On Jeffery Prior Distribution in Modified Double Stage Shrinkage-Bayesian Estimator for Exponential Mean

***Abbas Najim Salman, **Assel Hussein Ali , *Maha A. Mohammed**
*Department of Mathematics-Ibn Al-Haitham College of Education - University of Baghdad
**Computer Center - University of Baghdad

Abstract

This paper is concerned with **Modified** Double Stage Shrinkage Bayesian (DSSB) Estimator for lowering the mean squared error of classical estimator $\hat{\theta}$ for the scale parameter (θ) of an Exponential Distribution in suitable region (R) around available prior knowledge (θ_0) about the actual value (θ) as initial estimate as well as to reduce the cost of observation. In situation where the observations are time consuming or very costly, a "Double Stage procedure "can be used to reduce the Expected Sample Size needed to obtain the estimator. This estimator has been showing a smaller Mean Squared Error for certain choice of the shrinkage weight factor ψ(·) and for acceptance region R.

Expressions for Bias, Mean Square Error (MSE), Expected sample size [E(n/θ,R)], Expected sample size proportion [E(n/θ,R)/n], probability for avoiding the second sample $[p(\hat{\theta}_1 \in R)]$ and percentage of overall sample saved $[\frac{n_2}{n} p(\hat{\theta}_1 \in R) * 100]$ for the proposed estimator are derived. Numerical results and discussions are established when the consider estimator (DSSB) are testimator of level of significance α. Comparisons with the classical estimator as well as with some existing studies were made to shown the usefulness of the proposed estimator.

1. Introduction

1.1 The Model:

Exponential distribution is one of the most useful and widely exploited model, Epstein [1] remarks that the exponential distribution plays as important a role in life experiments as the part played by the normal distribution in agricultural experiments. It is applied in a very wide variety of statistical procedures. Among the most prominent applications are those in the field of life testing and reliability theory. The scale parameter (θ) is known as mean life time. The maximum likelihood estimator (MLE; $\hat{\theta}$) is the sample mean which is the minimum variance unbiased estimator.

The one parameter exponential distribution has the following probability density function (p.d.f.)

$$f(t;\theta) = \begin{cases} \frac{1}{\theta}\exp(\frac{-t}{\theta}) & ,t \geq 0,\ \theta > 0 \\ 0 & ,\text{o.w.} \end{cases} \qquad \ldots(1)$$

where θ is the average or the mean life or mean time to failure (MTTF) and it is also acts as scale parameter, see [1].

Furthermore, the Reliability function R(t) is defined as:

R(t) = exp(- t/θ), t > 0, θ >0.

Note that the maximum likelihood estimator $\hat{\theta}$ of the scale parameter θ of the mentioned distribution is $\bar{t} = \frac{\sum_{i=1}^{n} t_i}{n}$.

1.2 Jeffery Prior distribution (Bayesian Estimator):

Consider the one parameter Exponential Distribution which is define in (1).

Based on the rule proposed by Jeffery, one can get the prior distribution of θ [g(θ)] as below, see[2]

$g(\theta) \propto \sqrt{I(\theta)}$, where I(θ) is fisher information such that

$$I(\theta) = -n\,E\left(\frac{\partial^2 \ln f(t,\theta)}{\partial\theta^2}\right) = \frac{n}{\theta^2}, \qquad \ldots(2)$$

$$\therefore\ g(\theta) \propto \frac{\sqrt{n}}{\theta} \Rightarrow g(\theta) = k\frac{\sqrt{n}}{\theta}$$

$$L(t_1, t_2,\ldots,t_n) = \prod_{i=1}^{n} f(t_i \mid \theta) = \frac{1}{\theta^n}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)$$

The joint probability density function H (t_1, t_2,…, tn,θ) is given by

$$H(t_1, t_2,\ldots,t_n,\theta) = \prod_{i=1}^{n} f(t_i,\theta)\, g(\theta)$$

$$= L(t_1, t_2,\ldots,t_n \mid \theta)\, g(\theta)$$

$$= \frac{1}{\theta^n}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)\frac{k\sqrt{n}}{\theta}$$

$$= \frac{k\sqrt{n}}{\theta^{n+1}}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) \qquad \ldots(3)$$

The marginal probability density function of (t_1, t_2,…,t_n) is given by

$$p(t_1, t_2,\ldots,t_n) = \int_{\theta} H(t_1, t_2,\ldots,t_n,\theta)\, d\theta$$

$$= \int_0^{\infty} \frac{k\sqrt{n}}{\theta^{n+1}}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) d\theta$$

$$= \frac{\left(k\sqrt{n}\right)(n-1)!}{\left(\sum_{i=1}^{n} t_i\right)^n}$$

And the condition probability density function of θ given the data (t_1, t_2,…,t_n) is given by

$$\prod(\theta \mid t_1, t_2,\ldots,t_n) = \frac{H(t_1,t_2,\ldots,t_n,\theta)}{p(t_1,t_2,\ldots,t_n)}$$

$$= \frac{\exp\left[-\frac{\sum_{i=1}^{n} t_i}{\theta}\right]\left[\sum_{i=1}^{n} t_i\right]^n}{\theta^{n+1}\ (n-1)!}$$

Using squared error loss function

$$L(\hat\theta,\theta) = c(\hat\theta-\theta)^2$$

We can given Risk function, such that

$$R(\hat\theta,\theta) = E[L(\hat\theta,\theta)]$$

$$= \int_0^{\infty} L(\hat\theta,\theta)\prod(\theta \mid t_1,t_2,\ldots,t_n)\, d\theta$$

$$R(\hat\theta,\theta) = \int_0^{\infty} c(\hat\theta-\theta)^2\, \frac{\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)\left(\sum_{i=1}^{n} t_i\right)^n}{\theta^{n+1}\ (n-1)!} d\theta \qquad \ldots(4)$$

$$= c\hat\theta^2 - 2c\hat\theta\int_0^{\infty}\frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!}\,\theta^{-n}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) d\theta + \phi(\theta)$$

$$\frac{\partial R(\hat\theta,\theta)}{\partial\hat\theta} = 2c\hat\theta - 2c\frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!}\cdot$$

$$\int_0^{\infty}\theta^{-n}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) d\theta + \text{zero}$$

