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Weakly (resp., Closure, Strongly) Perfect Mappings

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الخلاصة

في هذا البحث عرفنا مفاهيم التطبيقات التامة الضعيفة (المغلقة، القوية) وأهم العلاقات التي درست:
(ا) المقارنة بين مختلف أشكال التطبيقات التامة.
(ب) العلاقة بين تركيب مختلف أشكال التطبيقات التامة.
(ج) أستقصاء العلاقات بين مختلف أشكال التطبيقات التامة والتطبيقات الراسمة لها.

ABSTRACT

In this paper the concepts of weakly (resp., closure, strongly) Perfect Mappings are defined and the important relationships are studied: (a) Comparison between deferent forms of perfect mappings. (b) Relationship between compositions of deferent forms of perfect mappings. (c) Investigate relationships between deferent forms of perfect mappings and their graphs mappings.

INTRODUCTION

In 1991 J. Chew and J. Tong (1) introduced the concept of weakly continuous mappings, in 1980 T. Noiri (2) introduced the concept of closure continuous mappings, in 1981 P. E. Long and L. L. Herrington (3) introduced the concept of strongly continuous mappings, in 1966 N.Bourbaki (4) defined perfect mappings and he stated and proved several theorems concerning perfect mappings, and in 1989 R.Engliking (5) introduced equivalent definition of perfect mappings. In this paper we introduce new concepts, which are the concepts of weakly (resp., closure, strongly) Perfect Mappings in which weakly (closure) Perfect Mappings are strictly weaker than perfect mappings, but strongly perfect mappings are stronger than perfect mappings. Also in this paper we introduce several results and examples concerning of deferent forms of perfect mappings.
For a subset A of a topological space X, the closure of A denoted by cl(A). For other notions or notations not defined here we follow closely R. Engelking (5).

Basic Definitions

Definition 1 (1, 6, and 7)

A mapping $f : X \rightarrow Y$ is weakly (resp., closure, strongly) continuous at a point $x \in X$ if given any open set V containing f(x) in Y, there exists an open set U containing x in X such that $f(U) \subseteq cl(V)$ (resp., $f(cl(U)) \subseteq cl(V)$, $f(cl(U)) \subseteq V$).

If this condition is satisfied at each point $x \in X$, then f is said to be weakly (resp., closure, strongly) continuous.

Definition 2 (5)

A space X is called Urysohn if for every $x \neq y \in X$, there exists an open set U containing x and an open set V containing y such that $cl(U) \cap cl(V) = \phi$.

Definition 3 (5)

Suppose we are given a topological space X, a family $\{Y_s\}_{s \in S}$ of topological spaces and a family of continuous mappings $\{f_s\}_{s \in S}$ where $f_s : X \to Y_s$. the mapping assigning to the point $x \in X$ the point $\{f_s(x)\} \in \Pi_{s \in S} Y_s$ is continuous ; it is called the diagonal of the mappings $\{f_s\}_{s \in S}$ and is denoted by $\Delta_{s \in S} f_s$ or by $f_1 \Delta f_2 \Delta \ldots \Delta f_k$ if $S = \{1,2,\ldots,k\}$.

Comparison between Deferent Forms of Perfect Mappings

The new concepts in this paper are given now.

Definition 1

A mapping $f : X \to Y$ is called weakly (resp., closure, strongly) perfect mapping if it is weakly (resp., closure, strongly) continuous, closed and for each $y \in Y$, $f^{-1}(y)$ is compact.

"It is well-known that if $f : X \to Y$ is perfect, then for any closed $A \subset X$ and any $B \subset Y$ the restrictions $f|_A : A \to Y$ and $f_B : f^{-1}(B) \to B$ are perfect, this is still the case in weakly (resp., closure, strongly) perfect, as it is shown in the next theorem."------------ *

Theorem 2

Let $f : X \to Y$ be a weakly (resp., closure, strongly) perfect, then for any closed $A \subset X$ and any $B \subset Y$ the restrictions $f|_A : A \to Y$ and $f_B : f^{-1}(B) \to B$ are weakly (resp., closure, strongly) perfect.

