Taylor Series

Theorem (Taylor's Theorem)

Suppose that a function f is analytic throughout an open disk $|z-z_0| < R_0$. Then, at each point z in that disk, $f(z)$ has the series representation

$$f(z) = \sum_{n=0}^{\infty} a_n (z-z_0)^n$$

where $a_n = \frac{f^{(n)}(z_0)}{n!}$. That is, the power series $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ converges to $f(z)$ in the disk $|z-z_0| < R_0$.

Subject
Date: / / الموافق
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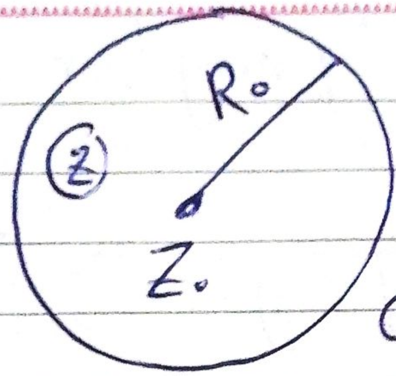

$$|z - z_0| < R$$

$$a_n = \frac{f^{(n)}(z_0)}{n!}$$

<u>Remark</u>: According to Taylor's Theorem, the Taylor series of function f about z_0 converges to $f(z)$ within the circle about z_0 whose radius is the distance from z_0 to the nearest point z_1 where f fails to be analytic.

<u>السؤال</u> : ينص النقطة الا التي قد تكون فيها الدالة التحليلية للتحويل

الوحيد والاجابة $|z - z_0| < R$.

Subject   موضوع الدرس
Date: / / الموافق   / / التاريخ

Example: show that the Maclaurian series expansion of $f(z) = e^{z}$ is $e^{z} = \sum_{n=0}^{\infty} \frac{z^n}{n!}$, $|z| < \infty$

Sol: $z_0 = 0$, e^{z} is the entire function $\Rightarrow R_0 = \infty$

الفرض عند من الصفر لحد ما تقف عند اول
نقطة تكون فيها الدالة غير تحليلية.

we want to write

$$f(z) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} z^n$$

المطلوب حساب المعاملات.

$f(z) = e^{z} \Rightarrow f(0) = 1$

$f'(z) = e^{z} \Rightarrow f'(0) = 1$

$\vdots$

$f^{(n)}(0) = 1 \quad \forall n$

Subject موضوع الدرس
Date: / / الموافق / / التاريخ

$$f(z) = e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

هذه السلسلة تكون متقاربة

$\forall\ |z| < \infty$ or $z \in \mathbb{C}$.

Example: Find The Taylor series of $f(z) = \cos z$ about $z_0 = 0$

Sol: $f(z) = \cos z$, $z_0 = 0$

$f(z)$ is entire

$$\cos z = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} z^n, \quad |z| < \infty$$

Subject
Date: / / الموافق / / التاريخ
موضوع الدرس

$f(z) = \cos z \Rightarrow f(0) = 1$

$f'(z) = -\sin z \Rightarrow f'(0) = 0$

$f''(z) = -\cos z \Rightarrow f''(0) = -1$

$f'''(z) = \sin z \Rightarrow f'''(0) = 0$

$f^{(2m+1)}(0) = 0$, when $n = 2m+1$ odd
$m = 0, 1, 2, \ldots$

$f^{(2m)}(0) = (-1)^m$, when $n = 2m$
$m = 0, 1, 2, \ldots$

$$\Rightarrow \cos z = \sum_{m=0}^{\infty} \cancel{\frac{f^{(2m+1)}(0)}{(2m+1)!} z^{2m+1}} \quad 0$$

$$+ \sum \frac{f^{(2m)}(0)}{(2m)!} z^{2m}$$

$$\Rightarrow \cos z = \sum \frac{(-1)^m}{(2m)!} z^{2m}, \quad z \in \mathbb{C}.$$