Let $\frac{\partial R(\hat\theta,\theta)}{\partial\hat\theta} = 0$, then

$$\hat\theta_B = \frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!}\int_0^{\infty}\theta^{-n}\exp\left[-\frac{\sum_{i=1}^{n} t_i}{\theta}\right] d\theta$$

$$= \frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!}\int_0^{\infty}\left[\frac{\sum_{i=1}^{n} t_i}{y}\right]^{-n}\exp(-y)\left(\frac{\sum_{i=1}^{n} t_i}{y^2}\right) dy$$

$$= \frac{\left(\sum_{i=1}^{n} t_i\right)}{(n-1)!}\int_0^{\infty} y^{n-2}\exp(-y)\, dy$$

$$\hat\theta_B = \frac{\left(\sum_{i=1}^{n} t_i\right)(n-2)!}{(n-1)!}$$

$$\therefore\ \hat{\theta}_B = \frac{\sum_{i=1}^{n} t_i}{n-1} \text{ (Bayes estimator)} \quad \ldots(5)$$

where $E(\hat{\theta}_B) = \frac{n}{n-1}\theta$

$$\text{Bias}(\hat{\theta}_B) = E(\hat{\theta}_B) - \theta = \frac{\theta}{n-1} \quad \ldots(6)$$

$$\text{MSE}(\hat{\theta}_B) = E(\hat{\theta}_B - \theta)^2 = \frac{(n+1)}{(n-1)^2}\theta^2 \quad \ldots(7)$$

1.3 Double Stage Shrinkage Procedure: -

A Double Stage Shrinkage Procedure is defined as follows; see [3], [4], [5], [6].

Let x_{1i}; i = 1, 2, …, n_1 be a random sample of n_1 from exponential distribution and $\hat{\theta}_1$ be a classical estimator (MLE) of θ based on n_1 observation. Construct a preliminary test region (R) in the parameter space based on prior estimate θ_0 and an appropriate criterion.

If $\hat{\theta}_1 \in$ R shrink $\hat{\alpha}_1$ towards α_0 by shrinkage weight factor $\psi(\hat{\theta}_1)$; $0 \le \psi(\hat{\theta}_1) \le 1$ and use the shrinkage estimator $\psi(\hat{\theta}_1)\ \hat{\theta}_1 + (1 - \psi(\hat{\theta}_1))\theta_0$, for estimate θ.

If $\hat{\theta}_1 \notin$ R, obtain x_{2i}; i = 1, 2, …, n_2, an additional sample of size n_2 and use a pooled estimator $\hat{\alpha}_p$ of α based on combined sample of size n = $n_1 + n_2$, i.e.; $\hat{\theta}_p = \frac{n_1\hat{\theta}_1 + n_2\hat{\theta}_2}{n}$.

Thus, the Double Stage Shrinkage Estimator (DSSE) will be

$$\tilde{\theta}_{DS} = \begin{cases} \psi(\hat{\theta}_1)\hat{\theta}_1 + (1-\psi(\hat{\theta}_1))\theta_0 & ;\text{if } \hat{\theta}_1 \in R \\ \hat{\theta}_p & ;\text{if } \hat{\theta}_1 \notin R \end{cases} \quad \ldots(8)$$

To motivation of this study was provided by the work of [3], [4], [5], [6], [7], [8],[9],[10] and others.

The aim of this paper is to employ Bayesian estimator which is defined in (5) in the form of double stage shrinkage estimator (DSSE) which is defined in (8) for estimate the scale parameter (θ) of Exponential Distribution.

2. Modified Double Stage Shrinkage-Bayesian Estimator

This section is concern with pooling approach between shrinkage estimation that uses a prior information about unknown parameter as initial value and Bayesian estimation that uses a prior information about unknown parameter as a prior distribution for the scale parameter (θ) of exponential distribution using special shrinkage weight factors as well suitable region R when a prior information about (θ) is available as initial value (θ_0).

In the present work we establish modified Double Stage Shrinkage-Bayesian Estimator (DSSBE) which has the following form:-

$$\tilde{\theta}_{DS} = \begin{cases} \psi(\hat{\theta}_1)\hat{\theta}_{1B} + (1-\psi(\hat{\theta}_1))\theta_0 & ;\text{if } \hat{\theta}_1 \in R \\ \hat{\theta}_{PB} = \frac{n_1\hat{\theta}_{1B} + n_2\hat{\theta}_{2B}}{n} & ;\text{if } \hat{\theta}_1 \notin R \end{cases} \quad \ldots(9)$$

where $\hat{\theta}_{iB}(i = 1,2)$ represent to Bayes estimator for θ on $n_{i(i=1,2)}$ observation, R is suitable region (say pretest region) and $\psi(\hat{\theta}_1)$; $0 \le \psi(\hat{\theta}_1) \le 1$ is shrinkage weight factor which may be a function of $\hat{\theta}_1$ or constant, see for example : [3], [4], [7] and [10].

The Expressions for Bias, Mean Square Error [MSE], Relative Efficiency [R.Eff(·)], Expected sample size, Expected sample size proportion, probability for avoiding the second sample and percentage of overall sample saved are derived and obtained for the proposed estimator.

Numerical results and discussions due mentioned expressions including some constants are performed and displayed in annexed tables.

Comparisons between the proposed estimator with the classical estimator $(\hat{\theta})$ and with some of the last studies are demonstrated.

2.1 Modified DSSBE($\tilde{\theta}_{DS}$) Using Constant Shrinkage Weight Factor:

Using the form (9) above, the proposed DSSBE $\tilde{\theta}_{DS}$ has the following forms:

$$\tilde{\theta}_{DS} = \begin{cases} \theta_0 & ,\text{ if } \hat{\theta}_1 \in R \\ \hat{\theta}_{PB} & ,\text{ if } \hat{\theta}_1 \notin R \end{cases} \quad \ldots(10)$$

i.e.; we place $\psi(\hat{\theta}_1) = 0$ (constant).

