Designed Algorithms for Compute the Tenser Product of Representation for the Special Linear Groups

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Abstract

The main objective of this paper is to designed algorithms and implemented in the construction of the main program designated for the determination the tenser product of representation for the special linear group.

Keywords:

Designed algorithms, representation for the group, degree of the representation, character of representation, tenser product.

1. INTRODUCTION

The set of all Z-valued class functions of a finite group G form an abelian group cf (G,Z) under point wise addition. Inside this group we have a subgroup of Z-valued generalized characters of G denoted by R(G).

The group of invertible $n\times n$ matrices over a field F denoted by GL(n,F). The determinant of these matrices is a homomorphism from GL(n,F) into F-{0} and we denote the kernel of this homomorphism by SL(n,F), the special linear group. Thus SL(n,F) is the subgroup of GL(n,F) which contains all matrices of determinant one over the field F, [1].

In section one wepresent some conceptswhich we neededit later,the second section include the algorithms designed for compute the tenser product of representation for the special linear groupSL(n,F).

2. BASIC CONCEPTS

We recall definition proposition and remark which we needed in the next section.

Definition 1.1 : [2]

The set of all n×n non-singular matrices over the field F this set forms a group under the operation of matrix multiplication. This group is called the general linear group of dimension n over the field F, denoted by GL(n,F).

Definition 1.2 : [2]

Let V be a vector space over the field F and let GL(V) denote the group of all linear isomorphisms of V onto itself.

A representation of a group G with representation space V is a homomorphism t: $g \rightarrow$ t(g) of G into GL(V).

Definition 1.3 : [2]

A matrix representationof a group Gis ahomomorphism T: $g \rightarrow$ T(g) of G into GL(n,F), where n is called the degree of the matrix representation.

Definition 1.4 : [3]

Let T be a matrix representation of a finite group G over the field F.

The character χ of T is the mapping χ: G $\rightarrow$ F defined by

$\chi(g) = \mathrm{Tr}(T(g))\ \forall g\in G$, where Tr (T($g$)) refers to the trace of the matrix T(g).

Definition 1.5 : [4]

The general linear group is the group of invertible n×n matrices over a field F denoted by GL(n,F_q). The determinant of these matrices is a homomorphism from GL(n,F_q) into F^* and we denote the kernel of this homomorphism by SL(n,F_q),the special linear group. Thus SL(n,F_q) is the subgroup of GL(n,F_q) which contains all matrices of determinant one.

The order of the group is

$$|SL(n,F_q)| = \frac{\prod_{i=0}^{n-1}(q^n - q^i)}{q-1}$$

Definition 1.6 : [5]

Let $A \in M_n(\mathbb{C})$, $B \in M_m(\mathbb{C})$, we defined a matrix $A \otimes B \in M_m(\mathbb{C})$, put

$$A \otimes B = \begin{bmatrix} \alpha_{11}B & \alpha_{12}B & \cdots & \alpha_{1n}B \\ \alpha_{21}B & \alpha_{22}B & \cdots & \alpha_{2n}B \\ M & M & & M \\ \alpha_{n1}B & \alpha_{n2}B & \cdots & \alpha_{nn}B \end{bmatrix}_{nm \times nm},$$

$$A = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ M & M & & M \\ \alpha_{n1} & \alpha_{n2} & \cdots & \alpha_{nn} \end{bmatrix}_{n \times n}, \text{ and}$$

$$B = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1m} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2m} \\ M & M & & M \\ \beta_{m1} & \beta_{m2} & \cdots & \beta_{mm} \end{bmatrix}_{m \times m}$$

