

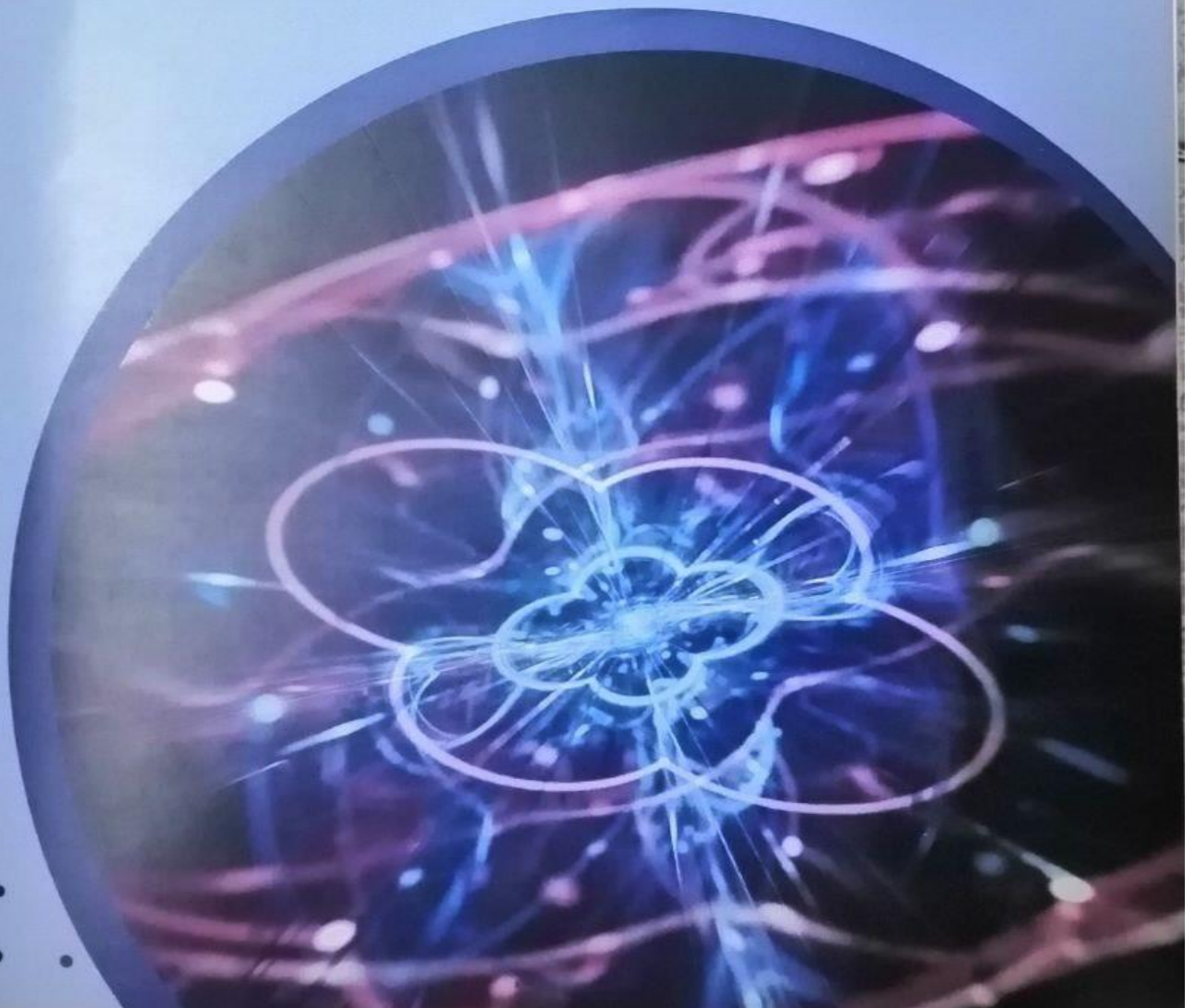
# **MULTIVARIATE ANALYSIS**

## **(SECOND EDITION)**

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# MULTIVARIATE ANALYSIS

THIS BOOK IS INTENDED TO BE TEXTBOOK STUDIED FOR UNDERGRADUATE COURSE IN MULTIVARIATE ANALYSIS. THIS BOOK IS DESIGNED TO BE USED IN SEMESTER SYSTEM. IN ORDER TO ACHIEVE THE GOALS OF THE BOOK, IT IS DIVIDED INTO THE FOLLOWING CHAPTERS. CHAPTER ONE INTRODUCES MATRIX ALGEBRA. CHAPTER TWO DEVOTES TO LINEAR EQUATION SYSTEM SOLUTION WITH QUADRATIC FORMS, CHARACTERISTIC ROOTS & VECTORS. CHAPTER THREE DISCUSSES PARTITIONED MATRICES AND HOW TO GET INVERSE, JACOBI AND HESSIAN MATRICES. CHAPTER FOUR DEALS WITH MULTIVARIATE NORMAL DISTRIBUTION (MVN). CHAPTER FIVE CONCERN WITH JOINT, MARGINAL AND CONDITIONAL NORMAL DISTRIBUTION, INDEPENDENCY AND CORRELATIONS. CHAPTER SIX INTRODUCES MEAN VECTOR ESTIMATION AND COVARIANCE MATRIX ESTIMATION. CHAPTER SEVEN DEVOTES TO TESTING CONCERNING MEAN: ONE SAMPLE MEAN, AND TWO SAMPLE MEAN. CHAPTER EIGHT DISCUSSES SPECIAL CASE OF FACTORIAL ANALYSIS WHICH IS PRINCIPAL COMPONENTS ANALYSIS. CHAPTER NINE DEALS WITH DISCRIMINANT ANALYSIS. WHILE CHAPTER TEN DEALS WITH CLUSTER ANALYSIS. MANY SOLVED EXAMPLES ARE INTERSEED IN THIS BOOK, IN ADDITION TO A VARIETY OF UNSOLVED RELIED PROBLEMS AT THE END OF EACH CHAPTER TO ENRICH THE STATISTICAL KNOWLEDGE OF THE READERS.



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