Where R is pretest region for acceptance of size α for testing the hypothesis H_0: $\theta = \theta_0$ Vs. the hypothesis H_A : $\theta \neq \theta_0$ using test statistic $T(\hat{\theta}_1|\theta) = \frac{2n_1\hat{\theta}_1}{\theta_0}$

In that,

$$R = \left[\frac{\theta_0}{2n_1}X^2_{1-\alpha/2,2n_1}, \frac{\theta_0}{2n_1}X^2_{\alpha/2,2n_1}\right] \quad \ldots(11)$$

Assume that, R=[a , b] , $a < b$.

i.e. $a = \frac{\theta_0}{2n_1}X^2_{1-\alpha/2,2n_1}$ and $b = \frac{\theta_0}{2n_1}X^2_{\alpha/2,2n_1}$ …(12)

where $X^2_{1-\alpha/2,2n_1}$ and $X^2_{\alpha/2,2n_1}$ are respectively lower and upper 100(α/2) percentile point of Chi-square distribution with degree of freedom ($2n_1$).

The Expression for Bias is given below

$$Bias(\tilde{\theta}_{DS}|\theta,R) = E(\tilde{\theta}_{DS}) - \theta$$

$$= \left\{\int_{\hat{\theta}_2=0}^{\infty}\int_{\hat{\theta}_1\in R}(\theta_0-\theta) + \int_{\hat{\theta}_2=0}^{\infty}\int_{\hat{\theta}_1\in \bar{R}}(\hat{\theta}_{pB}-\theta)\right\}\prod_{i=1}^{2} f(\hat{\theta}_i;\theta)d\hat{\theta}_i$$

where $\bar{R}$ is the complement region of R in real space and $f(\hat{\theta}_i;\theta)$ {for i=1,2} is a p.d.f. of $\hat{\theta}_i$ which has the following form:-

$$f(\hat{\theta}_i;\theta) = \begin{cases} \frac{[\hat{\theta}_i]^{n_i-1}\exp[-n_i\hat{\theta}_i/\theta]}{\Gamma(n_i)(\theta/n_i)^{n_i}} & ,\text{for } 0<\hat{\theta}_i<\infty \\ 0 & ,\text{o.w} \end{cases}$$

We conclude:

$$Bias(\tilde{\theta}_{DS}|\theta,R) = \theta\left\{(\lambda-1)J_1(a^*,b^*) + \left[\frac{1}{(1+u)(n_1-1)} + \frac{u}{(1+u)(un_1-1)}\right] - \left[\frac{1}{(1+u)}\left(\frac{n_1}{n_1-1}J_1(a^*,b^*) - J_0(a^*,b^*)\right)\right] - \left[\frac{u}{(1+u)(un_1-1)}J_0(a^*,b^*)\right]\right\} \quad \ldots(13)$$

where,

$\lambda = \theta_0/\theta$, $y = n_1\hat{\theta}_1|\theta$, $u = n_2/n_1$, $n = n_1 + n_2$,

$$J_\ell(a^*,b^*) = \frac{1}{n_1^\ell\Gamma(n_1)}\int_{a^*}^{b^*} y^\ell y^{n_1-1}e^{-y}dy \quad \ldots(14)$$

And $a^* = \lambda X^2_{1-\alpha/2,2n_1}$, $b^* = \lambda X^2_{\alpha/2,2n_1}$ …(15)

The Bias ratio $B(\cdot)$ of $\tilde{\theta}_{DS}$ is defined as below

$$B(\tilde{\theta}_{DS}|\theta,R) = \frac{Bias(\tilde{\theta}_{DS}|\theta,R)}{\theta} \quad \ldots(16)$$

See [6], [7] and [9].

The expression of mean square error [MSE] of $\tilde{\theta}_{DS}$ is as follows

$$MSE(\tilde{\theta}_{DS}|\theta,R) = E(\tilde{\theta}_{DS}-\theta)^2$$

$$= \theta^2\left\{(\lambda-1)^2J_0(a^*,b^*) + \frac{1}{(1+u)^2}\frac{n_1+1}{(n_1-1)^2} + \frac{u^2}{(1+u)^2}\frac{n_1u+1}{(n_1u-1)^2} + \frac{u^2}{(u+1)^2(n_1-1)(n_1u-1)} - \frac{1}{(1+u)^2}\left[\frac{n_1^2}{(n_1-1)^2}J_2(a^*,b^*) - 2\frac{n_1}{n_1-1}J_1(a^*,b^*) + J_0(a^*,b^*)\right] - \frac{u^2}{(u+1)^2(n_1u-1)}\left[\frac{n_1}{n_1-1}J_1(a^*,b^*) + J_0(a^*,b^*)\right] - \left(\frac{u}{1+u}\right)^2\frac{n_1u+1}{(n_1u-1)^2}J_0(a^*,b^*)\right\}$$

… (17)

The Relative Efficiency of Estimator $\tilde{\theta}_{DS}$ relative to classical estimator ($\hat{\theta}_1$) is defined as below:-

$$R.Eff(\tilde{\theta}_{DS}|\theta,R) = \frac{MSE(\hat{\theta})}{MSE(\tilde{\theta}_{DS}|\theta,R)[E(n|\theta,R)/n]} \quad \ldots(18)$$

where $E(n|\alpha,R)$ is the Expected sample size, which is defined as:

$$E(n|\theta,R) = n\left[1 - \frac{u}{1+u}J_0(a^*,b^*)\right].$$

See for example [3], [6], [7], [9] and [10].

As well as, the Expected sample size proportion { E(n | α,R)/n } equal to

$$1-\frac{u}{1+u}J_0(a^*,b^*) \qquad \ldots(19)$$

See [6],[7]and [9].

Also, it is necessary to define the percentage of the overall sample saved (P.O.S.S) of $\tilde{\theta}_{DS}$ as:

$$P.O.S.S.=\frac{n_2}{n}J_0(a^*,b^*)*100 \qquad \ldots(20)$$

See [6] ,[7]and [9].

And, finally, $p(\hat{\theta}_1 \in R)$ represent the probability of a voiding the second sample.