Also it is will-known that if $f : X \to Y$ is perfect, then $f_{f(X)} : X \to f(X)$ is perfect. This is not the case in weakly (closure) perfect even over an Urysohn space as it is shown in the next example, but it is true for strongly perfect as it is shown in theorem (3.4).

Example 3

Let P be the upper half of plane and L be the x-axis. Let $X = P \cup L$. If τ_{hdis} is the half disc topology on X and τ_r be the relative topology that X inherits by virtue of being a subspace of IR^2. The identity mapping $f : (X, \tau_r) \to (Y, \tau_{hdis})$ is weakly (closure) perfect but not perfect since it is not continuous. And f : $(L,\tau_r)\rightarrow(X, \tau_{hdis})$ is weakly (closure) perfect, but f : $(L,\tau_r)\rightarrow(L, \tau_{hdis})$ is not weakly (closure) perfect.

Theorem 4

Let $f: X\rightarrow Y$ be strongly perfect, then $f_{f(X)} : X\rightarrow f(X)$ is strongly perfect.
Proof: Let F be a closed subset of X, we have $f_{f(X)}(F\cap f^{-1}(f(X)))= f(F\cap f^{-1}(f(X)))=f(F)\cap f(X)$ is closed in f(X), hence $f_{f(X)}$ closed. Let $y\in f(X)$, then $f^{-1}_{f(X)}(y)=f^{-1}(y)$ is compact subset of X. Let $x\in X$ and let V be any open set containing f(x) in f(X), also in Y because $f(X)\subseteq Y$. Since f is strongly continuous, there is an open set U containing x in X such that $f(cl(U))\subseteq V$, hence $f_{f(X)}$ is strongly continuous. Therefore $f_{f(X)}$ is strongly perfect.

Now we will compare between deferent forms of perfect mappings.

Theorem 5

Let $f: X\rightarrow Y$ be a perfect on X, then f is closure perfect on X.
Proof: The proofs of f is closed and for each $y\in Y$, $f^{-1}(y)$ is compact are obvious. Let $x\in X$ and let V be any open set containing f(x) in Y. Since f is continuous, there is an open set U containing x in X such that $f(U)\subseteq V$. Hence $clf(U)\subseteq cl(V)$. By continuity of f, $f(clU)\subseteq cl(f(U))$, therefore $f(clU)\subseteq clV$ and f is closure continuous. Hence f is closure perfect on X.

The converse of the above theorem is not true, as it is shown in the next example.

Example 6

Let X=[0, 1] with topology τ_{cof} consisting of the empty set together with all sets whose complements are finite, let Y=[0, 1] with topology τ_{coco} consisting of the empty set together with all sets whose complements are countable. Let f : $(X,\tau_{cof})\rightarrow(Y,\tau_{coco})$ be the identity mapping, then f is closure perfect on X since for every nonempty open set U in Y, clU=X. It is clear that for every $x\in X$, f is not continuous at x. Hence f is not perfect.

If the range of f is a regular space then the converse of theorem (3.5) is also true.

Theorem 7

Let Y be a regular space and $f: X\rightarrow Y$. Then f is perfect on X iff f is closure perfect on X.
Proof: ($\Rightarrow$) Follows from theorem (3.5)
($\Leftarrow$) Suppose that f : $X\rightarrow Y$ closure perfect. It suffices to show that f continuous, let $x\in X$ and let V be an open set containing f(x) in Y. Since Y is regular there is an open set V_1 in Y such that $f(x)\in V_1$ and $cl(V_1)\subset V$, since f is closure continuous, there is an open set U containing x such that

$f(cl(U)) \subseteq cl(V_1)$, since $U \subseteq cl(U)$, so $f(U) \subseteq f(cl(U))$. It follows that $f(U) \subseteq V$, therefore f is continuous on X. Hence f is perfect on X.

