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Cite as: AIP Conference Proceedings **2414**, 040011 (2023); https://doi.org/10.1063/5.0115406
Published Online: 13 February 2023

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Fibrewise Slightly Topological Spaces

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Abstract. The primary objective of this paper is to introduce a new concept of fibrewise topological spaces over $\mathfrak{B}$ is said to be fibrewise slightly topological spaces over $\mathfrak{B}$. Also, we introduce the concepts of fibrewise slightly closed, fibrewise slightly open, fibrewise slightly locally sliceable and fibrewise slightly locally sectionable topological spaces over $\mathfrak{B}$. In addition, we state and prove several propositions related to these concepts.

2020 MSC. 54C08,54C10,55R70

Keywords. Fibrewise slightly topological spaces, fibrewise slightly closed topological spaces, fibrewise slightly open topological spaces, fibrewise slightly locally sliceable and fibrewise locally slightly sectionable topological spaces.

INTRODUCTION

To start the classification in the arrangement of fibrewise (briefly, $\mathcal{F}.\mathcal{W}.$) set, called the base set, which know by $\mathfrak{B}$. Then a $\mathcal{F}.\mathcal{W}.$ set over $\mathfrak{B}$ containing a set $\mathcal{H}$ with a function $\wp : \mathcal{H} \to \mathfrak{B}$ is called the projection function for each point $\mathfrak{b} \in \mathfrak{B}$, the subset $\mathcal{H}_{\mathfrak{b}} = \wp^{-1}(\mathfrak{b})$ of $\mathcal{H}$ namely the fibre over $\mathfrak{b}$. The fibers could be null because we don't need them $\wp$ to be subjective, in addition for any of the subset $\mathfrak{B}^*$ of . We regard $\mathcal{H}_{\mathfrak{B}^*} = \wp^{-1}(\mathfrak{B}^*)$ as a $\mathcal{F}.\mathcal{W}.$ set over $\mathfrak{B}^*$ for the projection function Specified by $\wp$. The concept of fibrewise set over a given set was introduced by James in [3], [4]. We built on some of the result in [1, 8, 9]. For other notations or notions which are not mentioned here we go behind closely I.M.James [3], R.Engelking [7] , and N. Bourbaki [6].

Definition 1[3]. Assume that $\mathcal{H}$ & $\mathcal{D}$ are $\mathcal{F}.\mathcal{W}.$ sets over $\mathfrak{B}$, for projections $\wp_{\mathcal{H}} : \mathcal{H} \to \mathfrak{B}$ & $\wp_{\mathcal{D}} : \mathcal{D} \to \mathfrak{B}$. A function $\eta : \mathcal{H} \to \mathcal{D}$ is called fibrewise if $\wp_{\mathcal{D}} \circ \eta = \wp_{\mathcal{H}}$. i.e., if $\eta(\mathcal{H}_{\mathfrak{b}}) \subset \mathcal{D}_{\mathfrak{b}}$ for each $\mathfrak{b} \in \mathfrak{B}$.

Note that a $\mathcal{F}.\mathcal{W}.$ function $\eta : (\mathcal{H}, \sigma) \longrightarrow (\mathcal{D}, \varrho)$ over $\mathfrak{B}$ determines by restriction, a $\mathcal{F}.\mathcal{W}.$ function $\eta_{\mathfrak{B}^*} : \mathcal{H}_{\mathfrak{B}^*} \to \mathcal{D}_{\mathfrak{B}^*}$ over $\mathfrak{B}^*$ $\forall$ $\mathfrak{B}^* \subseteq \mathfrak{B}$.

Definition 2[3]. Assume that $(\mathfrak{B}, \Gamma)$ is a topological space. A $\mathcal{F}.\mathcal{W}.$ topology on a $\mathcal{F}.\mathcal{W}.$ set $\mathcal{H}$ over $\mathfrak{B}$. Thus any topology on $\mathcal{H}$ over the projection function $\wp$ is continuous.

The Sixth Local Scientific Conference-The Third Scientific International
AIP Conf. Proc. 2414, 040011-1–040011-11; https://doi.org/10.1063/5.0115406
Published by AIP Publishing. 978-0-7354-4431-7/$30.00

Definition 3[3]. A $\mathcal{F}.\mathcal{W}.$ function $\eta : (\mathcal{H}, \sigma) \longrightarrow (\mathcal{D}, \varrho)$ where $\mathcal{H}$ & $\mathcal{D}$ are $\mathcal{F}.\mathcal{W}.\mathcal{T}$ spaces over $\mathfrak{B}$ is said to be:

i. continuous if for each point $\hbar \in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, the inverse image of each open set of $\eta(\hbar)$ is an open set of $\hbar$.

ii. open if for each point $\hbar \in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, the image of each open set of $\hbar$ is an open set of $\eta(\hbar)$.

Definition 4[3]. A $\mathcal{F}.\mathcal{W}.$ topological space $(\mathcal{H}, \sigma)$ over $(\mathfrak{B}, \Gamma)$ is named $\mathcal{F}.\mathcal{W}.$ open (resp., closed), if the projection $\wp$ is open (resp., closed).

Definition 5[5]. A function $\eta : (\mathcal{H}, \sigma) \longrightarrow (\mathcal{D}, \varrho)$ is slightly continuous if $\eta^{-1}(\mathcal{V})$ is open set in $\mathcal{H}$ for each clopen set $\mathcal{V}$ of $\mathcal{D}$.

Definition 6[7]. Suppose that we are given a topological space $\mathcal{H}$, a family $\{\eta_r\}_{r\in R}$ of continuous functions, and a family $\{ \mathcal{D}_r \}_{r\in R}$ of topological spaces where the function $\eta_r : \mathcal{H} \rightarrow \mathcal{D}_r$ that transfers $\hbar \in \mathcal{H}$ to the point $\{\eta_r (\hbar)\} \in \prod_{r\in R} \eta_r$ is continuous, it is said to be the diagonal of the functions $\{\eta_r\}_{r\in R}$ and is denoted by $\Delta_{r\in R}\, \eta_r$ or $\eta_1 \Delta\, \eta_2 \Delta \ldots \Delta\, \eta_k$ if $R = \{1, 2, \ldots, k\}$.

Definition 7[7]. For every topological space $\mathcal{H}^*$ and any subspace $\mathcal{H}$ of $\mathcal{H}^*$, the function $i_{\mathcal{H}} : \mathcal{H} \rightarrow \mathcal{H}^*$ define by $i_{\mathcal{H}}(\hbar) = \hbar$ is called embedding of the subspace $\mathcal{H}$ in the space $\mathcal{H}^*$. Observe that $i_{\mathcal{H}}$ is continuous, since $i_{\mathcal{H}}^{-1}(\mathcal{V}) = \mathcal{H} \cap \mathcal{V}$, where $\mathcal{V}$ is open set in $\mathcal{H}^*$. The embedding $i_{\mathcal{H}}$ is closed (resp., open) iff the subspace $\mathcal{H}$ is closed (resp., open).

Definition 8[7]. The graph function $\Phi : (\mathcal{H}, \sigma) \longrightarrow (\mathcal{D}, \varrho)$, we imply the subset of the Cartesian product $\mathcal{H} \times \mathcal{D}$ defined by $G(\Phi) = \{ (\hbar, d) \in \mathcal{H} \times \mathcal{D} ;\ d = \Phi(\hbar)\}$.
The fibrewise graph of a fibrewise function Φ of a fibrewise space $\mathcal{H}$ over $\mathfrak{B}$, we imply the subset of the Cartesian product $\mathcal{H} \times_{\mathfrak{B}} \mathcal{D}$ defined by $G_{\mathfrak{B}}(\Phi) = \{ (\hbar, d) \in \mathcal{H} \times_{\mathfrak{B}} \mathcal{D} ;\ d = \Phi(\hbar) \}$.

Definition 9[2]. The $\mathcal{F}.\mathcal{W}.$ function $\eta : (\mathcal{H}, \sigma) \longrightarrow (\mathcal{D}, \varrho)$ where $(\mathcal{H}, \sigma)$ and $(\mathcal{D}, \varrho)$ are $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ spaces over $(\mathfrak{B}, \Gamma)$ is said to be totally continuous if $\forall\, \hbar \in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, the $\eta^{-1}(\mathcal{V})$ is clopen in $\mathcal{H}$ of each open set $\mathcal{V}$ in $\mathcal{D}$.