3. Numerical Results and Discussion:

The computations of Relative Efficiency [R.Eff (·)] and Bias Ratio [B(·)], Expected sample size [E(n | θ,R)], Expected sample size proportion [E(n | θ,R)/n], Percentage of the overall sample saved (P.O.S.S.) and probability of avoiding the second sample $p(\hat{\theta}_1 \in R)$ were used for the estimator $\tilde{\theta}_{DS}$. These computations (using Mat. LAB programs) were performed for n_1 = 4, 6, 8, 10, 12, 16, u = (n_2/n_1) = 0.5, 1, 2, 3, 9, 12, λ = (θ_0/θ) = 0.25(0.25)2, α = 0.01, 0.05, 0.1.

Some of these computations are given in tables (1) - (12).

The observation mentioned in the tables leads to the following results:

i. The Relative Efficiency [R.Eff (·)] of $\tilde{\theta}_{DS}$ are adversely proportional with small value of α especially when λ = 1, i.e. α = 0.01 yield highest efficiency. see table (1) ,(2).

ii. The Relative Efficiency [R.Eff (·)] of $\tilde{\theta}_{DS}$ has maximum value when $\theta=\theta_0(\lambda=1)$, for each n_1 and α, and decreasing otherwise (λ≠1). This feature shown the important usefulness of prior knowledge which given higher Effects of proposed estimator as well as the important role of shrinkage technique and its philosophy. see table (1) ,(2).

iii. Bias Ratio [B(·)] of $\tilde{\theta}_{DS}$ are reasonably small when $\theta = \theta_0$ for each n_1, α, and increases otherwise. This property shown that the proposed estimator $\tilde{\theta}_{DS}$ is very closely to unbiased ness property especially when $\theta = \theta_0$. See table (1) ,(2).

iv. The Effective interval of $\tilde{\theta}_{DS}$ [the value of λ which makes R.Eff.(·) of $\tilde{\theta}_{DS}$ greater than one] is approximately [0.75, 1.25]. See table (1) ,(2).

v. Bias Ratio [B(·)] of $\tilde{\theta}_{DS}$ are reasonably small with small value of u. see table (1) ,(2).

vi. R.Eff ($\tilde{\theta}_{DS}$) is decreasing function with increasing of the first sample size n_1, for each α and λ. See table (1) ,(2).

vii. The Expected value of sample size of $\tilde{\theta}_{DS}$ is close to n_1, especially when $0.5 \le \lambda < 1$ and start faraway otherwise. see table (3) ,(4),(5),(8), (9) ,(10).

viii. Percentage of the overall sample saved $\left[\frac{n_2}{n}J_0(a^*,b^*)*100\right]$ is increasing value with increasing value of u ($u = n_2/n_1$) and decreasing value with increasing value of $\lambda \ge 0.5$. see table (6) ,(11).

ix. R.Eff ($\tilde{\theta}_{DS}$) is an increasing function with respect to u. This property shown the effective of proposed estimator using small n_1 relative to n_2 (or large n_2) which given higher efficiency and reduce the observation cost. See table (1) ,(2).

x. the Probability of avoiding second sample is very suitable especially when $\theta \approx \theta_0$, see table (7) , (12) .

xi. The considered estimator $\tilde{\theta}_{DS}$ is better than the classical estimator $\hat{\theta}$ especially when $\theta \approx \theta_0$, this will given the Effective of $\tilde{\theta}_{DS}$ Relative to $\hat{\theta}$ and also given an important weight of prior knowledge, and the augmentation of efficiency may be reach to tens times. see table (1) ,(2).

xii. The considered estimator $\tilde{\theta}_{DS}$ is more efficient than the estimators introduced by [6] and [7] in the sense of higher efficiency.

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Table (1)

Shown Bias Ratio [B(·)] and Relative Efficiency [R.Eff.(·)] of $\tilde{\theta}_{DS}$ w.r.t. u, n_1 and λ when α=0.01