Theorem 8

Let $f : X \to Y$ be a strongly perfect. Then f is perfect.

The converse of the above theorem is not true, as it is shown in the next example.

Example 9

Let (IR, τ) where τ is the topology with basis whose members are of the form (a, b) and (a, b)-N, $N=\{1/n ; n \in Z^+\}$. Then (IR,τ) is Hausdorff but not regular. Let $f : (IR,\tau) \to (IR,\tau)$, $f(x)=x$, then f is perfect but not strongly perfect.

If the domain of f is a regular space then the converse of theorem (3.8) is also true.

Theorem 10

Let X be a regular space and let $f : X \to Y$. Then f is perfect iff f is strongly perfect.

Proof: (⇐) Follows from theorem (3.8)

(⇒) Suppose that $f : X \to Y$ perfect. It suffices to show that f strongly continuous, let $x \in X$ and let V be an open set containing f(x) in Y. Since f is continuous, there is an open set U containing x in X such that $f(U) \subseteq V$, since X is regular there is an open set U_1 in X such that $x \in U_1$ and $cl(U_1) \subset U$, so $f(cl(U_1)) \subset f(U)$. It follows that $f(cl(U_1)) \subseteq V$, therefore f is strongly continuous. Hence f is strongly perfect.

Theorem 11

Let Y be a regular space and let $f : X \to Y$. Then f is closure perfect on X iff f is strongly perfect on X.

Proof: (⇒) Suppose that $f : X \to Y$ closure perfect. It suffices to show that f strongly continuous, let $x \in X$ and let V be an open set containing f(x) in Y. Since Y is regular there is an open set V_1 in Y such that $f(x) \in V_1$ and $cl(V_1) \subset V$, since f is closure continuous, there is an open set U containing x such that $f(cl(U)) \subseteq cl(V_1)$. It follows that $f(cl(U)) \subseteq V$, therefore f is strongly continuous. Hence f is strongly perfect on X.

(⇐) Follows from theorems (3.8) and (3.5)

Theorem 12

Let X be a regular space and let $f : X \to Y$. Then f is closure perfect on X iff f is weakly perfect on X.

Proof: (⇒) Suppose that $f : X \to Y$ closure perfect. It suffices to show that f weakly continuous, let $x \in X$ and let V be an open set containing f(x) in Y. Since f is closure continuous, there is an open set U containing x such that $f(cl(U)) \subseteq cl(V)$, since $U \subseteq cl(U)$, so $f(U) \subseteq f(cl(U))$. It follows that $f(U) \subseteq cl(V)$, therefore f is weakly continuous. Hence f is weakly perfect on X. (⇐) Suppose that $f : X \to Y$ weakly perfect. It suffices to show that f closure continuous, let $x \in X$ and let V be an open set containing f(x) in Y. Since f is weakly continuous, there is an open set U containing x such that $f(U) \subseteq cl(V)$, since X is regular there is an open set U_1 in X such that $x \in U_1$ and $cl(U_1) \subset U$, so $f(cl(U_1)) \subset f(U)$, It follows that $f(cl(U_1)) \subseteq cl(V)$ therefore f is closure continuous. Hence f is closure perfect on X.

Theorem 13

Let X or Y be a regular space and let $f : X \to Y$. Then f is perfect iff f is strongly perfect.

Proof: If X is regular space, then the proof follows from theorem (3.10).
If Y is regular space, then the proof follows from theorems (3.7) and (3.11).

Theorem 14

Let Y be a regular space and let $f : X \to Y$. Then f is perfect on X iff f is weakly perfect on X.

Proof: (⇒) Suppose that $f : X \to Y$ perfect. It suffices to show that f weakly continuous, let $x \in X$ and let V be an open set containing f(x) in Y. Since f is continuous, there is an open set U containing x such that $f(U) \subseteq V$, since $V \subseteq cl(V)$, It follows that $f(U) \subseteq cl(V)$, therefore f is weakly continuous. Hence f is weakly perfect on X.