Fibrewise Slightly Topological Space

In this segment we establish $\mathcal{F}.\mathcal{W}.$ slightly topological spaces (briefly, $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}$ space). Many topological characteristics on this space also definitions and discussed.

Definition 10. Assume that $(\mathfrak{B}, \Gamma)$ be a topological space. A $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ topology on a $\mathcal{F}.\mathcal{W}.$ set $\mathcal{H}$ over $\mathfrak{B}$ means any topology on $\mathcal{H}$ for which the projection $\wp$ is slightly continuous.

Example 1. Let $\mathfrak{B} = \{a, b, c\}$, $\Gamma = \{ \mathfrak{B}, \emptyset, \{a\}, \{b, c\}\}$, $\mathcal{H} = \{e, d, f\}$ and $\sigma = \{\mathcal{H}, \emptyset, \{e\}, \{d\}, \{e, f\}, \{e, d\} \}$. Let be $\wp : \mathcal{H} \rightarrow \mathfrak{B}$; defined as $\wp(c) = e$, $\wp(a) = d$, $\wp(b) = f$. Then (H, σ) is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ topology on $(\mathfrak{B}, \Gamma)$.

Remarks 1.

i. In $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ topology, we operate over $(\mathfrak{B}, \Gamma)$. The concept changes to that of normal topology, if $\mathfrak{B}$ is a point space.

ii. The $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ spaces over $\mathfrak{B}$ is Simply a topological space $(\mathcal{H}, \sigma)$ for the slightly continuous projection $\wp : (\mathcal{H}, \sigma) \longrightarrow (\mathfrak{B}, \Gamma)$.

iii. The $\mathcal{F}.\mathcal{W}.$ is said to be indiscrete $\mathcal{S}.\mathcal{T}.$ space, if the smallest such slightly topology is obtained by p.

iv. The $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space over $(\mathfrak{B}, \Gamma)$ is known to be a $\mathcal{F}.\mathcal{W}.$ set over $\mathfrak{B}$ for the $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ topology.

v. We regard the $\mathcal{S}.\mathcal{T}.$ product $\mathfrak{B} \times \Sigma$, for each topological space σ as a $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ spaces over $\mathfrak{B}$ by the primary projection.

Definition 11. The $\mathcal{F}.\mathcal{W}.$ function $\eta : (\mathcal{H},\sigma) \longrightarrow (\mathcal{D},\varrho)$ where $(\mathcal{H},\sigma)$ and $(\mathcal{D},\varrho)$ are $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ spaces over $(\mathfrak{B},\Gamma)$ is said to be to be :

i. Slightly continuous if $\forall\, \hbar \in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, the $\eta^{-1}(\mathcal{V})$ is open in $\mathcal{H}$ of each clopen set $\mathcal{V}$ in $\mathcal{D}$.

ii. Slightly open if $\forall\, \hbar \in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, the $\eta(\mathcal{V})$ is open set in $\mathcal{H}$ is clopen set of each clopen set $\mathcal{V}$ in $\mathcal{D}$

iii. Slightly closed if $\forall\, \hbar \in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, the $\eta(\mathcal{V})$ is closed set in $\mathcal{H}$ is clopen set of each clopen set $\mathcal{V}$ in $\mathcal{D}$.

Example 2. Let $\mathcal{H} = \{a, b, c\}$, $\sigma = \{\mathcal{H}, \emptyset, \{a\}, \{b, c\}\}$. Let $\mathcal{D} = \{e, f, g\}$, $\varrho = \{\mathcal{D}, \emptyset , \{e\} , \{f, g\}\}$. Let $\mathfrak{B} = \{1,2,3\}$, $\Gamma = \{\mathfrak{B}, \emptyset, \{1\} , \{2 , 3\} \}$. Define $\wp : \mathcal{H} \to \mathfrak{B}$ s.t $\wp(a) = 1$, $\wp(b) = 2$, $\wp(c) = 3$. Define $\wp : \mathcal{D} \to B$ s.t $\wp(e) = 1$, $\wp(f) = 2$, $\wp(g) = 3$. Let $\eta : \mathcal{H} \to \mathcal{D}$ s.t $\eta(a) = e$, $\eta(b) = f$, $\eta(c) = g$. Then η is slightly continuous , slightly open and slightly closed.

Example 3. Let $\mathcal{H} = \{\hbar_1, \hbar_2\}$, $\sigma = \{\mathcal{H}, \emptyset, \{\hbar_1\} , \{\hbar_2\}\}$. Let $\mathcal{D} = \{d_1, d_2\}$, $\varrho = \{\mathcal{D}, \emptyset\}$. Let $\mathfrak{B} = \{\mathfrak{b}_1, \mathfrak{b}_2\}$, $\Gamma = \{\mathfrak{B},\ \emptyset\ \}$. Define $p_H : \mathcal{H} \to \mathfrak{B}$ s.t $\wp(\hbar_1) = \mathfrak{b}_1$, $\wp(\hbar_2) = \mathfrak{b}_2$. Define $\wp : \mathcal{D} \to B$ s.t $\wp(d_2) = \mathfrak{b}_1$, $\wp(d_1) = \mathfrak{b}_2$. Let $\eta : (\mathcal{H} ,\ \sigma) \longrightarrow (\mathcal{D}, \Gamma)$ s.t $\eta(\hbar_1) = d_1$, $\eta(\hbar_2) = d_2$. Then η is slightly continuous , however not slightly open and not slightly closed.

Remark 2. Each $\mathcal{F}.\mathcal{W}.\mathcal{S}.\ \mathcal{T}.$ space is $\mathcal{F}.\mathcal{W}.\ \mathcal{T}.$ space , however the reverse does not need to be correct. .

Example 4. Let $\mathcal{H} = \{1,2, 3\}$, $\sigma = \{\mathcal{H}, \emptyset , \{1\}\}$, Let $\mathfrak{B} = \{r, q\}$, $\varrho = \{\mathfrak{B}, \emptyset, \{r\}\}$. The function define $\wp : \mathcal{H} \to \mathfrak{B}$ s.t $(1) = q$, $\wp(2)$, $\wp(3) = r$. Then $\wp$ is $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ space , but is not $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space. . Since $\{r\}$ is clopen in $\mathfrak{B}$, but $\{3\}$ is not open in $\mathcal{H}$. Then $\wp$ is not slightly continuous.

If $\eta : (\mathcal{H}, \sigma) \longrightarrow (\mathcal{D}, \varrho)$ where $(\mathcal{H}, \sigma)$ and $(\mathcal{D}, \varrho)$ are $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ spaces over $(\mathfrak{B},\Gamma)$. We can give the innovator topology, in the normal meaning and this is certainly an F.W. topology. We may point out to it, so like the innovator $\mathcal{F}.\mathcal{W}.$ topology and the next recommendations to remember.

Proposition 1. Assume that $\eta : (\mathcal{H},\sigma) \to (\mathcal{D},\varrho)$ be a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ continuous and slightly open function where $(\mathcal{D},\varrho)$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space over $(\mathfrak{B},\Gamma)$ and $(\mathcal{H},\sigma)$ has an induced $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space. Then for every $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space $(\mathcal{W},\delta)$ a $\mathcal{F}.\mathcal{W}.$ function $\vartheta : (\mathcal{W},\delta) \to (\mathcal{H},\sigma)$ is continuous iff the composition $\eta \circ \vartheta : (\mathcal{W},\delta) \to (\mathcal{D},\varrho)$ is slightly continuous.

Proof. $(\Rightarrow)$ Suppose that ϑ is continuous. Let $w \in W_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$ and let $\mathcal{V}$ be clopen set of $(\eta\ o\ \vartheta)(w) = d \in \mathcal{D}_{\mathfrak{b}}$ in $\mathcal{D}$. Since η is slightly continuous, then $\eta^{-1}(\mathcal{V})$ is an open set containing $\vartheta(w) = \hbar \in \mathcal{H}_{\mathfrak{b}}$ in $\mathcal{H}$.Since ϑ is continuous, then $\vartheta^{-1}(\eta^{-1}(\mathcal{V}))$ is an open set containing $w \in \mathcal{W}_{\mathfrak{b}}$ in $\mathcal{W}$ and $\vartheta^{-1}(\eta^{-1}(\mathcal{V})) = (\eta\ o\ \vartheta)^{-1}(\mathcal{V})$ is a $\delta-$ open set containing $w \in \mathcal{W}_{\mathfrak{b}}$ in $\mathcal{W}$. Then $\eta\ o\ \vartheta$ is slightly continuous.

