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Some Results on Epiform Modules.

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Abstract: The concept of epiform modules is a dual of the notion of monoform modules. In this work we give some properties of this class of modules. Also, we give conditions under which every hollow (copolyform) module is epiform.

Keywords: monoform modules, rational submodules, essential submodules polyform modules, copolyform modules, hollow modules.

§1. Introduction:

Let R be an associative ring with 1, and let M be a unitary (left) R-module. A non-zero module M is called monoform, if every non-zero homomorphism f: N→ M with N a submodule of M, is a monomorphism [١].In [2] we introduced the dual of this concept which we call epiform modules. In this paper we study some properties of this class of modules, and we give characterizations of this concept, we start by the following definition.

Definition (1.1): [2] A nonzero R - module M is called epiform module if every nonzero homomorphism f: $M \longrightarrow \frac{M}{K}$ with K a proper submodule of M is an epimorphism.

Remark (1.2): If a module M is epiform, then every non zero endomorphism of M is an epimorphism.

Examples (1.3):

Every simple module is epiform.

Z_{p^∞} as Z - module is epiform. In fact, every almost finitely generated module is an epiform module, where an R - module M is called almost finitely generated if M is not finitely generated and every proper submodule of M is finitely generated [3], and the result follows from the following propositions [4, 1.4 and 1.7].

§2. Basic results of epiform modules :

In this section we give some properties of epiform modules, and we give conditions under which every hollow module is epiform.

We prove in [2] that if f: $M \longrightarrow M'$ be an epimorphism with M epiform module, then M′ is epiform module.

Now we have the following direct consequence:

Remark (2.1): A direct summand of an epiform module is epiform.

Remark (2.2): The direct sum of epiform modules is not epiform module. In fact both of the modules Z2 and Z3 are epiform modules, but Z6 which is isomorphic to Z2⊕ Z3 is not.

A submodule N of an R-module M is called small submodule of M (denoted by $N \ll M$), if $N+L \neq M$ for every proper submodule L of M [5], and a nonzero module M is called a hollow module if every proper submodule of M is a small submodule of M [6].

Note that not every nonzero module has a submodule which is epiform module. For example, the Z-module Z dose not contain an epiform module.

The following proposition deals with the existences of epiform modules in nonzero Artinian modules.

Proposition (2.3): Let M be a nonzero Artinian module, then M has a submodule which is an epiform.

Proof: Let N be a nonzero submodule of M. If N is epiform, then we are done. Otherwise there exists a proper submodule K1 of N and a nonzero homomorphism f1: N $\longrightarrow \frac{N}{K_1}$ with f1(N)= $\frac{N_1}{K_1} \neq \frac{N}{K_1}$ for some proper submodule N1 of N which contains K1 properly. Now, if N1 is epiform, we are through, otherwise there exists a proper submodule K2 of K1 and a nonzero homomorphism f2: N1 $\longrightarrow \frac{N_1}{K_2}$ with f 2(N1) = $\frac{N_2}{K_2} \neq \frac{N_1}{K_2}$ for some proper submodule N2 of N1 which contains K2 properly. If we continue in this way we will arrive at an epiform submodule of M in a finite number of steps, for otherwise there exists an infinite descending chain:

N ⊃ N1 ⊃ N2 ⊃ . . . . .

of submodules of M, contrary to our assumption.

Corollary (2.4): Let M be a nonzero Artinian module, then M has a submodule which is hollow.

It was shown in [2] that every epiform module is hollow module but the converse is not true, for example the Z-module Z4 is a hollow module but it is not epiform module. However, the converse is true uner certain conditions as the next two propositions shows. Before that, Let us recall that an R-module M is called noncosingular module if for any nonzero module N and for every nonzero homomorphism f : M $\longrightarrow$ N, Im f is not a small submodule of N [7].

Proposition (2.5): Let M be a hollow noncosingular module, then M is epiform module.

