⑤ Exponential Form.

let r & θ be the polar Coordinate of the point (x, y) that corresponds to the non zero Complex number

$Z = x + iy$ since $x = r\cos\theta$
$y = r\sin\theta$

Z can be written in polar form as $Z = r(\cos\theta + i\sin\theta)$

Remark: ① If $Z = 0$ then θ is not defined and hence we can not write $Z = 0$ in polar form.

② In Complex analysis we assume $r \geq 0$ and $r = |Z| = \sqrt{x^2 + y^2}$.

③ The real number θ represent The angle, measured in radians that Z makes with the positive real axis.

(4) θ has an infinite number valus, both positive and negative, that differ by integer ~~multipl~~ multiplucatian 2π.

(5) The value of θ can be determined by specifying the quadrant containing $z = x + iy$ and by using one of the following three equation:

$$\tan\theta = \frac{y}{x}, \quad \cos\theta = \frac{x}{|z|}, \quad \sin\theta = \frac{y}{|z|}$$

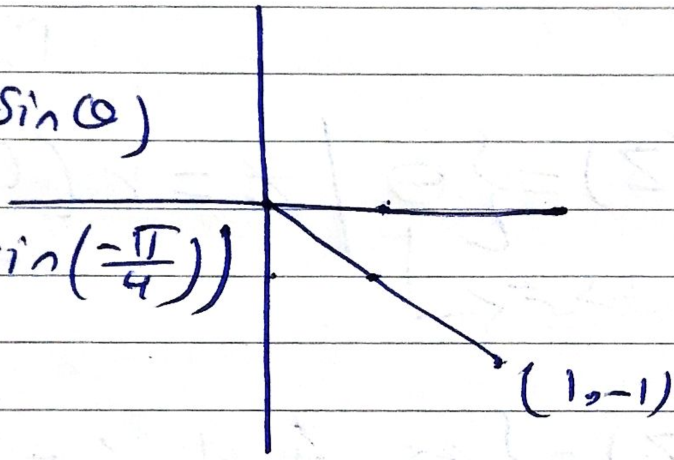
Example: write $z = 1 - i$ in polar form.

Sol: $r = |z| = \sqrt{1+1} = \sqrt{2}$

$\tan\theta = \frac{y}{x} = \frac{-1}{1} = -1 \Rightarrow \theta = \frac{-\pi}{4}$

$\therefore z = r(\cos\theta + i\sin\theta)$

$= \sqrt{2}\left(\cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)\right)$



Def: let z be a nonzero complex number

① Each values of θ s.t.

$z = x + iy = r(\cos\theta + i\sin\theta)$

is called an argument of z.

② The set of all argument of z is denoted by $\arg(z)$

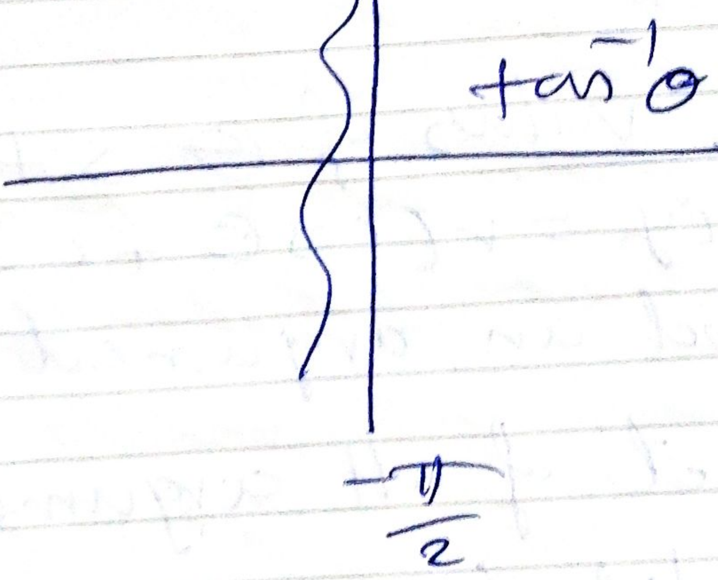
$Arg(Z) = \pi + \tan^{-1}(\frac{y}{x})$

③ The principle value of arg Z is the unique value θ s.t $-\pi < \theta \leq \pi$, and is denoted by Arg Z. (احد القيم المحصورة بين $-\pi$ و π من خلالها نجد بقية القيم.)

$$\arg(Z) = \{\theta \mid Z = r(\cos\theta + i\sin\theta)\}$$

مجموعة من الاعداد

$$\arg(Z) = \{\text{Arg } Z + 2n\pi \mid n \in \mathbb{Z}\}$$



Example ①: Find $Arg(5)$, $Arg(-5)$, $Arg(2i)$, $Arg(-3i)$.

Sol: $Arg(5) = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(0) = 0$

$Arg(-5) = \pi + \underbrace{\tan^{-1}(0)}_{=0} = \pi$

$Arg(2i) = \frac{\pi}{2}$ & $Arg(-3i) = -\frac{\pi}{2}$

Example ②: Find $arg(Z)$ if $Z = 1+i$

Sol: $arg(Z) = Arg(Z) + 2n\pi$, $n \in \mathbb{Z}$

$Arg(Z) = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

$\therefore$ $arg(Z) = \frac{\pi}{4} + 2n\pi$, $n \in \mathbb{Z}$.

الموضوع :　　　　　　　　التاريخ　/　/ ٢٠٢

Example ③: Find the principle argument $(Arg(Z))$ to $Z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$

Sol:

$$Z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}} \cdot \frac{1+i\sqrt{3}}{1+i\sqrt{3}}$$

$$Z = \frac{1-3+i2\sqrt{3}}{4} = \frac{-2+i2\sqrt{3}}{4}$$

$$= \frac{-1+i\sqrt{3}}{2}$$

$$Arg(Z) = \pi + \tan^{-1}\left(\frac{\sqrt{3}/2}{-1/2}\right)$$

$$Arg(Z) = \pi + \tan^{-1}(-\sqrt{3})$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}.$$

ملاحظة: اذا كان $Z=0$ فان θ غير معرفة لان $\theta = \tan^{-1}\frac{y}{x} \Leftarrow \theta = \frac{0}{0}$ وهذا مقدار غير معروف وعليه لا يمكن تمثيل $Z=0$ بالاحداثيات القطبية.