$(\Leftarrow)$ Suppose that $\eta\ o\ \vartheta$ is slightly continuous. Let $w \in \mathcal{W}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$ and $\mathcal{U}$ be open set of $\vartheta(w) = \hbar \in \mathcal{H}_{\mathfrak{b}}$ in $\mathcal{H}$. Since η is slightly open then , $\eta(\mathcal{U})$ is an clopen set containing $\eta(\hbar) = d \in \mathcal{D}_{\mathfrak{b}}$ in $\mathcal{D}$. Since $\eta o \vartheta$ is slightly continuous, then $(\eta\ o\ \vartheta)^{-1}(\ \eta(\mathcal{U})) = \vartheta^{-1}(\mathcal{U})$ is $\delta -$ open set containing $w \in \mathcal{W}_{\mathfrak{b}}$ in $\mathcal{W}$. Then ϑ is continuous.

Proposition 2. Let $\eta : (\mathcal{H},\sigma) \to (\mathcal{D},\varrho)$ be an $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ continuous and slightly open function , where, (D,ϱ) a $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space over $(\mathfrak{B},\Gamma)$ and $(\mathcal{H},\sigma)$ has an induced $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ topology. Then for each $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ topological space $(\mathcal{W},\delta)$, the surjective $\mathcal{F}.\mathcal{W}.$ function $\vartheta : (\mathcal{W},\delta) \to (\mathcal{H},\sigma)$ is open iff the composition $\eta \circ \vartheta : (\mathcal{W},\delta) \to (\mathcal{D},\varrho)$ is an slightly open.

Proof. $(\Rightarrow)$ Suppose that ϑ slightly open, let $w \in \mathcal{W}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$ and let $\mathcal{V}$ be an δ-open set of w in $\mathcal{W}$. By of ϑ is open then $\vartheta(\mathcal{V})$ is open set containing $\vartheta(w) = \hbar \in \mathcal{H}_{\mathfrak{b}}$ in $\mathcal{H}$. Since η is slightly open, then $\eta(\vartheta(\mathcal{V}))$ is clopen set containing $\eta(\hbar) = d \in \mathcal{D}_{\mathfrak{b}}$ in $\mathcal{D}$ and $\eta(\vartheta(\mathcal{V})) = (\eta o \vartheta)(\mathcal{V}) \Rightarrow \eta o \vartheta$ is an slightly open.

(⇐) Suppose that $\eta\ o\ \vartheta$ is slightly open . Let $w \in W_b$; $\mathfrak{b} \in \mathfrak{B}$. Let $\mathcal{V}$ be an δ-open set of w in $\mathcal{W}$. By $\eta\ o\ \vartheta$ is slightly open, then $(\eta o \vartheta)(\mathcal{V})$ is clopen set containing $(\eta\ o\ \vartheta)(w) = d \in \mathcal{D}_{\mathfrak{b}}$. Since η is slightly continuous, then $\eta^{-1}((\eta\ o\ \vartheta)\ (\mathcal{V}))$ is open set of $\vartheta(w) = \hbar \in \mathcal{H}_{\mathfrak{b}}$ in $\mathcal{H}$, but $\eta^{-1}\big((\eta\ o\ \vartheta)(\mathcal{V})\big) = \vartheta(V) \Rightarrow \vartheta$ is an open.

Fibrewise Slightly Closed and Fibrewise Slightly Open Topologacal Spaces

In this segment we establish $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed and $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open topological spaces over $\mathfrak{B}$. So many topological characteristics on this space also definitions and discussed.

Definition 12. A $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ space $(\mathcal{H},\sigma)$ over $(\mathfrak{B}, \Gamma)$ is called $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed if the projection $\wp$ is slightly closed.

Example 5. Let $\mathcal{H} = \{a, b, c\}$, $\sigma = \{\mathcal{H}, \emptyset , \{a\} , \{b, c\}\}$. Let $\mathfrak{B} = \{1,2,3\}$, $\Gamma = \{\mathfrak{B}, \emptyset, \{1\}, \{2, 3\}\}$. Define $\wp : \mathcal{H} \to \mathfrak{B}$ s.t $\wp(a) = 1$, $\wp(b) = 2$, $\wp(c) = 3$. Then $\wp$ is slightly closed . Therefore $(\mathcal{H}, \sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed

Remark 3. Every $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed is $\mathcal{F}.\mathcal{W}.$ closed , however the reverse does not need to be correct.

Example 6. Let $\mathcal{H} = \{1 , 2 , 3\}$, $\sigma = \{\mathcal{H} ,\ \emptyset , \{1\}\}$. Let $\mathfrak{B} = \{a, b, c\}$, $\Gamma = \{\mathfrak{B}, \emptyset , \{a, b\}\}$. Define $\wp : \mathcal{H} \to \mathfrak{B}$ s.t $\wp(\hbar) = c\ \forall\ \hbar \in \mathcal{H}$. Then $\wp$ is closed, therefore $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.$ closed but is not $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed , since $\wp(\{2 , 3\})$ is closed in $\mathcal{H}$, but $\{c\}$ is not clopen in $\mathfrak{B}$.

Proposition 3. Let $\eta : \mathcal{H} \to \mathcal{D}$ be a closed $\mathcal{F}.\mathcal{W}.$ function where $(\mathcal{H},\sigma)$ & $(\mathcal{D} , \varrho)$ are $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ spaces over $(\mathfrak{B}, \Gamma)$.

i. If $\mathcal{D}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed. So that, $\mathcal{H}$ it's a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed.

ii. If $\mathcal{D}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed. So that, $\mathcal{H}$ it's a $\mathcal{F}.\mathcal{W}.$ closed.

Proof. The proofs for these two cases are same, so we will only prove the truth.

i. Since $\mathcal{D}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed, we have $\wp_{\mathcal{D}} : \mathcal{D} \to \mathfrak{B}$ is slightly closed. To prove that $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed . i.e., To prove that $\wp_{\mathcal{H}} : \mathcal{H} \to \mathfrak{B}$ is slightly closed. let $\in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, and $\mathbb{F}$ be σ closed set of h. Since η is closed $\mathcal{F}.\mathcal{W}.$ function, then $\eta(\mathbb{F})$ is a closed set of $\eta(\hbar) = d \in \mathcal{D}_{\mathfrak{b}}$ in $\mathcal{D}$. Since $\wp_{\mathcal{D}}$ is slightly closed, then $\wp_{\mathcal{D}}(\eta(\mathbb{F}))$ is clopen set at $(\mathfrak{B}, \Gamma)$. But , $(\wp_{\mathcal{D}}\ o\ \eta)(\mathbb{F}) = \wp_{\mathcal{H}}(\mathbb{F})$ is Γ – clopen set of $\mathbb{F}$. Thus , $\wp_{\mathcal{H}}$ is slightly closed and $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed .

Proposition 4. Let $\eta: \mathcal{H} \longrightarrow \mathcal{D}$ be a surjection $\mathcal{F}.\mathcal{W}.$ continuous where $(\mathcal{H},\sigma)$ and $(\mathcal{D},\varrho)$ are $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ spaces over $(\mathfrak{B}, \Gamma)$.

i. If $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed. So that, $\mathcal{D}$ it's a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed.

ii. If $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed. So that, $\mathcal{D}$ it's a $\mathcal{F}.\mathcal{W}.$ closed.