(⇐) Suppose that $f : X \to Y$ weakly perfect. It suffices to show that f continuous, let $x \in X$ and let V be an open set containing f(x) in Y. Since Y is regular there is an open set V_1 in Y such that $f(x) \in V_1$ and $cl(V_1) \subset V$, since f is weakly continuous, there is an open set U containing x such that $f(U) \subseteq cl(V_1)$. It follows that $f(U) \subseteq V$, therefore f is continuous on X. Hence f is perfect on X.

Remark 15

In (1) it is shown that weak continuity does not imply closure continuity. Therefore, strongly perfect ⇒ perfect ⇒ closure perfect ⇒ weakly perfect, but not conversely. While the four conditions are equivalent if the range is regular space.

Relationship between Compositions of Deferent Forms of Perfect Mappings

It is well-known that the composition of perfect mappings is perfect. Similar results hold for closure and strongly perfect but it is not true for weakly perfect.

Theorem 1

Let $f : X \to Y$ be weakly perfect and let $g : Y \to Z$ be closure perfect. Then $gof : X \to Z$ is weakly perfect.

Proof: Since the composition of closed mappings is closed, then gof is closed. Let $z \in Z$, to show that $(gof)^{-1}(z)$ is compact, since g is closure perfect, then $g^{-1}(z)$ is compact, now let $\{U_\alpha : \alpha \in \Lambda\}$ be a family of open sets of X such that $(gof)^{-1}(z) \subset \cup_{\alpha \in \Lambda} U_\alpha$. If $y \in g^{-1}(z)$, then there exists a finite subset M(y) of Λ such that $f^{-1}(y) \subset \cup_{\alpha \in M(y)} U_\alpha$. Since f is a closed mapping, by theorem (1.4.12) in (5) there exists an open set V_y of Y such that $y \in V_y$ and $f^{-1}(V_y) \subset \cup_{\alpha \in M(y)} U_\alpha$. Since $g^{-1}(z)$ is compact, there exists a finite subset B of $g^{-1}(z)$ such that $g^{-1}(z) \subset \cup_{y \in B} V_y$. Hence $f^{-1}(g^{-1}(z)) \subset \cup_{y \in B} f^{-1}(V_y) \subset \cup_{y \in B} \cup_{\alpha \in M(y)} U_\alpha$. Thus if $M = \cup_{y \in B} M(y)$, then M is a finite subset of Λ and $f^{-1}(g^{-1}(z)) \subset \cup_{\alpha \in M} U_\alpha$. Thus $f^{-1}(g^{-1}(z)) = (gof)^{-1}(z)$ is compact. Let $x \in X$ and let W open set containing $(gof)(x)$ in Z, since g is closure continuous, there is an open set V containing f(x) in Y such that $g(cl(V)) \subseteq cl(W)$. Since f is weakly continuous, then for every $x \in X$ and every open set V of $f(x)=y$, there exists an open set U of x in X such that $f(U) \subseteq cl(V)$, so $g(f(U)) \subseteq g(cl(V))$ and $(gof)(U) \subseteq g(cl(V))$, then we have $(gof)(U) \subseteq cl(W)$ and gof is weakly continuous. Hence f is weakly perfect.

Theorem 2

Let $f : X \to Y$ be perfect and let $g : Y \to Z$ be weakly perfect. Then $gof : X \to Z$ is weakly perfect.

Proof: The proofs of gof is closed and for each $z \in Z$, $(gof)^{-1}(z)$ is compact are similar to the proof of theorem (4.1), so it is omitted. Let $x \in X$ and let W open set containing $(gof)(x)$ in Z, since g is weakly continuous, there is an open set V containing f(x) in Y such that $g(V) \subseteq cl(W)$. Since f is continuous, then for every $x \in X$ and every open set V of $f(x)=y$, there exists an open set U of x in X such that $f(U) \subseteq V$, so $g(f(U)) \subseteq g(V)$ and $(gof)(U) \subseteq g(V)$, then we have $(gof)(U) \subseteq cl(W)$ and gof is weakly continuous. Hence f is weakly perfect.