Thus

$$A \otimes B = \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1k} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2k} \\ M & M & & M \\ \delta_{k1} & \delta_{k2} & \cdots & \delta_{kk} \end{bmatrix}_{nm \times nm}$$

where

$$\delta_{11} = \begin{bmatrix} \alpha_{11}\beta_{11} & \alpha_{11}\beta_{12} & \cdots & \alpha_{11}\beta_{1m} \\ \alpha_{11}\beta_{21} & \alpha_{11}\beta_{22} & \cdots & \alpha_{11}\beta_{2m} \\ M & M & & M \\ \alpha_{11}\beta_{m1} & \alpha_{11}\beta_{m2} & \cdots & \alpha_{11}\beta_{mm} \end{bmatrix}_{m \times m}, \ldots,$$

$$\delta_{1k} = \begin{bmatrix} \alpha_{1n}\beta_{11} & \alpha_{1n}\beta_{12} & \cdots & \alpha_{1n}\beta_{1m} \\ \alpha_{1n}\beta_{21} & \alpha_{1n}\beta_{22} & \cdots & \alpha_{1n}\beta_{2m} \\ M & M & & M \\ \alpha_{1n}\beta_{m1} & \alpha_{1n}\beta_{m2} & \cdots & \alpha_{1n}\beta_{mm} \end{bmatrix}_{m \times m}, \ldots$$

$$\delta_{kk} = \begin{bmatrix} \alpha_{nn}\beta_{11} & \alpha_{nn}\beta_{12} & \cdots & \alpha_{nn}\beta_{1m} \\ \alpha_{nn}\beta_{21} & \alpha_{nn}\beta_{22} & \cdots & \alpha_{nn}\beta_{2m} \\ M & M & & M \\ \alpha_{nn}\beta_{m1} & \alpha_{nn}\beta_{m2} & \cdots & \alpha_{nn}\beta_{mm} \end{bmatrix}_{m \times m}$$

and $k = nm$.

Example 1.7 :[5]

$$A = \begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix}_{2 \times 2}, \quad B = \begin{bmatrix} 1 & -2 & -1 \\ 3 & 1 & 2 \\ 6 & 4 & 5 \end{bmatrix}_{3 \times 3}$$

$$A \otimes B = \begin{bmatrix} 1 & -2 & -1 & M & -3 & 6 & 3 \\ 3 & 1 & 2 & M & -9 & -3 & -6 \\ 6 & 4 & 5 & M & -18 & -12 & -15 \\ L & L & L & L & L & L & L \\ 2 & -4 & -2 & M & 0 & 0 & 0 \\ 6 & 2 & 4 & M & 0 & 0 & 0 \\ 12 & 8 & 10 & M & 0 & 0 & 0 \end{bmatrix}$$

Proposition 1.8 : [5]

Let $A, A', B, B' \in M_m(K)$, then

(1) $(A + A') \otimes B = (A \otimes B) + (A' \otimes B)$

(2) $(A \otimes B)(A' \otimes B') = AA' \otimes BB'$

Remark 1.9 :[5]

Let S and T be two representations of degree n and m of the group, for each $x \in SL(n,F_q)$ define $U(x) = S(x) \otimes T(x)$. Then U is representation of degree nm, we write $U = S \otimes T$.

Now, let χ_S, χ_T be two character of S and T respectively then $\chi_U = \chi_S \chi_T$.

3. THE ALGORITHMS

This section contains a collection of the computer ready FORTRAN algorithms for many standard methods of number theory installed in our main program.

Algorithm (1):
The Number of Degree of Representation for the Group SL(*n*,F$_q$)

Input: n (the order of the group $SL(n,F_q)$)
Step 1: To evaluate m where
$T:SL(n,F_q) \longrightarrow S(F)$,

$$S_m(F) = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1m} \\ s_{21} & s_{22} & \cdots & s_{2m} \\ M & M & & M \\ s_{m1} & s_{m2} & \cdots & s_{mm} \end{bmatrix}_{m \times m}$$

Step 2: Do I = 1 to m
Do J = 1 to m
Print IA(I,J)
End J-loop
End I-loop
Output: The number of degree of representation for groups $SL(n,F_q)$ is m.