Proof. The proofs for these two cases are same, so we will only prove the truth.

i. Suppose that $(\mathcal{H} ,\ \sigma)$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed, then $\wp_{\mathcal{H}}: \mathcal{H} \longrightarrow \mathfrak{B}$ is slightly closed. To prove that $\mathcal{D}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed topological space over $(\mathfrak{B} , \Gamma)$. i.e., the projection $\wp_{\mathcal{D}} : (D, \varrho) \to (\mathfrak{B}, \Gamma)$ is slightly closed. Suppose that $\in \mathcal{D}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$. Let $\mathbb{F}$ be $\varrho -$ closed set of d. Since η is continuous, then $\eta^{-1}(\mathbb{F})$ is $\sigma -$closed set of $\eta^{-1}(d) = \hbar \in \mathcal{H}_{\mathfrak{b}}$ in $\mathcal{H}$. Since $\wp_{\mathcal{H}}$ is slightly closed , then $p_H(\eta^{-1}(\mathbb{F}))$ is slightly clopen set in $\mathfrak{B}$. But, $\wp_{\mathcal{H}}\big(\eta^{-1}(\mathbb{F})\big) = \wp_{\mathcal{D}}(\mathbb{F})$. Thus $\wp_{\mathcal{D}}$ is slightly closed and $\mathcal{D}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed.

Proposition 5. If $(\mathcal{H},\sigma)$ is a $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ space over $(\mathfrak{B}, \Gamma)$. Suppose that $(\mathcal{H},\sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed over $(\mathfrak{B}, \Gamma)$. Then $(\mathcal{H}_{\mathfrak{B}^*} ,\ \sigma_{\mathfrak{B}^*})$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed over $(\mathfrak{B}^*, \Gamma^*)$ for each subspace $(\mathfrak{B}^*, \Gamma^*)$ of $(\mathfrak{B}, \Gamma)$.

Proof. Assume that $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed . So that the projection $\wp_{\mathcal{H}}: \mathcal{H} \to B$ is slightly closed . To prove that $\mathcal{H}_{B^*}$ is slightly closed . i.e., the projection $\wp_{\mathfrak{B}^*}:\mathcal{H}_{\mathfrak{B}^*} \longrightarrow \mathfrak{B}^*$ is slightly closed . Now, Let $\hbar \in \mathcal{H} \mid B^*$, $\mathcal{U}$ be $\sigma -$ closed set of h,. $\mathcal{U} \cap \mathcal{H}_{B^*}$ is $\sigma_{\mathfrak{B}^*} -$ closed set of $\mathcal{H}_{\mathfrak{B}^*}$. $\wp_{\mathfrak{B}^*}(\mathcal{U} \cap \mathcal{H}_{\mathfrak{B}^*}) = \wp(\mathcal{U} \cap \mathcal{H}_{\mathfrak{B}^*}) = \wp(\mathcal{U}) \cap \wp(\mathcal{H}_{\mathfrak{B}^*}) = p(\mathcal{U}) \cap \mathfrak{B}^*$ this is $\Gamma_{\mathfrak{B}^*} -$ clopen set in $\mathfrak{B}^*$. $\wp_{\mathfrak{B}^*}$ is slightly closed . Therefore, $\mathcal{H}_{\mathfrak{B}^*}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed.

Proposition 6. Let $(\mathcal{H}, \sigma)$ be a $\mathcal{F}.\mathcal{W}.\ \mathcal{T}.$ space over $(\mathfrak{B}, \Gamma)$. If $(\mathcal{H}_{\mathfrak{B}_j}, \sigma_{\mathfrak{B}_j})$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed topological spaces over $(\mathfrak{B}_j, \Gamma_{\mathfrak{B}_j})$ for all member of a $\Gamma_{\mathfrak{B}_j}$- clopen covering of $\mathfrak{B}$. Then H is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed topological space over $\mathfrak{B}$.

Proof. Let $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\ \mathcal{T}.$ space over $\mathfrak{B}$ so, the projection $\wp: \mathcal{H} \to \mathfrak{B}$ existing . To show that $\wp$ is closed. By $\mathcal{H}_{\mathfrak{B}j}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed over $\mathfrak{B}_j$ for every member $\Gamma -$ clopen covered of $\mathfrak{B}$, then the projection $\wp_{\mathfrak{B}j} : \mathcal{H}_{\mathfrak{B}j} \to \mathfrak{B}_j$ is closed . Now, let $\mathcal{U}$ be σ-closed set of $\mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, $\wp(\mathcal{U}) = \cup\, \wp_{\mathfrak{B}j}(\mathcal{U} \cap \mathcal{H}_{\mathfrak{B}j})$ this union is specific of Γ – clopen sets of $\mathfrak{B}$. Therefore, $\wp$ is slightly closed and $\mathcal{H}$ is closed $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ topological space over $\mathfrak{B}$.

Proposition 7. If $(\mathcal{H}, \sigma)$ is a $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ space over $(\mathfrak{B}, \Gamma)$. Let $\mathcal{H}_i$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed for every member $\mathcal{H}_i$ of a finite covering of $\mathcal{H}$. Then $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed.

Proof. Assume that $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\ \mathcal{T}.$ space over $\mathfrak{B}$, then the projection $\wp : \mathcal{H} \to \mathfrak{B}$ existing. To prove that $\wp$ is slightly closed. By $\mathcal{H}_i$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed , then the projection $\wp_{\mathcal{H}_i} : \mathcal{H}_i \to \mathfrak{B}$ is slightly closed for all member $\mathcal{H}$ of a specific covering. Let $\mathbb{F}$ be σ - closed subset of $\mathcal{H}_i$. So $\wp(\mathbb{F}) = \cup\, \wp(\mathcal{H} \cap \mathbb{F})$ this isa specific union of clopen sets and so $\wp$ is slightly closed. Therefore $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed .

Proposition 8. Let $(\mathcal{H}, \sigma)$ be a $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ space over $(\mathfrak{B}, \Gamma)$. Then $(\mathcal{H}, \sigma)$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed iff for all fibre $\mathcal{H}_{\mathfrak{b}}$, $\mathfrak{b} \in \mathfrak{B}$ of $\mathcal{H}$ and every $\sigma -$ open set $\mathcal{V}$ of $\mathcal{H}_{\mathfrak{b}}$ in $\mathcal{H}$, there is a $\Gamma -$ clopen set $\mathcal{L}$ of $\mathfrak{b}$ where $\mathcal{H}_{\mathcal{L}} \subset \mathcal{V}$.

Proof. (⇒) Assume that $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed. i.e., $\wp: \mathcal{H} \to \mathfrak{B}$ is slightly closed. Now, let $\mathfrak{b} \in \mathfrak{B}$ and $\mathcal{V}$ be $\sigma -$open set of $\mathcal{H}_{\mathfrak{b}}$. Thus we have $\mathcal{H} - \mathcal{V}$ is $\sigma -$ closed set and $\wp(\mathcal{H} - \mathcal{V})$ is $\Gamma -$ clopen set. Let $\mathcal{L} = \mathfrak{B} - \wp(\mathcal{H} - \mathcal{V})$ is $\Gamma -$ clopen set of $\mathfrak{b}$. Hence, $\mathcal{H}_{\mathcal{L}} = \wp^{-1}(\mathfrak{B} - \wp(\mathcal{H} - \mathcal{V}))$ is a subset of $\mathcal{V}$.

(⇐) Suppose that the other direction is hold, to show that $(\mathcal{H}, \sigma)$ is a $\mathcal{F}.\mathcal{W}.$ slightly closed . Let $\mathbb{F}$ be $\sigma -$closed set in $\mathcal{H}$. Let $\mathfrak{b} \in \mathfrak{B} - \wp(\mathbb{F})$ and every $\sigma -$open set $\mathcal{V}$ of $\mathcal{H}_{\mathfrak{b}}$ in $\mathcal{H}$. By assumption there is $\Gamma -$ clopen set $\mathcal{L}$ of $\mathfrak{b}$ such that $\mathcal{H}_{\mathcal{L}} \subset \mathcal{V}$. It's easy to show that $\mathcal{L} \subset \mathfrak{B} - \wp(\mathbb{F})$. Hence, $\mathfrak{B} - \wp(\mathbb{F})$ is a $\Gamma -$ clopen set in $\mathcal{H}$. Hence, $\wp(\mathbb{F})$ is a $\Gamma -$ clopen in $\mathfrak{B}$, $\wp$ is slightly closed, and $(\mathcal{H}, \sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed .

