General motion of Particles in 3D ①

The force $\vec{F} = \frac{d\vec{P}}{dt}$ in which $\vec{P} = m\vec{V}$ is Linear momentum.

F: is known as explicit function of time

Linear moment $\vec{P}$ can be found by finding The Impulse.

$$\int_0^t \bar{F}(t) \cdot dt = \int_0^t \vec{P}(t) \Rightarrow \int_0^t F(t) \cdot dt = P(t) - P(0) = mV(t) - mV(0)$$

The special case of zero force, the moment and velocity are constant

The angular momentum: $\int F \cdot dt = \int dP$

Since $F = \frac{d\vec{P}}{dt} \Rightarrow$ نضرب ب $\vec{r}$

$$\bar{r} \times \bar{F} = \vec{r} \times \frac{d\bar{P}}{dt} \Rightarrow \frac{d}{dt}(\bar{r} \times \bar{P}) = \frac{dr}{dt} \times P + r \times \frac{dP}{dt}$$

$$\bar{r} \times \bar{F} = V \times P + r \times \frac{dP}{dt}, \quad \text{But } V \times P = V \times mV$$

$$V \times V = 0$$

$$\therefore \bar{r} \times \bar{F} = r \times \frac{dP}{dt},$$

the quantity $\bar{r} \times \bar{P}$ is called the angular moment of particle about the origion.

The work principle.

$$\text{The } F = \frac{dP}{dt} \qquad * (V)$$

$$F \cdot V = \frac{dP}{dt} \cdot V = \frac{d}{dt}(m \cdot V) \cdot V$$

$$F \cdot V = m\frac{d}{dt}(V \cdot V) \Rightarrow m\left(\frac{d}{dt}(V^2)\right) = \underline{2mV\frac{dV}{dt}}$$

$$T = \frac{1}{2}mV^2 \Rightarrow \frac{1}{2}m\,V.V \Rightarrow \frac{d}{dt}\left(\frac{1}{2}mV^2\right)$$

$$= \frac{\cancel{2}}{\cancel{2}}mV\frac{dV}{dt} \Rightarrow mV\frac{dV}{dt}$$

$$\therefore \quad F \cdot V = \frac{dT}{dt}$$

The potential Energy Function in three Dimension motion. The Del-operator.

If aparticle moves under the action of aconservative force F.

$$F = -\frac{du}{dr} \Rightarrow F \cdot dr = -du(r)$$

$$E = \frac{1}{2}mv^2 + U(r) \qquad \text{قانون حفظ الطاقة}$$

$$F \cdot dr = F_x \cdot dx + F_y \cdot dy + F_z \cdot dz$$

$$\therefore F_x = -\frac{\partial u}{\partial x}, \quad F_y = -\frac{\partial u}{\partial y}, \quad F_z = -\frac{\partial u}{\partial z}$$

If the force field is conservative, then the components of the force are given by negative partial derivatives of the potential energy function.

The force can be expressed as

$$F = -i\frac{\partial u}{\partial x} - j\frac{\partial u}{\partial y} - k\frac{\partial u}{\partial z}$$

$\therefore F = -\nabla \cdot u$

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

∇ = Del operator