = ma$, $F = m\ddot{r}$

$\therefore m\ddot{r} = -Kx - j4Ky$

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(3)

The component of Differential equation of motion

$m\ddot{x} = -Kx \Rightarrow m\ddot{x} + Kx = 0$

$m\ddot{y} = -4Ky \Rightarrow m\ddot{y} + 4Ky = 0$

x-motion has angular frequency $\omega_x = \left(\frac{K}{m}\right)^{1/2}$

while the y motion has angular frequency

$$W_y = \left(\frac{4K}{m}\right)^{1/2} = 2\left(\frac{K}{m}\right)^{1/2} = 2W_x .$$

$W_y = 2W$

The equation of motion

$x = A_1 \cos\omega t + B_1 \sin\omega t$

$y = A_2 \cos 2\omega t + B_2 \sin 2\omega t$

$\dot{x} = -A_1 w \sin wt + B_1 w \cos wt$

$\dot{y} = -2A_2 w \sin 2wt + w 2 B_2 \cos 2wt$

at $t = 0$ $x = a$ $\therefore A_1 = a$, $A_2 = 0$

$B_1 w = 0$ $B_1 = 0$ $V_0 = 2 B_2 W \Rightarrow B_2 = \frac{V_0}{2W}$

The final equation

$x = a \cos\omega t$

$y = \frac{V_0}{2w} \sin 2\omega t$

Find the force for each of the following potential energy functions. (4)

(a) $V = cxyz + C$

$F_x = -\frac{\partial V}{\partial x}$, $F_y = -\frac{\partial V}{\partial y}$, $F_z = -\frac{\partial V}{\partial z}$

$F_x = -cyz + 0$, $F_y = -cxz$, $F_z = -cxy$

$F = -cyz\,i - cxz\,j - cxy\,k$

(b) $V = \alpha x^2 + \beta y^2 + \gamma z^2 + C$

$F_x = -\frac{\partial V}{\partial x} = -2x\alpha + 0 + 0 + 0$

$F_y = -\frac{\partial V}{\partial y} = -0 - 2\beta y + 0 + 0$

$F_z = -\frac{\partial V}{\partial z} = -2\gamma z + 0$

$\therefore F = -2\alpha x\,i - 2\beta y\,j - 2\gamma z\,k$

(c) $V = c\,e^{-(\alpha x - \beta y - \gamma z)}$ H.W

Ex
By finding the curl. determine which of the following forces are conservative.

(a) $F = ix + jy + kz$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = \left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right)i + \left(\frac{\partial x}{\partial z} - \frac{\partial z}{\partial x}\right)j + \left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right) = 0$$

(b) $F = iy + jx + kx^2$

(c) $F = iy + jx + kz^3$

Ex Find the value of constant (c) such that the force is conservative

$$F = ixy + jcx^2 + kz^3$$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^3 \end{vmatrix} =$$

$$= \left(\frac{\partial z^3}{\partial y} - \frac{\partial cx^2}{\partial z}\right)i + \left(\frac{\partial xy}{\partial z} - \frac{\partial z^3}{\partial x}\right)j +$$

$$\left(\frac{\partial cx^2}{\partial x} - \frac{\partial xy}{\partial y}\right)k$$

$$= (0-0)i + (0-0)j + (2cx - x)k$$

$$0 = 2(cx - x)k$$

$$2cx = x \Rightarrow \boxed{c = \frac{1}{2}}$$