u	n_1	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	4	R.Eff.$_C$(·)	0.3305	0.9384	3.7891	35.8892	3.3465	0.9428	0.4402	0.2596
		R.Eff.$_B$(·)	0.4857	1.3788	5.5677	52.7352	4.9173	1.3853	0.6468	0.3814
		B(·)	−0.4257	−0.4877	−0.2489	−0.0078	0.2163	0.4145	0.5809	0.7124
	6	R.Eff.$_C$(·)	0.2587	0.6464	2.5308	19.3121	2.1616	0.6568	0.3375	0.2271
		R.Eff.$_B$(·)	0.3335	0.8334	3.2629	24.8982	2.7869	0.8468	0.4351	0.2928
		B(·)	−0.3848	−0.4906	−0.2488	−0.0142	0.1830	0.3256	0.4093	0.4416
	8	R.Eff.$_C$(·)	0.2161	0.4903	1.8921	12.1306	1.6177	0.5593	0.3406	0.2814
		R.Eff.$_B$(·)	0.2613	0.5927	2.2874	14.6646	1.9557	0.6761	0.4117	0.3401
		B(·)	−0.3351	−0.4918	−0.2477	−0.0193	0.1493	0.2398	0.2643	0.2500
	12	R.Eff.$_C$(·)	0.1778	0.3297	1.2551	6.3488	1.1810	0.5852	0.5170	0.5677
		R.Eff.$_B$(·)	0.2016	0.3739	1.4235	7.2009	1.3395	0.6638	0.5864	0.6439
		B(·)	−0.2405	−0.4930	−0.2436	−0.0241	0.0959	0.1245	0.5170	0.1021
3	4	R.Eff.$_C$(·)	0.3128	0.9642	3.7634	69.9797	3.1479	0.7822	0.3355	0.1843
		R.Eff.$_B$(·)	0.3782	1.1656	4.5496	84.5976	3.8054	0.9456	0.4055	0.2228
		B(·)	−0.5147	−0.4915	−0.2473	−0.0046	0.2205	0.4186	0.5830	0.7102
	6	R.Eff.$_C$(·)	0.2075	0.6482	2.4611	33.5512	1.8257	0.4678	0.2174	0.1372
		R.Eff.$_B$(·)	0.2353	0.7352	2.7914	38.0543	2.0708	0.5306	0.2466	0.1556
		B(·)	−0.4599	−0.4928	−0.2458	−0.0076	0.1915	0.3319	0.4081	0.4285
	8	R.Eff.$_C$(·)	0.1572	0.4876	1.8036	18.8733	1.2417	0.3611	0.2046	0.1661
		R.Eff.$_B$(·)	0.1727	0.5358	1.9819	20.7391	1.3645	0.3968	0.2249	0.1825
		B(·)	−0.4023	−0.4934	−0.2436	−0.0101	0.1591	0.2417	0.2516	0.2203
	12	R.Eff.$_C$(·)	0.1150	0.3258	1.1428	7.9895	0.8251	0.3769	0.3567	0.4562
		R.Eff.$_B$(·)	0.1224	0.3469	1.2167	8.5068	0.8785	0.4013	0.3798	0.4857
		B(·)	−0.2961	−0.4940	−0.2376	−0.0124	0.0999	0.1103	0.0842	0.0618
9	4	R.Eff.$_C$(·)	0.1565	0.7168	2.3860	22.2987	1.3441	0.3380	0.1561	0.0975
		R.Eff.$_B$(·)	0.1687	0.7729	2.5727	24.0433	1.4493	0.3645	0.1684	0.1052
		B(·)	−0.4296	−0.4696	−0.2313	−0.0037	0.1743	0.2938	0.3582	0.3783
	6	R.Eff.$_C$(·)	0.1032	0.4806	1.4152	10.4846	0.7834	0.1821	0.1521	0.1817
		R.Eff.$_B$(·)	0.1085	0.5053	1.4880	11.0237	0.8237	0.2598	0.1599	0.1385
		B(·)	−0.3581	−0.4717	−0.2240	−0.0045	0.1306	0.2471	0.1779	0.1485
	8	R.Eff.$_C$(·)	0.0812	0.3612	0.9582	6.0224	0.6107	0.2624	0.2256	0.2697
		R.Eff.$_B$(·)	0.0843	0.3751	0.9949	6.2530	0.6341	0.2724	0.2343	0.2800
		B(·)	−0.2937	−0.4726	−0.2258	−0.0042	0.0936	0.1046	0.0817	0.0572
	12	R.Eff.$_C$(·)	0.0671	0.2413	0.5456	2.8836	0.5842	0.46.39	0.6103	0.7962
		R.Eff.$_B$(·)	0.0688	0.2474	0.5594	2.9567	0.5990	0.4756	0.6258	0.8164
		B(·)	−0.1914	−0.4734	−0.1979	−0.0022	0.0451	0.0344	0.0231	0..0129
12	4	R.Eff.$_C$(·)	0.01596	0.8999	3.2827	170.9368	2.0479	0.4202	0.1579	0.0787
		R.Eff.$_B$(·)	0.1691	0.9536	3.4783	181.1234	2.1699	0.4452	0.1673	0.0834
		B(·)	−0.5723	−0.4939	−0.2458	−0.0015	0.2252	0.4243	0.5885	0.7142
	6	R.Eff.$_C$(·)	0.0917	0.6006	2.0033	67.1863	0.9563	0.1988	0.0827	0.0489
		R.Eff.$_B$(·)	0.0953	0.6242	2.0820	69.8266	0.9941	0.2066	0.0859	0.0508
		B(·)	−0.5096	−0.4943	−0.2435	−0.0024	0.1989	0.3387	0.4108	0.4239
	8	R.Eff.$_C$(·)	0.0637	0.4506	1.3679	31.5402	0.5703	0.1379	0.0727	0.0577
		R.Eff.$_B$(·)	0.0656	0.4638	1.4080	32.4648	0.5870	0.1419	0.0749	0.0594
		B(·)	−0.4474	−0.4945	−0.2406	−0.0032	0.1671	0.2451	0.2454	0.2030
	12	R.Eff.$_C$(·)	0.0423	0.3005	0.7548	10.2308	0.3508	0.1446	0.1437	0.0354

		R.Eff.$_B$(·)	0.0431	0.3063	0.7695	10.4296	0.3576	0.1474	0.1465	0.2199
		B(·)	−0.3337	−0.4947	−0.2333	−0.0039	0.1036	0.1018	0.0645	0.2157

Table (2)

Shown Bias Ratio [B(·)] and Relative Efficiency [R.Eff. (·)] of $\tilde{\theta}_{DS}$ w.r.t. u, n_1 and λ when α=0.05