Proof : Let M be a hollow noncosingular module. Let f: M $\longrightarrow \frac{M}{K}$ be a nonzero homomorphism with K a proper submodule of M. But M is noncosingular module thus f (M) is not a small submodule of $\frac{M}{K}$. Also since M is a hollow module, then $\frac{M}{K}$ is a hollow module [7], thus f (M) = $\frac{M}{K}$, and we are done.

An R - module M is called cosemisimple if Rad($\frac{M}{K}$) = 0, for all submodules K of M [8].

Proposition (2.6) : Every hollow cosemisimple module is epiform module.

Proof: Let M be a hollow cosemisimple module, and let f: M $\longrightarrow \frac{M}{K}$ be a nonzero homomorphism with K a proper submodule of M. If f(M) ≠ $\frac{M}{K}$, then since $\frac{M}{K}$ Is a hollow module, then f(M) is a small submodule of $\frac{M}{K}$, and hence f(M)⊆ Rad($\frac{M}{K}$). But M is cosemisimple module, this implies that f (M) = 0 which is a contradiction. Therefore f is an epimorphism.

§3. Small cover of epiform modules

We prove in [2] that a homomorphic image of epiform module is epiform module. In this section we give conditions under which the converse of this statement is true.

Definition (3.1): [9] A module M is called a small cover for a module N, if there exists an epimorphism ϕ : M $\longrightarrow$ N such that kerϕ is small submodule of M.

Proposition (3.2): Let M a small cover of N. If N is a hollow module and M is cosemisimple module then M is epiform module.

Proof: Letϕ : M → N be a small cover of N, then By the first isomorphism theorem, $\frac{M}{\ker \phi} \cong N$. Since N is a hollow module then $\frac{M}{\ker \phi}$ is hollow module. On the other hand ker ϕ « M implies that M is hollow module [6]. But M is cosemisimple module, so by (2.6), we get the result.

Corollary (3.3): Let M be cosemisimple small cover of N. Then M is epiform module if and only if N is epiform module.

Theorem (3.4): Let M be an noncosingular small cover of a hollow module N, then M is an epiform module.

Proof: Since M is a small cover of N, then there exists an epimorphism f: M→ N with ker f « M. By the first isomorphism theorem, $\frac{M}{\ker f} \cong N$. Since N is a hollow module then $\frac{M}{\ker f}$ is hollow module. On the other hand ker f « M implies that M is hollow module [6]. But M is noncosingular module, so by (2.5), we get the result.

Corollary (3.5): Let M be an noncosingular small cover of a module N. Then M is epiform if and only if N is epiform module.

§4 Epiform modules and copolyform module :

In this section we give conditions under which a copolyform module is epiform. We start by the definition of copolyform modules.

Definition (4.1): [6] An R-module M is called copolyform if $Hom_R(M, \frac{N}{K}) = 0$ for all submodule N of M with $K \subseteq N \ll M$.

We prove in [2] that every epiform module is copolyform. The converse is false, to see this, just take Z as Z - module which is a copolyform module, but it is not epiform, since the homomorphism $f : Z \longrightarrow \frac{Z}{6Z}$ defined by $f(n) = 3n + 6Z$ for all $n \in Z$ is not epimorphism In the following proposition we give a condition under which the converse of this statement is true.

Proposition (4.2): Every hollow copolyform module is epiform module.

Proof: Let N be a proper submodule of M. Since M is hollow module, then N is a small submodule of M. But M is copolyform module, thus $Hom_R(M, \frac{N}{K}) = 0$ for all $K \leq N$, and hence for every proper submodule N of M we have $Hom_R(M, \frac{N}{K}) = 0$. This implies that any nonzero homomorphism $f: M \to \frac{M}{L}$ where L is a proper submodule of M must be an epimorphism. Thus M is an epiform module.

As a corollary of (4.2) we have the following.

Corollary (4.3): Let M be a copolyform module such that every nonzero factor module of M is indecomposable. Then M is epiform module.

Proof: Since every nonzero factor module of M is indecomposable then M is a hollow module [10]. But M is copolyform module, so by (4.2), M is epiform module

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بعض النتائج حول مقاسات الصيغة الشاملة

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