Theorem 3

Let $f : X \to Y$ be a closure perfect and let $g : Y \to Z$ be closure perfect. Then $gof : X \to Z$ is closure perfect.

Proof: The proofs of gof is closed and for each $z \in Z$, $(gof)^{-1}(z)$ is compact are similar to the proof of theorem (4.1), so it is omitted. Let $x \in X$ and let W open set containing $(gof)(x)$ in Z, since g is closure continuous, there is an open set V containing f(x) in Y such that g(cl(V))⊆cl(W). Since f is closure continuous, then for every x∈X and every open set V of f(x)=y, there exists an open set U of x in X such that f(cl(U))⊆cl(V), so g(f(cl(U)))⊆g(cl(V)) and (gof)(cl(U))⊆g(cl(V)), then we have (gof)(cl(U))⊆cl(W) and gof is closure continuous. Hence f is closure perfect.

Theorem 4

Let f : X→Y be a closure perfect and let g : Y→Z be strongly perfect. Then gof : X→Z is strongly perfect.

Proof: The proofs of gof is closed and for each z∈Z, $(gof)^{-1}(z)$ is compact are similar to the proof of theorem (4.1), so it is omitted. Let x∈X and let W open set containing (gof)(x) in Z, since g is strongly continuous, there is an open set V containing f(x) in Y such that g(cl(V))⊆W. Since f is closure continuous, then for every x∈X and every open set V of f(x)=y, there exists an open set U of x in X such that f(cl(U))⊂cl(V), so g(f(cl(U)))⊆g(cl(V)) and (gof)(cl(U))⊆g(cl(V)), then we have (gof)(cl(U))⊆W and gof is strongly continuous. Hence f is strongly perfect.

Theorem 5

Let f : X→Y be weakly perfect and let g : Y→Z be strongly perfect. Then gof : X→Z is perfect.

Proof: The proofs of gof is closed and for each z∈Z, $(gof)^{-1}(z)$ is compact are similar to the proof of theorem (4.1), so it is omitted. Let x∈X and let W open set containing (gof)(x) in Z, since g is strongly continuous, there is an open set V containing f(x) in Y such that g(cl(V))⊆W. Since f is weakly continuous, then for every x∈X and every open set V of f(x)=y, there exists an open set U of x in X such that f(U)⊆cl(V), so g(f(U))⊆g(cl(V)) and (gof)(U)⊆g(cl(V)), then we have (gof)(U)⊆W and gof is continuous. Hence f is perfect.

The next example shows that the perfect of f in theorem (4.2) can not be weakened into closure perfect, and it also shows that the composition of weakly perfect is not to be weakly perfect.

Example 6

Let X={x, y, z, w} with topology τ_X={ϕ, {z}, {z, w}, {x, y, z}, X}and let Y={a, b, c, d} with topology τ_Y={ϕ, {b}, {d}, {a, b}, {b, d}, {a, b, d}, {b, c, d}, Y}. Define g : (X, τ_X)→(Y, τ_Y), by g(x)=a, g(y)=b, g(z)=c, g(w)=d . Then g is weakly perfect but not closure perfect. Define f : (IR, τ_u)→(X,τ_X), where τ_u is the usual topology on IR by f(rational)=y, f(irrational)=w. Then f is closure perfect but not perfect, and gof is not weakly perfect.

It is well-known that, if the composition gof of continuous mappings f : X→Y and g : Y→Z, where Y is a Hausdorff space, is perfect, then the mappings $g|_{f(X)}$ and f are perfect. Similar results hold for deferent forms of continuity mappings and perfectly mappings, as it is shown in the next theorems.

Theorem 7

If the composition gof of continuous mapping f : X→Y and weakly continuous mapping g : Y→Z, where Y is a Hausdorff space, is weakly perfect, then the mapping $g|_{f(X)}$ is weakly perfect and f is perfect.