Definition 13. The $\mathcal{F}.\mathcal{W}.$ topological space $(\mathcal{H}, \sigma)$ over $(\mathfrak{B}, \Gamma)$ is called $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open if the projection $\wp$ is slightly open.

Example 7. Let $\mathcal{H} = \mathfrak{B}$ = R with usual topology . Define : $(\mathcal{H}, \mathcal{U}) \to (\mathfrak{B}, \mathcal{U})$, s.t $\wp(\hbar) = \hbar\ \forall \hbar \in R$. Then $\wp$ is slightly open and $(\mathcal{H}, \mathcal{U})$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open space.

Example 8. Let $\mathcal{H} = \{a, b\}$, σ = $\{\mathcal{H}, \emptyset\}$. Let $\mathfrak{B}$ = {c , d}, Γ = $\{\mathfrak{B}, \emptyset\, \{c\}, \{d\}\}$. Define $\wp : \mathcal{H} \to \mathfrak{B}$ s.t $\wp(a) = c$, $\wp(b) = d$. Then $\wp$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open but is not $\mathcal{F}.\mathcal{W}.\ \mathcal{T}.$ spaces over $(\mathfrak{B}, \Gamma)$. Since {d} is clopen in , but $\wp^{-1}(d)$ is not open in $\mathcal{H}$.

Remark 4. Every $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open is $\mathcal{F}.\mathcal{W}.$ open , however the reverse does not need to be correct.

Example 9. Let $\mathcal{H} = \{1,2,3\}$, σ = $\{\mathcal{H}, \emptyset, \{1\}\}$. Let $\mathfrak{B}$ = {a, b , c} , Γ = $\{\mathfrak{B}, \emptyset, \{a\}\}$. Define $\wp : \mathcal{H} \to \mathfrak{B}$ s.t $\wp(1) = \wp(2) = \wp(3) = a$. Then $\wp$ is open , therefore $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.$ open, but is not $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open , since $\wp(\{2,3\})$ is open in $\mathcal{H}$, but {a} is not clopen in $\mathfrak{B}$.

Proposition 9. Let $\eta\ :\mathcal{H}\ \rightarrow \mathcal{D}$ be an open $\mathcal{F}.\mathcal{W}.$ function where $(\mathcal{H},\sigma\ )$ and $(\mathcal{D},\varrho)$ are $\mathcal{F}.\mathcal{W}.$ topological spaces over $(\mathfrak{V}\ ,\ \Gamma)$.

i. If $\mathcal{D}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open. So that, $\mathcal{H}$ it's $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open.
ii. If $\mathcal{D}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open. So that, $\mathcal{H}$ it's a $\mathcal{F}.\mathcal{W}.$ open.

Proof. The proofs for these two cases are same, so we will only prove the truth.
i. By $\mathcal{D}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open, therefore, $\wp_{\mathcal{D}} : \mathcal{D} \rightarrow \mathfrak{V}$ is slightly open. To prove that $\mathcal{H}$ it's a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open . i.e., To show that $\wp_{\mathcal{H}} : \mathcal{H}\ \rightarrow\ \mathfrak{V}$ is slightly open. let $\in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{V}$, and $\mathbb{F}$ be σ open set of h. since η is open $\mathcal{F}.\mathcal{W}.$ function then $\eta(\mathbb{F})$ is an open set of $\eta(\hbar)\ =\ d \in \mathcal{D}_{\mathfrak{b}}$ in $\mathcal{D}$. Since $\wp_{\mathcal{D}}$is slightly open , then $\wp_{\mathcal{D}}(\eta(\mathbb{F}))$ is clopen set at $(\mathfrak{V},\Gamma)$. But , $(\wp_{\mathcal{D}} o\ \eta)(\mathbb{F}) =\ \wp_{\mathcal{H}}(\mathbb{F})$ is Γ – clopen set of $\mathbb{F}$. Thus , $\wp_{\mathcal{H}}$ is slightly open and $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open.

Proposition 10. Let $\eta{:}\,\mathcal{H} \longrightarrow \mathcal{D}$ be a surjection $\mathcal{F}.\mathcal{W}.$ continuous where $(\mathcal{H},\sigma)$ and $(\mathcal{D},\varrho)$ are $\mathcal{F}.\mathcal{W}.$ topological spaces over $(\mathfrak{V},\Gamma)$.

i. If $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open. So that, $\mathcal{D}$ it's a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open.
ii. If $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open. So that, $\mathcal{D}$ it's a $\mathcal{F}.\mathcal{W}.$ open.

Proof. The proofs for these two cases are same, so we will only prove the truth.
i. Suppose that $(\mathcal{H},\sigma)$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open, then $\wp_{\mathcal{H}} : \mathcal{H} \longrightarrow \mathfrak{V}$ is slightly closed. To prove that $\mathcal{D}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open topological space over $(\mathfrak{V},\Gamma)$. i.e., the projection $\wp_{\mathcal{D}} : (\mathcal{D}\ ,\ \varrho) \rightarrow (\mathfrak{V}\ ,\ \Gamma\ )$ is slightly open. Suppose that $\in \mathcal{D}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{V}$. Let $\mathbb{F}$ be $\varrho -$ open set of d. Since η is continuous, then $\eta^{-1}(\mathbb{F})$ is $\sigma -$ open set of $\eta^{-1}(d) = \hbar \in \mathcal{H}_{\mathfrak{b}}$ in $\mathcal{H}$. Since $\wp_{\mathcal{H}}$ is slightly open , then $p_H(\eta^{-1}(\mathbb{F}))$ is slightly clopen set in $\mathfrak{V}$. But, $\wp_{\mathcal{H}}\left(\eta^{-1}(\mathbb{F})\right) = \wp_{\mathcal{D}}(\mathbb{F})$. Thus $\wp_{\mathcal{D}}$ is slightly open and $\mathcal{D}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open.

Proposition 11. If $(\mathcal{H},\sigma)$ is a $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ space over $(\mathfrak{V},\Gamma)$. Suppose that $(\mathcal{H},\sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open over $(\mathfrak{V}\ ,\ \Gamma)$. Then $(\mathcal{H}_{\mathfrak{V}^*}\ ,\ \sigma_{\mathfrak{V}^*})$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open over $(\mathfrak{V}^*,\Gamma^*)$ for each subspace $(\mathfrak{V}^*,\ \Gamma^*)$ of $(\mathfrak{V},\Gamma)$.

Proof. Assume that $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open. So that the projection $\wp_{\mathcal{H}} : \mathcal{H}\ \rightarrow \mathfrak{V}$ is slightly open . To prove that $\mathcal{H}_{B^*}$ is slightly open. i.e., the projection $\wp_{\mathfrak{V}^*} : \mathcal{H}_{\mathfrak{V}^*} \longrightarrow \mathfrak{V}^*$ is slightly open . Now, Let $\hbar \in \mathcal{H} \mid B^*$, $\mathcal{U}$ be $\sigma -$ open set of h,. $\mathcal{U} \cap \mathcal{H}_{B^*}$ is $\sigma_{\mathfrak{V}^*} -$ open set of $\mathcal{H}_{\mathfrak{V}^*}$. $\wp_{\mathfrak{V}^*}(\mathcal{U} \cap \mathcal{H}_{\mathfrak{V}^*}) = \wp\ (\mathcal{U} \cap \mathcal{H}_{\mathfrak{V}^*}) = \wp(\mathcal{U}) \cap \wp(\mathcal{H}_{\mathfrak{V}^*}) = p(\mathcal{U}) \cap \mathfrak{V}^*$ this is $\Gamma_{\mathfrak{V}^*} -$ clopen set in $\mathfrak{V}^*$. $\wp_{\mathfrak{V}^*}$ is slightly open . Therefore, $\mathcal{H}_{\mathfrak{V}^*}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open.

Proposition 12. Let $(\mathcal{H},\sigma)$ be a $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ space over $(\mathfrak{V},\Gamma)$. Assume that $(\mathcal{H}_{\mathfrak{V}_j},\sigma_{\mathfrak{V}_j})$ it's a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed topological spaces over $(\mathfrak{V}_j\ ,\Gamma_{\mathfrak{V}_j})$ for each member of a $\Gamma_{\mathfrak{V}_j}$- clopen covering of $\mathfrak{V}$. So that, $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open topological space over $\mathfrak{V}$.