u	**n_1**	**λ**	**0.25**	**0.5**	**0.75**	**1**	**1.25**	**1.5**	**1.75**	**2**
1	4	**R.Eff.$_C$(·)**	0.3087	0.8177	3.1909	9.8719	2.4474	0.8913	0.4849	0.3305
		R.Eff.$_B$(·)	0.4536	1.2015	4.6887	14.5057	3.5962	1.3096	0.7125	0.4856
		B(·)	−0.2283	−0.4460	−0.2377	−0.0119	0.1739	0.3105	0.3997	0.4494
	6	**R.Eff.$_C$(·)**	0.2704	0.6025	2.2648	6.6277	1.7150	0.7266	0.4783	0.4010
		R.Eff.$_B$(·)	0.3486	0.7768	2.9199	8.5448	2.2111	0.9367	0.6166	0.5170
		B(·)	−0.2044	−0.4575	−0.2361	−0.0182	0.1321	0.2125	0.2423	0.2441
	8	**R.Eff.$_C$(·)**	0.2460	0.4685	1.7350	4.7315	1.3649	0.6991	0.5538	0.5349
		R.Eff.$_B$(·)	0.2974	0.5664	2.0974	5.7199	1.6500	0.8451	0.6695	0.6466
		B(·)	−0.1668	−0.4625	−0.2309	−0.0181	0.1044	0.1510	0.1586	0.1540
	12	**R.Eff.$_C$(·)**	0.2334	0.3214	1.1921	2.8392	1.0528	0.7480	0.7065	0.7128
		R.Eff.$_B$(·)	0.2647	0.3645	1.3521	3.2202	1.1941	0.8427	0.8013	0.8084
		B(·)	−0.0990	−0.4670	−0.2161	−0.0089	0.0747	0.0917	0.0921	0.0913
3	4	**R.Eff.$_C$(·)**	0.2823	0.8630	3.1404	15.4862	2.1773	0.6526	0.3252	0.2116
		R.Eff.$_B$(·)	0.3413	1.0433	3.7964	18.7211	2.6321	0.7889	0.3931	0.2558
		B(·)	−0.3579	−0.4612	−0.2346	−0.0086	0.1711	0.2958	0.3683	0.3984
	6	**R.Eff.$_C$(·)**	0.2055	0.5918	2.0429	8.4952	1.3600	0.4978	0.3182	0.2740
		R.Eff.$_B$(·)	0.2331	0.6712	2.3171	9.6353	1.5425	0.5646	0.3609	0.3107
		B(·)	−0.3024	−0.4665	−0.2290	−0.0107	0.1293	0.1909	0.1989	0.1809
	8	**R.Eff.$_C$(·)**	0.1702	0.4485	1.4838	5.3742	1.0547	0.5028	0.4208	0.4559
		R.Eff.$_B$(·)	0.1871	0.4928	1.6304	5.9054	1.1589	0.5525	0.4624	0.5010
		B(·)	−0.2473	−0.4689	−0.2218	−0.0103	0.0963	0.1203	0.1086	0.0915
	12	**R.Eff.$_C$(·)**	0.1476	0.3015	0.9378	2.8385	0.8846	0.6729	0.7304	0.8016
		R.Eff.$_B$(·)	0.1571	0.3210	0.9985	3.0222	0.9418	0.7165	0.7777	0.8535
		B(·)	−0.1574	−0.4711	−0.2049	−0.0052	0.0555	0.0550	0.0481	0.0452
9	4	**R.Eff.$_C$(·)**	0.1565	0.7168	2.3860	22.2987	1.3441	0.3380	0.1561	0.0975
		R.Eff.$_B$(·)	0.1687	0.7729	2.5727	24.0433	1.4493	0.3645	0.1684	0.1052
		B(·)	−0.4296	−0.4696	−0.2313	−0.0037	0.1743	0.2938	0.3582	0.3783
	6	**R.Eff.$_C$(·)**	0.1032	0.4806	1.4152	10.4846	0.7834	0.2471	0.1521	0.1317
		R.Eff.$_B$(·)	0.1085	0.5053	1.4880	11.0237	0.8237	0.2598	0.1599	0.1385
		B(·)	−0.3581	−0.4717	−0.2240	−0.0045	0.1306	0.1821	0.1779	0.1485
	8	**R.Eff.$_C$(·)**	0.0812	0.3612	0.9582	6.0224	0.6107	0.2624	0.2256	0.2697
		R.Eff.$_B$(·)	0.0843	0.3751	0.9949	6.2530	0.6341	0.2724	0.2343	0.2800
		B(·)	−0.2937	−0.4726	−0.2158	−0.0042	0.0936	0.1046	0.0817	0.0572
	12	**R.Eff.$_C$(·)**	0.0671	0.2413	0.5456	2.8836	0.5842	0.4639	0.6103	0.7962
		R.Eff.$_B$(·)	0.0688	0.2474	0.5594	2.9567	0.5990	0.4756	0.6258	0.8164
		B(·)	−0.1914	−0.4734	−0.1979	−0.0022	0.0451	0.0344	0.0231	0.0189
12	4	**R.Eff.$_C$(·)**	0.1263	0.6524	2.1050	23.9545	1.1174	0.2717	0.1231	0.0767
		R.Eff.$_B$(·)	0.1338	0.6913	2.2304	25.3820	1.1840	0.2878	0.1311	0.0813
		B(·)	−0.4405	−0.4708	−0.2307	−0.0029	0.1749	0.2937	0.3569	0.3755
	6	**R.Eff.$_C$(·)**	0.0820	0.4365	1.2179	10.9075	0.6437	0.1971	0.1204	0.1043
		R.Eff.$_B$(·)	0.0853	0.4537	1.2657	11.3362	0.6690	0.2049	0.1251	0.1084
		B(·)	−0.3666	−0.4724	−0.2232	−0.0035	0.1309	0.1809	0.1748	0.1437
	8	**R.Eff.$_C$(·)**	0.0641	0.3279	0.8096	6.1513	0.5038	0.2114	0.1824	0.2221
		R.Eff.$_B$(·)	0.0660	0.3375	0.8334	6.3316	0.5186	0.2176	0.1878	0.2286
		B(·)	−0.3007	−0.4731	−0.2149	−0.0033	0.0932	0.1023	0.0776	0..0521
	12	**R.Eff.$_C$(·)**	0.0526	0.2188	0.4496	2.8942	0.4990	0.3987	0.5541	0.7710
		R.Eff.$_B$(·)	0.0536	0.2231	0.4584	2.9505	0.5087	0.4064	0.5649	0.7859
							0.0436	0.0313	0.0193	0.0149

		B(·)	− 0.1966	− 0.4738	− 0.1968	− 0.0017				

Table (3)

Shown Expected Sample Size [E] and Expected Sample Size Proportion [Ep] of $\tilde{\theta}_{DS}$ w.r.t. u and λ when n_1 = 4, α=0.01

u	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	EP	0.6017	0.5050	0.5098	0.5238	0.5451	0.5729	0.6057	0.6417
	E	4.0400	0.0400	4.0783	4.1906	4.3612	4.5835	4.8458	5.1338
3	EP	0.4025	0.2575	0.2647	0.2857	0.3177	0.3594	0.4086	0.462
	E	6.4406	4.1200	4.2348	4.5719	5.0835	5.7506	6.5374	7.4013
9	EP	0.2830	0.1090	0.1176	0.1429	0.1813	0.2313	0.2903	0.3551
	E	11.3218	4.3600	4.7043	5.7156	7.2506	9.2518	11.6122	14.2038
12	EP	0.264	0.0862	0.0950	0.1209	0.1603	0.2116	0.2721	0.3386
	E	13.7624	4.4800	4.9391	6.2875	8.3341	11.0024	14.1496	17.601

Table (4)

