Double Stage Shrinkage Estimator in Pareto Distribution

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Abstract:

This paper proposes using preliminary test double stage shrinkage estimator (PTDSSE) for estimating the shape parameter (α) of Pareto distribution when the scale parameter equal to the smallest loss in a region (R) around available prior knowledge (α_0) about the actual value (α) as initial estimator as well as to reduce the mean squared error and the cost of experimentations.

In situation where the experimentation time consuming and sample cost are very costly and expensive a double stage procedure can be used to reduce the expected sample size which are needed to obtain the estimator which minimize these costs.

This estimator is shown to have smaller mean squared error for certain choice of the shrinkage weight factor $\psi(\cdot)$ and for acceptance suitable region (R).

Expressions for Bias $B(\cdot)$, Mean Square Error (MSE), Relative Efficiency [R.Eff(·)], Expected sample size [$E(n/\alpha,R)$], Expected sample size proportion [$E(n/\alpha,R)/n$], probability for avoiding the second sample saved $[\frac{n_2}{n_1} p(\hat{\alpha}_1 \in R) * 100]$ for the proposed estimator are derived.

Numerical results and conclusions are established when the consider estimator (PTDSSE) are testimator of level of significance Δ.

Comparisons between the proposed estimator with the classical and the existing estimators were curried out to show the usefulness of the proposed estimator.

Keywords: Pareto distribution, Double stage shrinkage estimator, Preliminary test region, Bias, Mean square error and Relative Efficiency.

1. Introduction

1.1 Previous Researches

The Pareto distribution has been in a variety of fields, especially in economics. It is often used to model the distribution in income [1], [2] and [3]. Various estimation methods for the parameter of the Pareto distribution have been discussed in the literature. The least squares and the moment estimators and their properties have been discussed by [4].

The maximum likelihood estimator is discussed by [5]. Estimators based on order statistics are discussed by [6] and [7] among others. Minimax, Bayesian and other issues of estimation in this model are reviewed in [8] and[9].

In this paper, PTDSSE has been considered for the shape parameter of the Pareto distribution considered.

1.2. The Model

The Pareto model is very often used as a basis of excess of loss quotation as it gives a petty good description of the random behavious of large losses, and often used to model the distribution of icome, [3], [5] and [10].

Assume that a random sample of size (n) from a pareto distribution, with the following p.d.f.:

$$f(x;\alpha,\lambda)=\begin{cases}\alpha k^{\alpha}x^{-(\alpha+1)} & \text{for } \alpha,k>0, x\geq k\\ 0 & \text{o.w.}\end{cases}$$

In conventional notation, we write x~Par(α,k) where α and k are the shape and scale parameters respectively.

In this work, we suggest the problem of estimating the shape parameter (α) when some prior estimate (α_0) regarding (α) is a available as well as entity an acceptance region (R) for test the hypothesis $H_0:\alpha = \alpha_0$ vs. $H_1:\alpha = \alpha_0$.

According to [11], such prior estimate mentioned above many arise for any one of a number of reasons, e.g., we are estimating α and

(i) We believe α_0 is closed to the true value of α; or
(ii) We fear that α_0 may be near the true value of α; i.e. something bad happens if $\alpha = \alpha_0$, and we do not know about it.

In such a situation, this prior guess value may be utilized to improve the estimation procedure with usual estimator $\hat{\alpha}$ (MLE) via shrinkage estimator $\tilde{\alpha}$ which including shrinkage weight factor $\psi(\hat{\alpha}), 0\leq\psi(\hat{\alpha})\leq 1$ as follows;

$$\tilde{\alpha}=\psi(\hat{\alpha})\hat{\alpha}+(1-\psi(\hat{\alpha}))\alpha_0$$
…(1)

See [12].

This method of constructing these estimators performs better than usual estimator in the term of mean square error when the guess value is close to the true value, [11].

Preliminary test double stage shrunken estimator (PTDSSE) for the shape parameter that utilizes a prior estimator (α_0) is represents as follows steps:

1. Select two positive integers (n_1) and (n_2).
2. Obtain a random sample of size (n_1) on x (first stage sample), and determine usual estimator $(\hat{\alpha}_1)$ for this sample.
3. Choose a suitable region (R) around α_0.
4. If $\hat{\alpha}_1 \in R$ take the shrinkage estimator of the form which is defined in equation (1), using $\hat{\alpha}_1$ and shrinkage weight factor $\psi(\hat{\alpha})$.