Proof. Assume that $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\ \mathcal{T}.$ space over $\mathfrak{V}$ so that, the projection $\wp : \mathcal{H}\ \rightarrow \mathfrak{V}$ existing .To show that $\wp$ is open . By $\mathcal{H}_{\mathfrak{V}j}$ is open over $\mathfrak{V}_j$ for every member $\Gamma -$ clopen covered of $\mathfrak{V}$,, then the projection $\wp_{\mathfrak{V}j} : \mathcal{H}_{\mathfrak{V}j} \rightarrow\ \mathfrak{V}_j$ is closed . Now, let $\mathcal{U}$ be σ- open set of $\mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{V}$, $\wp(\mathcal{U}) = \cup\ \wp_{\mathfrak{V}j}(\mathcal{U} \cap \mathcal{H}_{\mathfrak{V}j})$ this is a specific union of Γ – clopen sets of $\mathfrak{V}$. Therefore, $\wp$ is slightly open and $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open.

Proposition 13. Assume that $\eta\ :\mathcal{H}\ \rightarrow \mathcal{D}$ be a $\mathcal{F}.\mathcal{W}.$ function where $(\mathcal{H},\sigma)$ and $(\mathcal{D}\ ,\ \varrho)$ are $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ spaces over $(\mathfrak{V},\Gamma)$. Assume that the product: $id_{\mathcal{H}} \times \eta\ :\ \mathcal{H} \times_{\mathfrak{V}} \mathcal{H} \longrightarrow \mathcal{H} \times_{\mathfrak{V}} \mathcal{D}$ is slightly open and $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open . Then η itself is slightly open.

Proof. Consider the following figure:

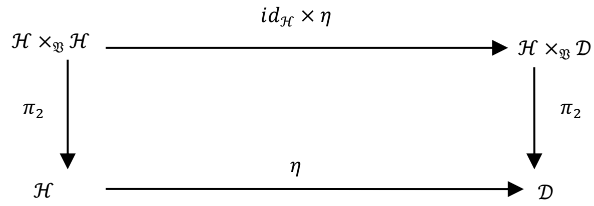

FIGURE 1. Diagram of Proposition 13**.**

The projection on the left is surjective while the projection on the right is slightly open because $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open topological space. Thus, $\pi_2 o(id_{\mathcal{H}} \times \eta) = \eta o \pi_2$ is slightly open and thus, η is slightly open

Fibrewise Locally Sliceable and Fibrewise Locally Sectionable Slightly Topological Spaces

In this section, we are generalizing $\mathcal{F}.\mathcal{W}.$ locally sliceable and $\mathcal{F}.\mathcal{W}.$ locally sectionable slightly topological spaces over $(\mathfrak{B}, \Gamma)$ and many topological characteristics on this space also definitions and discussed.

Definition 14. A $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space $(\mathcal{H}, \sigma)$ over $(\mathfrak{B}, \Gamma)$ is said to be locally sliceable slightly topological space if for all $\hbar \in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, so there is a $\Gamma -$ clopen set $\mathcal{K}$ of $\mathfrak{b}$ and a section $\mathfrak{s} : \mathcal{K} \to \mathcal{H}_{\mathcal{K}}$ s.t $\mathfrak{s}(\mathfrak{b}) = \hbar$.

Example 10. Let $\mathcal{H} = \{a, b, c\}$, $\sigma = \{\mathcal{H}, \varphi, \{b\}\}$, $\mathfrak{B} = \{e, d, f\}$, $\Gamma = \{ \mathfrak{B}, \emptyset, \{e\}, \{d, f\}\}$. Let $\wp : \mathcal{H} \to \mathfrak{B}$ s.t $\wp(a) = f$, $\wp(b) = e$, $\wp(c) = d$. We have $\mathcal{H}_e = \{b\}$, $\mathcal{H}_d = \{c\}$, $\mathcal{H}_f = \{a\}$, $\mathfrak{s} : \mathcal{K} \to \mathcal{H}_{\mathcal{K}}$ where $\mathfrak{s}(e) = b$, $\mathfrak{s}(d) = c$, $\mathfrak{s}(f) = a$. So $\mathcal{H}$ it's a $\mathcal{F}.\mathcal{W}.$ locally sliceable slightly topological space .

Thus it will lead to $\wp$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}$ open , since if $\mathcal{V}$ is a $\sigma -$open set of $\hbar$ in $\mathcal{H}$, so $\mathfrak{s}^{-1}(\mathcal{H}_{\mathcal{K}} \cap \mathcal{V}) \subset \wp(V)$ is a $\Gamma -$ clopen set of $\mathfrak{b}$ in $\mathcal{K}$, also in $\mathfrak{B}$.

The category of $\mathcal{F}.\mathcal{W}.$ locally sliceable slightly topological spaces is finitely multiplicative as mentioned in.

Proposition 14. Assume that $\{ (\mathcal{H}_m, \sigma_m) \}$ $(m = 1, \dots\dots, n)$ be a finite family of $\mathcal{F}.\mathcal{W}$ locally sliceable slightly topological space over $(\mathfrak{B}, \Gamma)$. So the $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ product $\mathcal{H} = \prod_{\mathfrak{B}} \mathcal{H}_m$ is locally sliceable slightly topological space over $(\mathfrak{B}, \Gamma)$.

Proof. Assume that $\hbar = (\hbar_m)$ be a point of $\mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, so that $\hbar_m = \pi_m(\hbar)$ for each index m. By $\mathcal{H}_m$ is a $\mathcal{F}.\mathcal{W}.$ locally sliceable slightly topological space over $(\mathfrak{B}, \Gamma)$, so there exist a $\Gamma -$ clopen set $\mathcal{K}_m$ of $\mathfrak{b}$ and a section $\mathfrak{s}_m : \mathcal{K}_m \to \mathcal{H}_m \mid \mathcal{K}_m$, s.t $\mathfrak{s}_m(\mathfrak{b}) = \hbar_m$. So $\mathcal{K} = \cap \mathcal{K}_m$ $(m = 1, \dots\dots, n)$ is a $\Gamma -$ clopen set of $\mathfrak{b}$, also a section $\mathfrak{s} : \mathcal{K} \to \mathcal{H}_{\mathcal{K}}$ is written by $(\pi_m \, o \, \mathfrak{s})\,(\alpha) = \mathfrak{s}_m(\alpha)$ for each $\alpha \in \mathcal{K}$.

Proposition 15. Suppose that $\eta : \mathcal{H} \to \mathcal{D}$ be a $\mathcal{F}.\mathcal{W}.$ continuous , surjection, where $(\mathcal{H}, \sigma)$ & $(\mathcal{D}, \varrho)$ are $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ spaces over $(\mathfrak{B}, \Gamma)$. Then $\mathcal{D}$ is locally sliceable slightly topological space , when $\mathcal{H}$ is locally sliceable slightly topological space over $(\mathfrak{B}, \Gamma)$.

Proof. Take $d \in \mathcal{D}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, so $d = \eta(\hbar)$, for some $\hbar \in H_{\mathfrak{b}}$. Since $\mathcal{H}$ is locally sliceable slightly topological spaces over $(\mathfrak{B}, \Gamma)$, so there is a $\Gamma -$ clopen set $\mathcal{K}$ of $\mathfrak{b}$ and a section $\mathfrak{s} : \mathcal{K} \to \mathcal{H}_{\mathcal{K}}$ where $\mathfrak{s}(\mathfrak{b}) = \hbar$. So that $\eta o \mathfrak{s} : \mathcal{K} \to \mathcal{D}_{\mathcal{K}}$ is a section s.t $\mathfrak{s}(\mathfrak{b}) = d$.

Definition 15. Let $(\mathcal{H}, \sigma)$ and $(\mathcal{D}, \varrho)$ be topological spaces. A function $\eta : (\mathcal{H}, \sigma) \longrightarrow (\mathcal{D}, \varrho)$ is said to be a slightly local homeomorphism if for each $\hbar$ in $\mathcal{H}$ there exists an open set $\mathcal{V}$ containing $\hbar$, such that the image is clopen in $\mathcal{D}$ and the restriction is a slight homeomorphism.