Algorithm (2):
The Tenser Product of Two Representations for the Group SL(*n*,F$_q$)

Input: n (the order of the group $SL(n,F_q)$)
Step 1: Do C is the matrix of dimension $mn \times mn$
C(0,0) = 0
Do I = 1 to n
Do J = 1 to n

T(x) = A(I,J)
End J-loop
End I-loop
Step 2: Do I = 1 to m
Do J = 1 to m
Set T(x) = B(I,J)
End J-loop
End I-loop
Step 3: call algorithm 1
Step 4: To evaluate C where C(I,J) = A(I,J)∗B
Step 5: Set C(1,1) = A(1,1)∗B
C(1,2) = A(1,2)∗B
⋮
C(I,n) = A(I,n)∗B

$$\text{where } B = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1m} \\ B_{21} & B_{22} & \dots & B_{2m} \\ M & M & & M \\ B_{m1} & B_{m2} & \dots & B_{mm} \end{bmatrix}_{m\times m}$$

$$\text{Step 6: Set } C = \begin{bmatrix} C_{11} & C_{12} & \dots & C_{1nm} \\ C_{21} & C_{22} & \dots & C_{2nm} \\ M & M & & M \\ C_{nm1} & C_{nm2} & \dots & C_{nmmn} \end{bmatrix}_{nm\times nm}$$

Output: The tenser product of two representations of SL(n,F_q) is C(mn,mn)

Algorithm (3):
The Tenser Product of Three Representations for the Group SL(n,F_q)

Input: n (the order of the group SL(n,F_q))
Step 1: Call algorithm 2
Step 2: Do I = 1 to k
Do J = 1 to k
D(I,J)
End J-loop
End I-loop
Step 3: To evaluate R where R(I,J) = C(I,J) ∗ D
Step 4: Set
R(1,1) = C(1,1) ∗ D
R(1,2) = C(1,2) ∗ D

$$\text{where } D = \begin{bmatrix} D_{11} & D_{12} & \dots & D_{1k} \\ D_{21} & D_{22} & \dots & D_{2k} \\ M & M & & M \\ D_{k1} & D_{k2} & \dots & D_{kk} \end{bmatrix}_{k\times k}$$

Step 5: Set

$$R = \begin{bmatrix} R_{11} & R_{12} & \dots & R_{1s} \\ R_{21} & R_{22} & \dots & R_{2s} \\ M & M & & M \\ R_{s1} & R_{s2} & \dots & R_{ss} \end{bmatrix}_{s\times s}$$

where s = nm×k
Step 6: Do I = 1 to s
Do J = 1 to s
Print R(I,J)
End J-loop
End I-loop
Output: The tenser product of three representations of SL(n,F_q)is R(s,s)

Algorithm (4):
The Character of Representations for the Group SL(n,F_q)

Input: n (the order of the group SL(n,F_q))
Step 1: $\chi(0) = 0$
Step 2: Do I = 1 to m
χ_I
End I-loop
Step 3: Do J = 1 to n
χ_J
End J-loop
Step 4: Do I = 1 to m
Do J = 1 to n
$\chi_{(k)} = \chi_I * \chi_J$
End J-loop
End I-loop
Print χ_k

$$\text{Step 5: Set } \chi_k = \begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ M \\ \chi_s \end{bmatrix}, \; s = (nm)/2$$

Step 6: Call algorithm 3
Step 7: Call algorithm 4
Output: The character of representation for SL(n,F_q) is $\chi(k)$, k = 1 to s.

The Algorithm of the Main Program:
The Tenser Product of Representations for Group SL(n,F_q)

Input: n (the order of the group SL(n,F_q))
Step 1: Call algorithm 1
Step 2: Call algorithm 2
Step 3: Call algorithm 3
Step 4: Call algorithm
Output: (T(I), I = 1 to m) To evaluate the tenser product of representation for the group SL(n,F_q)
End

4. REFERENCES

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