Proof: (i) For every point z∈Z, $(g|_{f(X)})^{-1}(z)=f(X)\cap g^{-1}(z)=f((gof)^{-1}(z))$ is compact, because $(gof)^{-1}(z)$ is compact. Any closed subset of f(X) is of the form A∩f(X), where A is closed in Y. As the inverse image $f^{-1}(A)$ is closed in X and gof is a closed mapping, the set $(g|_{f(X)})(A\cap f(X))=g(A\cap f(X))=(gof)(f^{-1}(A))$ is closed in Z, we have $g|_{f(X)}$ is a closed mapping and thus the mapping $g|_{f(X)}$ is weakly perfect.

(ii) For every point y∈Y, $f^{-1}(y)=[(gof)^{-1}(g(y))]\cap f^{-1}(y)$ is compact. For every closed set F⊂X the mapping $(gof)|_F$ is weakly perfect, so that by the first part of our proof the restriction $g|_{f(F)}$ is weakly perfect, since the latter mapping can be continuously extended over cl(f(F)), it follows by lemma (3.7.4) in (5) that f(F)=cl(f(F)), we have f is a closed mapping and thus the mapping f is perfect.

Theorem 8

If the composition gof of continuous mapping f : X→Y and closure continuous mapping g : Y→Z, where Y is a Hausdorff space, is closure perfect, then the mapping $g|_{f(X)}$ is closure perfect and f is perfect.

Proof: The proof is similar to the proof of theorem (4.7), so it is omitted.

Theorem 9

If the composition gof of continuous mapping f : X→Y and strongly continuous mapping g : Y→Z, where Y is a Hausdorff space, is strongly perfect, then the mapping $g|_{f(X)}$ is strongly perfect and f is perfect.

Proof: The proof is similar to the proof of theorem (4.7), so it is omitted.

Theorem 10

If the composition gof of continuous mapping f : X→Y and closure continuous mapping g : Y→Z, where Y is a Hausdorff space, is weakly perfect, then the mapping $g|_{f(X)}$ is closure perfect and f is perfect.

Proof: The proof is similar to the proof of theorem (4.7), so it is omitted.

Theorem 11

If the composition gof of continuous mapping $f : X \to Y$ and strongly continuous mapping $g : Y \to Z$, where Y is a Hausdorff space, is strongly perfect, then the mapping $g|_{f(X)}$ is strongly perfect and f is closure perfect.

Proof: The proof is similar to the proof of theorem (4.7), so it is omitted.

Theorem 12

If the composition gof of continuous mapping $f : X \to Y$ and strongly continuous mapping $g : Y \to Z$, where Y is a Hausdorff space, is perfect, then the mapping $g|_{f(X)}$ is strongly perfect and f is perfect.

Proof: The proof is similar to the proof of theorem (4.7), so it is omitted.

Investigate Relationships between Deferent Forms of Perfect Mappings and their Graphs Mappings

The following theorem shows that a mapping f is weakly perfect iff its graph mapping G(f) is weakly perfect.

Theorem 1

Let $f : X \to Y$ be a mapping with Y a regular space and let $g : X \to XY$ be the graph mapping of f given by $G(f)=g(x)=(x, f(x))$ for every point $x \in X$. Then $g : X \to XY$ is weakly perfect iff $f : X \to Y$ is weakly perfect.

Proof: ($\Rightarrow$) Suppose g is weakly perfect. Let $x \in X$ and let V be an open set containing f(x) in Y, then XV is an open set containing g(x) in XY. Since g is weakly continuous, there exists an open set U containing x in X such that $g(U) \subseteq cl(XV)=Xcl(V)$. Since g is the graph mapping of f, we have $f(U) \subseteq cl(V)$, this shows that f is weakly continuous. Now the graph G(f) is the image of X under the homeomorphic embedding $id_X \Delta f$: $X \to XY$. The restriction $p_Y|_{G(f)}$ of the projection $P_Y : XY \to Y$ is homeomorphism , since Y is Hausdorff, the G(f) is a closed subset of XY. By (*), $p_Y|_{G(f)}$ is perfect and by remark (3.15), $p_Y|_{G(f)}$ is closure perfect. Since $f=(p_Y|_{G(f)})og$, by theorem (4.1) we have f is weakly perfect.