Definition 16. The $\mathcal{F}.\mathcal{W}.\mathcal{S}\,\mathcal{T}.$ space $(\mathcal{H},\sigma)$ over $(\mathfrak{B},\Gamma)$ is said to be $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ discrete if the projection p is a slightly local homeomorphism.

Example 11. Let $\mathcal{H} = \{1, 2,3\}$, $\sigma = \{\mathcal{H},\varphi,\{1\},\{2\},\{3\},\{1,3\},\{1,2\},\{2,3\}\}$, $\mathfrak{B}$={a, b, c}, $\Gamma =\{\mathfrak{B}, \emptyset,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$. Let $\wp\colon \mathcal{H} \to \mathfrak{B}$ such that $\wp(1) = a$, $\wp(2) = b$, $\wp(3) = c$. We have $\mathcal{H}_a = \{1\}$, $\mathcal{H}_b = \{2\}$, $\mathcal{H}_c = \{3\}$. $\mathfrak{s}\colon \mathcal{K} \to \mathcal{H}_{\mathcal{K}}$ where $\mathfrak{s}(a) = 1$, $\mathfrak{s}(b) = 2$, $\mathfrak{s}(c) = 3$. Therefore $\wp$ is slightly local homeomorphism, this implies that $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ discrete.

This implies that, that for all $\mathfrak{b} \in \mathfrak{B}$, and for each $\hbar \in \mathcal{H}_{\mathfrak{b}}$ there is a $\sigma -$ open set $\mathcal{V}$ of $\hbar$ in $\mathcal{H}$ and a $\Gamma -$ clopen set $\mathcal{K}$ of $\mathfrak{b}$ in $\mathfrak{B}$ where $\wp$ maps $\mathcal{V}$ slightly homeomorphically onto $\mathcal{K}$; in that situation implies that $\mathcal{K}$ is Uniformly covered by $\mathcal{V}$. It is clear that $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ discrete topological spaces are $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ locally sectionable topological spaces and $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open.

The category of $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ discrete topological spaces is finitely multiplicative as mentioned in .

Proposition 16. Assume that $\{ (\mathcal{H}_m,\sigma_m) \}$ $(m = 1, \dots\dots, n)$ be a finite family of $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ discrete topological spaces over $(\mathfrak{B},\Gamma)$. Therefore the $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ product $(\mathcal{H} = \prod_{\mathfrak{B}}\mathcal{H}_m,\sigma_m)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ discrete topological spaces over $(\mathfrak{B},\Gamma)$.

Proof. Take $\hbar \in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$,, so there is for all index m a $\sigma_m -$ open set $\mathcal{V}_m$ of $\pi_m\,(\hbar)$ in $\mathcal{H}_m$ s.t, the projection $\wp_m = \pi_m(\hbar)$ maps $\mathcal{V}_m$ slightly homeomorphically onto the $\Gamma -$ clopen $\wp_m(\mathcal{V}_m) = \mathcal{K}_m$ of $\mathfrak{b}$. So that $\sigma -$ open $\prod_{\mathfrak{B}}\mathcal{V}_m$ of $\hbar$ is mapped slightly homeomorphically onto the intersection $\mathcal{K} = \cap\, \mathcal{K}_m$ which is a $\Gamma -$ clopen of $\mathfrak{b}$.

An appealing description of $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ discrete topological spaces, it is presented by the following suggestions.

Proposition 17. If $(\mathcal{H},\sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space over $(\mathfrak{B},\Gamma)$. So, $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ discrete topological spaces over $(\mathfrak{B},\Gamma)$ iff :

i. $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open.
ii. The diagonal embedding $\Delta : \mathcal{H} \to \mathcal{H} \times_{\mathfrak{B}} \mathcal{H}$ is slightly open.

Proof. $(\Leftarrow)$ Let (i) & (ii) are satisfied. Given a point $\hbar \in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, so that $\Delta(\hbar) = (\hbar,\hbar)$ admits a $\sigma \times \sigma -$ clope*n* set in $\mathcal{H} \times_{\mathfrak{B}} \mathcal{H}$ which is completely belonging in $\Delta\,(\mathcal{H})$. Without real lacking in general, we may suppose the $\sigma \times \sigma -$ clopen set is the form $\mathcal{V} \times_{\mathfrak{B}} \mathcal{V}$, where $\mathcal{V}$ is a $\sigma -$ open set of $\hbar$ in $\mathcal{H}$. So $\wp|\mathcal{V}$ is a slight homeomorphism. So that, $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.$ discrete slightly topological spaces over $(\mathfrak{B},\Gamma)$.

$(\Rightarrow)$ Let $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ discrete topological spaces over $(\mathfrak{B},\Gamma)$, implies that $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open. To prove that Δ is slightly open. i.e., to prove that $\Delta(\mathcal{H})$ is $\sigma \times \sigma -$ clopen in $\mathcal{H} \times_{\mathfrak{B}} \mathcal{H}$. let $\hbar \in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, & $\mathcal{V}$ be a $\sigma -$ open set of $\hbar$ in $\mathcal{H}$, such that $\mathcal{K} = \wp(\mathcal{V})$ is a $\Gamma -$ clopen set of $\mathfrak{b}$ in $\mathfrak{B}$ and $\wp$ maps $\mathcal{V}$ slightly homeomorphically onto $\mathcal{K}$. So that, $\mathcal{V} \times_{\mathfrak{B}} \mathcal{V}$ is belonging in $\Delta(\mathcal{H})$ since if not, then there is distinct $\zeta, \zeta^* \in \mathcal{H}_{\mathcal{K}}$, where $\alpha \in \mathcal{K}$ and $\zeta, \zeta^* \in \mathcal{V}$, which is absurd.

Slightly open subset of $\mathcal{F}.\mathcal{W}.$ discrete slightly topological spaces are also $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ discrete. In truth, we have .

Proposition 18. Assume that $\eta\colon \mathcal{H} \to \mathcal{D}$ is a totally continuous $\mathcal{F}.\mathcal{W}.$ injection, where $(\mathcal{H},\sigma)$ and $(\mathcal{D},\varrho)$ are $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ open topological spaces over $(\mathfrak{B},\Gamma)$. If $\mathcal{D}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ discrete. So $\mathcal{H}$ is so.

Proof. Notice the graph below.

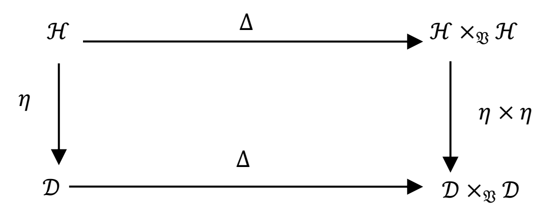

FIGURE 2. Diagram of Proposition 18.

Since η is totally continuous, so is $\eta \times \eta$. Now $\Delta(\mathcal{D})$ is $\varrho \times \varrho -$ clopen in $\mathcal{D} \times_B \mathcal{D}$, by Proposition (17) and $\mathcal{D}$ is a $\mathcal{F.W.S.}$ discrete, then $\Delta(\mathcal{H}) = \Delta\left(\left(\eta^{-1}(\mathcal{D})\right)\right) = (\eta \times \eta)^{-1}\left(\Delta(\mathcal{D})\right)$ is a $\sigma \times \sigma -$ clopen in $\mathcal{H} \times_{\mathfrak{B}} \mathcal{H}$. So the explanation comes from the Proposition (2.3.9.).

Proposition 19. Assume that $\eta: \mathcal{H} \to \mathcal{D}$ be an open $\mathcal{F.W.S.}$ surjection, where $(\mathcal{H}, \sigma)$ & $(\mathcal{D}, \varrho)$ are $\mathcal{F.W.S.}$ open topological spaces over $(\mathfrak{B}, \Gamma)$. If $\mathcal{H}$ is a $\mathcal{F.W.S.}$ discrete. So $\mathcal{D}$ is so.

Proof. From **FIGURE 2**, with the assumption on η, if $\mathcal{H}$ is a $\mathcal{F.W.S.}$ discrete, so $\Delta(\mathcal{H})$ is an $\sigma \times \sigma -$clopen in $\mathcal{H} \times_{\mathfrak{B}} \mathcal{H}$, by Proposition (17). Hence $\Delta(D) = \Delta\left(\left(\eta(\mathcal{H})\right)\right) = (\eta \times \eta)\left(\Delta(\mathcal{H})\right)$ is an $\varrho \times \varrho -$clopen in $\mathcal{D} \times_{\mathfrak{B}} \mathcal{D}$. So the explanation comes again from the Proposition (17).