Shown Expected Sample Size [E] and Expected Sample Size Proportion [Ep] of $\tilde{\theta}_{DS}$ w.r.t. u and λ when n_1 = 6, α=0.01

u	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	EP	0.6457	0.5050	0.5151	0.5458	0.5954	0.6580	0.7251	0.7889
	E	7.7484	6.0600	6.1818	6.5492	7.1444	7.8965	8.7016	9.4664
3	EP	0.4685	0.2575	0.2727	0.3187	0.3930	0.4871	0.5877	0.6833
	E	11.2451	6.1800	6.5453	7.6477	9.4331	11.6896	14.1047	16.3991
9	EP	0.3623	0.1090	0.1273	0.1824	0.2717	0.3845	0.5052	0.6200
	E	21.7354	6.5400	7.6358	10.9432	16.2992	23.0687	30.3141	37.1973
12	EP	0.3459	0.0862	0.1049	0.1614	0.2530	0.3687	0.4925	0.6102
	E	26.9806	6.7200	8.1810	12.5909	19.7323	28.7583	38.4189	47.5964

Table (5)

Shown Expected Sample Size [E] and Expected Sample Size Proportion [Ep] of $\tilde{\theta}_{DS}$ w.r.t. u and λ when n_1 = 8, α=0.01

u	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	EP	0.6885	0.5050	0.5215	0.5743	0.6584	0.7532	0.8380	0.9022
	E	11.0165	8.0800	8.3445	9.1891	10.5337	12.0509	13.4077	14.4345
3	EP	0.5328	0.2575	0.2823	0.3615	0.4875	0.6298	0.7570	0.8532
	E	17.0495	8.2400	9.0336	11.5674	15.6010	20.1526	24.2232	27.3035
9	EP	0.4394	0.1090	0.1388	0.2338	0.3850	0.5557	0.7084	0.8239
	E	35.1483	8.7200	11.1008	18.7021	30.8029	44.4577	56.6697	65.9105
12	EP	0.4250	0.0862	0.1167	0.2141	0.3693	0.5443	0.7009	0.8194
	E	44.1980	8.9600	12.1344	22.2694	38.4039	56.6102	72.8930	85.2141

Table (6)

Shown the Percentage of overall Sample Saved (P.O.S.S.) of $\tilde{\theta}_{DS}$ w.r.t. u, n_1 and λ when α=0.01

u	n_1	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	4	POSS	39.8308	49.5000	49.0218	47.6172	45.4853	42.7058	39.4275	35.8280
	6		35.4302	49.5000	48.4854	45.4230	40.4637	34.1956	27.4869	21.1136
	8		31.1469	49.5000	47.8467	42.5680	34.1646	24.6822	16.7843	9.7843
	12		23.3572	49.5001	46.2829	35.4833	21.0689	9.8173	3.7270	1.1963
3	4	POSS	59.7462	74.2500	73.5327	71.4259	68.2279	64.0587	59.1412	53.7420
	6		53.1453	74.2500	72.7281	68.1344	60.6956	51.2934	41.2304	31.6704
	8		46.7203	74.2500	71.7700	63.8520	51.2469	37.0233	24.3024	14.6765
	12		35.0358	74.2501	69.4244	53.2249	31.6033	14.7260	5.5905	1.7944
9	4	POSS	71.6954	89.1000	88.2392	85.7110	81.8735	76.8704	70.9694	64.4904
	6		63.7743	89.1000	87.2737	81.7613	72.8347	61.5521	49.4764	38.0045
	8		56.0643	89.1000	86.1240	76.6224	61.4963	44.4279	29.1628	17.6118
	12		42.0430	89.1001	83.3093	63.8699	37.9240	17.6712	6.7086	2.1533
12	4	POSS	73.5338	91.3846	90.5017	87.9087	83.9728	78.8414	72.7892	66.1440
	6		65.4095	91.3846	89.5115	83.8578	74.7022	63.1304	50.7451	38.9790
	8		57.5019	91.3846	88.3323	78.5871	63.0731	45.5671	29.9106	18.0634
	12		43.1210	91.3848	85.4454	65.5076	38.8964	18.1243	6.8807	2.2085

Table (7)

Shown the Probability of avoiding Second Sample [Av] w.r.t. u, n_1 and λ when α=0.01

u	n_1	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	4	AV	0.7966	0.9900	0.9804	0.9523	0.9097	0.8541	0.7885	0.7166
	6		0.7086	0.9900	0.9697	0.9085	0.8093	0.6839	0.5497	0.4223
	8		0.6229	0.9900	0.9569	0.8514	0.6833	0.4936	0.3240	0.1957
	12		0.4671	0.9900	0.9257	0.7097	0.4214	0.1963	0.0745	0.0239
3	4	AV	0.766	0.9900	0.9804	0.9523	0.9097	0.8541	0.7885	0.7166
	6		0.7086	0.9900	0.9697	0.9085	0.8093	0.6839	0.5497	0.4223
	8		0.6229	0.9900	0.9569	0.8514	0.6833	0.4936	0.3240	0.1957
	12		0.4671	0.9900	0.9257	0.7097	0.4214	0.1963	0.0745	0.0239
9	4	AV	0.7966	0.9900	0.9804	0.9523	0.9097	0.8541	0.7885	0.7166
	6		0.7086	0.9900	0.9697	0.9085	0.8093	0.6839	0.5497	0.4223
	8		0.6229	0.9900	0.9569	0.8514	0.6833	0.4936	0.3240	0.1957

Table (8)

Shown Expected Sample Size [E] and Expected Sample Size Proportion [Ep] of $\tilde{\theta}_{DS}$ w.r.t. u and λ when n_1 = 4, α=0.05

u	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	EP	0.6823	0.5250	0.5423	0.5883	0.6457	0.7065	0.7647	0.8167
	E	5.4587	4.2000	4.3384	4.7068	5.1654	5.6517	6.1179	6.5337
3	EP	0.5235	0.2875	0.3135	0.3825	0.4685	0.5597	0.64171	0.7251
	E	8.3761	4.5999	5.0153	6.1204	7.4961	8.9551	10.3537	11.6012
9	EP	0.4282	0.1450	0.1762	0.2590	0.3622	0.4716	0.5765	0.6701
	E	17.1284	5.7998	7.0460	10.3611	14.4884	18.8653	23.0610	26.8035
12	EP	0.4135	0.1231	0.1550	0.2400	0.3459	0.4581	0.5657	0.6616
	E	21.5045	6.3998	8.0613	12.4814	17.9845	23.8204	29.4147	34.4046