($\Leftarrow$) Suppose f is weakly perfect. Let $x \in X$ and W be an open set containing g(x) in XY, then there exist open sets $R \subset X$ and $V \subset Y$ such that $g(x)=(x, f(x)) \in RV \subseteq W$. Since f is weakly continuous, there exists an open set U containing x in X such that $U \subseteq R$ and $f(U) \subseteq cl(V)$, therefore we have $g(U) \subseteq Rcl(V) \subseteq cl(RV) \subseteq cl(W)$, this shows that g is weakly continuous. Now the mapping $g=id_X \Delta f : X \to XY$ maps X homeomorphically onto the graph G(f) which is closed subset of XY, therefore g is perfect, by remark (3.15), g is weakly perfect.

The following theorem shows that a mapping f is closure perfect iff its graph mapping G(f) is closure perfect.

Theorem 2

Let $f : X \to Y$ be a mapping and let $g : X \to XY$ be the graph mapping of f given by $G(f)=g(x)=(x, f(x))$ for every point $x \in X$. Then $g : X \to XY$ is closure perfect iff $f : X \to Y$ is closure perfect.

Proof: ($\Rightarrow$) If g is closure perfect, then it follows from theorem (4.3) that f is closure perfect.

($\Leftarrow$) Suppose f is closure perfect. Let $x \in X$ and let W be an open set containing g(x) in XY, then there exist open sets $R \subset X$ and $V \subset Y$ such that $g(x)=(x, f(x)) \in RV \subseteq W$. Since f is closure continuous, there exists an open set U containing x in X such that $U \subseteq R$ and $f(cl(U)) \subseteq cl(V)$, therefore we have $g(cl(U)) \subseteq cl(R)cl(V) \subseteq cl(RV) \subseteq cl(W)$, this shows that g is closure continuous. Now the mapping $g=id_X \Delta f : X \to XY$ maps X homeomorphically onto the graph G(f) which is closed subset of XY, therefore g is perfect, by remark (3.15), g is closure perfect.

The following theorem shows that if the graph mapping G(f) is strongly perfect, then f is strongly perfect.

Theorem 3

Let $f : X \to Y$ be a mapping and let $g : X \to XY$ be the graph mapping of f given by $G(f)=g(x)=(x, f(x))$ for every point $x \in X$. If g is strongly perfect, then $f : X \to Y$ is strongly perfect. Moreover, X is a regular space.

Proof: The strongly perfectness of f follows from theorem (4.4). To prove the regularity, let $x \in X$ and let U be an open set containing x , then UY is an open set containing f(x). The strong continuity of the graph mapping of f guarantees the existence of an open set W containing x such that $g(cl(W) \subseteq UY$. Thus $x \in cl(W) \subseteq U$, proving that X is regular.

The converse of the above theorem is not true in general, as it is shown in the next example.

Example 4

Let $X=Y=\{1, 2, 3\}$ with topologies $\tau_X=\{\phi, \{1\}, \{2\}, \{1, 2\}, X\}$, $\tau_Y=\{\phi, \{3\}, Y\}$. Define $f : (X, \tau_X) \to (Y, \tau_Y)$ by $f(x)=3$ for all x. Then f is strongly perfect but the graph mapping G(f), where $g(x)=(x, f(x))$ is not strongly perfect because it is not strongly continuous at 1 and 2.

If the domain of f is a regular space then the converse of theorem (5.3) is also true.

Theorem 5

Let $f : X \to Y$ be a mapping with X a regular space and let $g : X \to XY$ be the graph mapping of f given by $G(f)=g(x)=(x, f(x))$ for every point $x \in X$. If $f : X \to Y$ is strongly perfect, then $g : X \to XY$ is strongly perfect.