Proposition 20. If $\eta, \psi: \mathcal{H} \to \mathcal{D}$ is a $\mathcal{F.W.}$ totally continuous functions, where $(\mathcal{H}, \sigma)$ is a $\mathcal{F.W.S.T.}$ and $(\mathcal{D}, \varrho)$ is a $\mathcal{F.W.S.}$ discrete topological space over $(\mathfrak{B}, \Gamma)$. So the coincidence set $K(\eta, \psi)$ of η and ψ is slightly open in $\mathcal{H}$.

Proof. The coincidence set is exactly $\Delta^{-1}(\eta \times \psi)^{-1}\left(\Delta(\mathcal{D})\right)$, where:

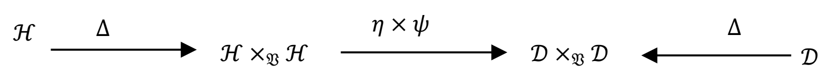

FIGURE 3. Diagram of Proposition 20.

Hence the immediately desired result of the Proposition (17). Specially, let $\mathcal{H} = \mathcal{D}$, $\eta = id_{\mathcal{H}}$, and $\psi = \mathfrak{s} o \wp$ where $\mathfrak{s}$ is a section. We conclude that $\mathfrak{s}$ is a slightly open embedding when $\mathcal{H}$ is a $\mathcal{F.W.S.}$ discrete.

Proposition 21. If $\eta: \mathcal{H} \to \mathcal{D}$ is a $\mathcal{F.W.}$ totally continuous functions, where $(\mathcal{H}, \sigma)$ is a $\mathcal{F.W.S.}$ open and $(\mathcal{D}, \varrho)$ is a $\mathcal{F.W.S.}$ discrete topological space over $(\mathfrak{B}, \Gamma)$. So, the $\mathcal{F.W.S.}$ graph $\Phi : \mathcal{H} \to \mathcal{H} \times_{\mathfrak{B}} \mathcal{D}$ of η is an slightly open embedding.

Proof. The $\mathcal{F.W.S.}$ graph is defined in the same way as the ordinary graph, but with values in the $\mathcal{F.W.S.T.}$ product. Therefore, the diagram shown below is commutative.

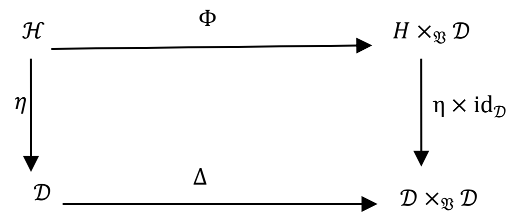

FIGURE 4. Diagram of Proposition 21.

Since $\Delta(\mathcal{D})$ is a slightly open , then $\Delta(\mathcal{D})$ is $\varrho \times \varrho -$ clopen in $\mathcal{D} \times_B \mathcal{D}$, by Proposition (17), so $\Phi(\mathcal{H}) = (\eta \times id_{\mathcal{D}})^{-1}\big(\Delta(\mathcal{D})\big)$ is an $\sigma \times \varrho -$ clopen in $H \times_{\mathfrak{B}} \mathcal{D}$.

Remark 5. If (H, σ) is a $\mathcal{F.W.S.}$ discrete topological space over $(\mathfrak{B}, \Gamma)$ So for all point $\hbar \in \mathcal{H}_{\mathfrak{b}}$; $\mathfrak{b} \in \mathfrak{B}$, there is a $\Gamma -$ clopen set K of $\mathfrak{b}$ and a unique section $\mathfrak{s}\colon \mathcal{K} \to \mathcal{H}_k$ exist satisfying $\mathfrak{s}(\mathfrak{b}) = \hbar$. We may refer to ψ as the section through $\hbar$.

Definition 17. The $\mathcal{F.W.S.T.}$ space $(\mathcal{H}, \sigma)$ over $(\mathfrak{B}, \Gamma)$ is said to be locally sectionable if for all $\mathfrak{b} \in \mathfrak{B}$, admits an $\Gamma -$ clopen set $\mathcal{K}$ and a section $\mathfrak{s}\colon \mathcal{K} \to \mathcal{H}_{\mathcal{K}}$.

Example 12. Let $\mathcal{H}$= {1, 2}, $\sigma = \big\{\mathcal{H}, \emptyset, \{1\}\big\}$. Let $\mathfrak{B}$ = {a, b}, $\Gamma = \big\{\mathfrak{B}, \emptyset, \{a\}\big\}$. Let $p : \mathcal{H} \to \mathfrak{B}$ where $\wp$ (1) = a, $\wp$ (2) = b. We have $\mathcal{H}_a = \{1\}$, $\mathcal{H}_{\mathrm{b}} = \{2\}$. Therefore $(\mathcal{H}, \sigma)$ is $\mathcal{F.W.S.}$ locally sectionable topological space over $(\mathfrak{B}, \Gamma)$

Remark 6. The $\mathcal{F.W.S.}$ non-empty locally sliceable topological spaces are locally sectionable, however the reverse does not need to be correct. In fact, slightly locally sectionable topological spaces are not necessarily $\mathcal{F.W.S.}$ open. For example. take $\mathcal{H} = (-1,1] \subset \mathbb{R}$ with $(\mathcal{H}, \sigma)$:with the natural projection onto $\mathfrak{B} = \mathbb{R} \mid \mathbb{Z}$; $(\mathfrak{B}, \Gamma)$.

The category of slightly locally sectionable topological spaces is finitely multiplicative. We will show it as follows.

Proposition 22. If $\{(\mathcal{H}_m, \sigma)\}$ is a finite family of slightly locally sectionable topological spaces over $(\mathfrak{B}, \Gamma)$. So the $\mathcal{F.W.S.T.}$ product $\mathcal{H} = \prod_{\mathfrak{B}} H_m$ is locally sectionable.

Proof. Given a point $\mathfrak{b}$ of $\mathfrak{B}$, there exist an $\Gamma -$ clopen set $\mathcal{K}_r$ of $\mathfrak{b}$ and a section $\mathfrak{s}_m\colon \mathcal{K}_m \to \mathcal{H}_m \mid \mathcal{K}_m$ for every index m. Since there are finite number of indices, the intersection $\mathcal{K}$ of the $\Gamma -$ clopen sets $\mathcal{K}_m$ is also a $\Gamma -$ clopen set of b, and a section $\mathfrak{s}\colon \mathcal{K} \to (\prod_{\mathfrak{B}} \mathcal{H}_m)_{\mathcal{K}}$ is given by $\pi_m\ o\ \mathfrak{s}(\mathcal{K}) = \mathfrak{s}_m(\alpha)$, for $\alpha \in \mathcal{K}$.

For each of the three suggestions above, our last two observations apply well.

Proposition 23. If $(\mathcal{H}, \sigma)$ is a $\mathcal{F.W.S.T.}$ space over $(\mathfrak{B}, \Gamma)$. Assume that $(\mathcal{H}, \sigma)$ is slightly locally sliceable topological spaces , $\mathcal{F.W.S.}$ discrete topological spaces or slightly locally sectionable topological spaces over $(\mathfrak{B}, \Gamma)$. So is $\mathcal{H}_{\mathfrak{B}^*}$ over $\mathfrak{B}^*$ for each $\Gamma -$ clopen set $\mathfrak{B}^*$ of $\mathfrak{B}$.

Proposition 24. Assume that $(\mathcal{H}, \sigma)$ be a $\mathcal{F.W.S.T.}$ space over $(\mathfrak{B}, \Gamma)$. Let $\mathcal{H}_{\mathfrak{B}_j}$ is a is slightly locally sliceable topological spaces , $\mathcal{F.W.S.}$ discrete topological spaces or slightly locally sectionable topological spaces over $\mathfrak{B}_j$ for each member $\mathfrak{B}_j$ of an $\Gamma -$ clopen covering of $\mathfrak{B}$. So is $\mathcal{H}$ over $\mathfrak{B}$.

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