Table (9)

Shown Expected Sample Size [E] and Expected Sample Size Proportion [Ep] of $\tilde{\theta}_{DS}$ w.r.t. u and λ when n_1 = 6, α=0.05

u	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	EP	0.7368	0.5250	0.5590	0.6404	0.7359	0.8231	0.8902	0.9361
	E	8.8418	6.3000	6.7076	7.6844	8.8314	9.8768	10.6823	11.2327
3	EP	0.6052	0.2875	0.3384	0.4606	0.6039	0.7346	0.8353	0.9041
	E	14.5255	6.9000	8.1228	11.0532	14.4941	17.6304	20.0470	21.6981
9	EP	0.5263	0.1450	0.2061	0.3527	0.5247	0.6815	0.8024	0.8849
	E	31.5766	8.7000	12.3684	21.1596	31.4822	40.8912	48.1410	53.0942
12	EP	0.5141	0.1231	0.1858	0.3361	0.5125	0.6734	0.7973	0.8820
	E	4.01022	9.6000	14.4912	26.2128	39.9763	52.5216	62.1881	68.7923

Table (10)

Shown Expected Sample Size [E] and Expected Sample Size Proportion [Ep] of $\tilde{\theta}_{DS}$ w.r.t. u and λ when n_1 = 8, α=0.05

u	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	EP	0.7838	0.5250	0.5766	0.6938	0.8158	0.9053	0.9571	0.9825
	E	12.5415	8.4000	9.2254	11.1003	13.0529	14.4847	15.3143	15.7201
3	EP	0.6758	0.2875	0.3649	0.5406	0.7237	0.8579	0.9357	0.9738
	E	21.6244	9.2000	11.6762	17.3008	23.1586	27.4542	29.9429	31.1604
9	EP	0.6109	0.1450	0.2379	0.4488	0.6684	0.8295	0.9229	0.9685
	E	48.8733	11.6000	19.0285	35.9023	53.4759	66.3627	73.8286	77.4812
12	EP	0.6009	0.1231	0.2183	0.4346	0.6599	0.8252	0.9209	0.9677
	E	62.4977	12.8000	22.7047	45.2030	68.6345	85.8170	95.7715	100.6416

Table (11)

Shown the Percentage of overall Sample Saved (P.O.S.S.) of $\tilde{\theta}_{DS}$ w.r.t. u, n_1 and λ when α=0.05

u	n_1	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	4	POSS	31.7661	47.5002	45.7694	41.1652	35.4328	29.3537	23.5264	18.3285
	6		26.3179	47.5000	44.1034	35.9633	26.4053	17.6933	10.9805	6.3942
	8		21.6158	47.5000	42.3413	30.6234	18.4195	9.4703	4.2857	1.7492
	12		14.2597	47.5001	38.6404	20.8292	7.6855	2.0880	0.4471	0.0795
3	4	POSS	47.6492	71.2503	68.6542	61.7478	53.1492	44.0305	35.2896	27.4928
	6		39.4769	71.2500	66.1550	53.9450	39.6080	26.5400	16.47089	9.5914
	8		32.4237	71.2500	63.5120	45.9351	27.6293	14.2055	6.4285	2.6238
	12		213896	71.2501	57.9605	31.2438	11.5283	3.1321	0.6707	0.1192
9	4	POSS	5731790	85.5004	82.3850	74.0973	63.7790	52.8367	42.3475	32.9913
	6		47.3723	85.5000	79.3860	64.7340	47.5296	31.8480	19.7949	11.5096
	8		38.9084	85.5000	76.2144	55.1222	33.1552	17.0466	7.7142	3.1485
	12		25.6675	85.5001	69.5526	37.4925	13.8340	3.7585	0.8049	0.1431
12	4	POSS	58.6452	87.6927	84.4974	75.9972	65.4143	54.1914	43.4334	33.8372
	6		48.5870	87.6923	81.4216	66.3938	48.7483	32.6646	20.2717	11.8048
	8		39.9060	87.6923	78.1686	56.5356	34.0053	17.4837	7.9120	3.2293
	12		26.3256	87.6924	71.3360	38.4539	14.1887	3.8549	0.8255	0.1467

Table (12)
Shown the Probability of a Voiding Second Sample [Av] w.r.t. u, n_1 and λ when α=0.05

u	n_1	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	4	AV	0.6353	0.95500	0.9154	0.8233	0.7087	0.5871	0.4705	0.3666
	6		0.5264	0.9500	0.8821	0.7193	0.5281	0.3539	0.2196	0.1279
	8		0.4323	0.9500	0.8468	0.6125	0.3684	0.1894	0.0857	0.0350
	12		0.2852	0.9500	0.7728	0.4166	0.1537	0.0418	0.0089	0.0016
3	4	AV	0.6353	0.9500	0.9154	0.8233	0.7087	0.5871	0.4705	0.3666
	6		0.5264	0.9500	0.8821	0.7193	0.5281	0.3539	0.2196	0.1279
	8		0.4323	0.9500	0.8468	0.6125	0.3684	0.1894	0.0857	0.0350
	12		0.2852	0.9500	0.7728	0.4166	0.1537	0.0418	0.0089	0.0016
9	4	AV	0.6353	0.9500	0.9154	0.8233	0.7087	0.5871	0.4705	0.3666
	6		0.5264	0.9500	0.8821	0.7193	0.5281	0.3539	0.2196	0.1279
	8		0.4323	0.9500	0.8468	0.6125	0.3684	0.1894	0.0857	0.0350
	12		0.2852	0.9500	0.7728	0.4166	0.1537	0.0418	0.0089	0.0016
12	4	AV	0.6353	0.9500	0.9154	0.8233	0.7087	0.5871	0.4705	0.3666
	6		0.5264	0.9500	0.8821	0.7193	0.5281	0.3539	0.2196	0.1279
	8		0.4323	0.9500	0.8468	0.6125	0.3684	0.1894	0.08057	0.0350
	12		0.2852	0.9500	0.7728	0.4166	0.1537	0.0418	0.0089	0.0016