Proof: Suppose f is strongly perfect. Let $x \in X$ and let W be an open set containing g(x) in $X \times Y$, then there exist open sets $R \subset X$ and $V \subset Y$ such that $g(x)=(x, f(x)) \in R \times V \subseteq W$. Since f is strongly continuous and X is regular, there exists an open set U containing x in X such that $cl(U) \subseteq R$ and $f(cl(U)) \subseteq (V)$, therefore we have $g(clU) \subseteq R \times V \subseteq W$, this shows that g is strongly continuous. Now the mapping $g = id_X \Delta f : X \rightarrow X \times Y$ maps X homeomorphically onto the graph G(f) which is closed subset of $X \times Y$, therefore g is perfect and since X regular we have $X \times Y$ is regular, by remark (3.15), g is strongly perfect.

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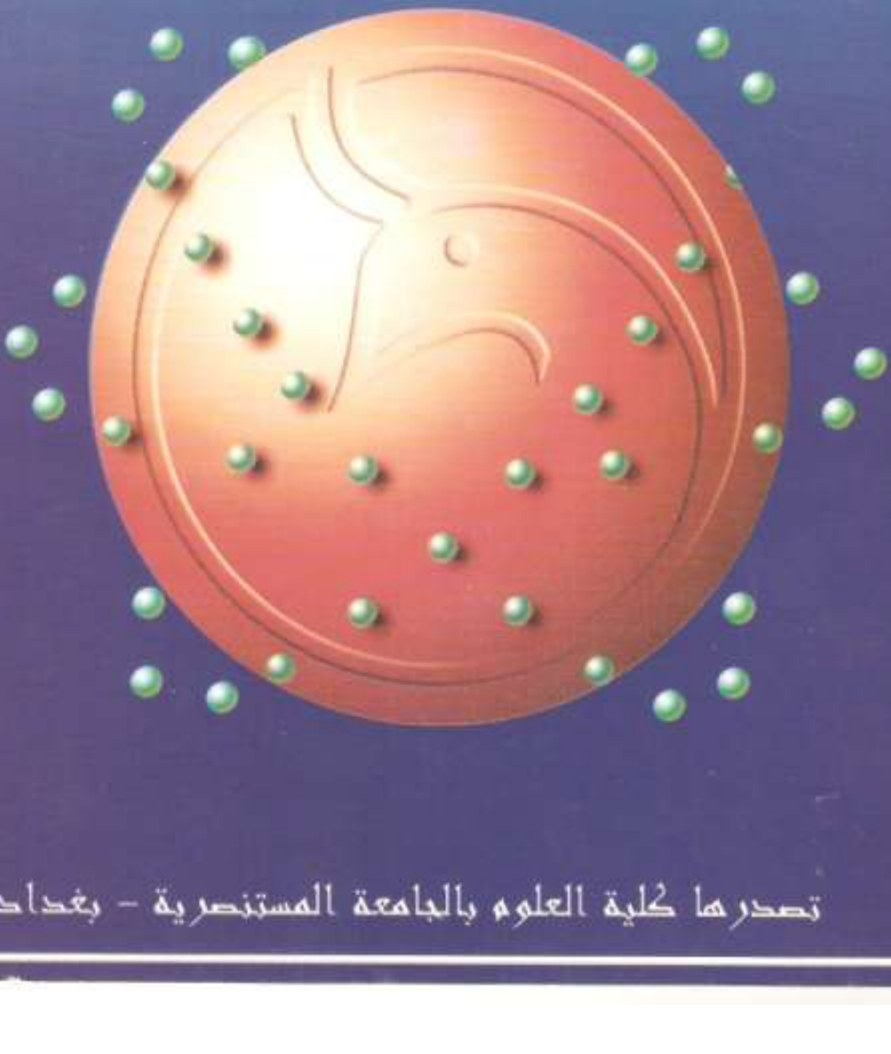

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