If $\hat{\alpha}_1 \notin R$, obtain a second stage random sample of size (n_2) on x and advise the estimator $\hat{\alpha}_p$ of α as polling estimator of two sample estimators.

i.e.; $\hat{\alpha}_p = (n_1\hat{\alpha}_1 + n_2\hat{\alpha}_2)/n$,

where $\hat{\alpha}_2$ is the usual estimator of the second sample of size n_2 and $n_1 + n_2 = n$.

Thus, the general form of (PTDSSE) has the following form

$$\tilde{\alpha}_{DS} = \begin{cases} \psi(\hat{\alpha}_1)\hat{\alpha}_1 + (1-\psi(\hat{\alpha}_1))\alpha_0 & \text{if } \hat{\alpha}_1 \in R \\ \hat{\alpha}_p & \text{if } \hat{\alpha}_1 \notin R \end{cases} \quad \ldots(2)$$

Several authors have been studied (DSSE) for different distributions, for example [13], [14] and [15].

2. Usual Estimation Of The Shape Parameter (α), [9]

Consider $x_{11}, x_{12}, \ldots, x_{1n}$ be identically independent Pareto distribution random variables.

The maximum likelihood estimator of α when $k = \min(x_{1i})$; (the smallest loss) is defined in [8] as below:

$$\hat{\alpha}_{1MLE} = \frac{n_1}{\sum_{i=1}^{n_1} \ln\left(\frac{x_{1i}}{\min(x_{1i})}\right)}$$

It follows easily that $\ln\left(\frac{x_{1i}}{\min(x_{1i})}\right)$ will be exponentially distributed with mean $\frac{1}{\alpha}$. Then $T = \sum_{i=1}^{n_1} \ln\left(\frac{x_{1i}}{\min(x_{1i})}\right)$ will be Gamma distributed with (n_1) and $\left(\frac{1}{\alpha}\right)$ parameters.

Now, when $\hat{\alpha}_{1MLE} = \frac{n_1}{T}$, we get:

$$E(\hat{\alpha}_{1MLE}) = \frac{n_1}{n_1 - 1}\alpha \quad \text{and} \quad var(\hat{\alpha}_{1MLE}) = \frac{n_1^2\alpha^2}{(n_1-1)^2(n_1-2)}.$$

The maximum likelihood estimator $\hat{\alpha}_{1MLE}$ is biased estimator but the following $\hat{\alpha}_1$ is unbiased estimator.

$$\hat{\alpha}_1 = \frac{n_1 - 1}{n}\hat{\alpha}_{1MLE} = \frac{n_1 - 1}{T}$$

i.e.; $E(\hat{\alpha}_1) = \alpha$ and $var(\hat{\alpha}_1) = \frac{\alpha^2}{n_1 - 2}$.

Furthermore,

$$var(\hat{\alpha}_1) = \frac{\alpha^2}{n_1 - 2} < var(\hat{\alpha}_{1MLE}).$$

Thus, $\hat{\alpha}_1$ is a better estimator of α than $\hat{\alpha}_{1MLE}$.

Therefore, we used $\hat{\alpha}_1$ estimator above in (PTDSSE) (2) as a usual estimator in the next section.

3. Preliminary Test Double Stage Shrunken Estimator (PTDSSE)

In this section, recall the estimator which has the form (2).

Where R is the pre-test region for testing the hypothesis H_0: $\hat{\alpha}_1 = \alpha_0$ against H_A: $\hat{\alpha}_1 \neq \alpha_0$ with level of significance (α) using test statistic

$$T(\hat{\alpha}_1 / \alpha_0) = \frac{2(n_1 - 1)\alpha_0}{\hat{\alpha}_1}$$

…(3)

i.e.; $R = \left[a \leq \frac{2(n_1 - 1)\alpha_0}{\hat{\alpha}_1} \leq b \right]$

or $R = \left[\frac{2(n_1 - 1)\alpha_0}{b}, \frac{2(n_1 - 1)\alpha_0}{a} \right]$.

Where a and b are the lower and upper $100(\Delta/2)$ percentile point of chi-square distribution with degree of freedom ($2n_1$).

The expressions for Bais [B(·)] and mean square Error [MSE(·)] of $\tilde{\alpha}_{DS}$:

$$Bias(\tilde{\alpha}_{DS} | \hat{\alpha}_1, R) = E[\tilde{\alpha}_{DS} - \alpha]$$

$$= \int_{\hat{\alpha}_2 = 0}^{\infty} \int_{\hat{\alpha}_1 \in R}^{\infty} [\psi(\hat{\alpha}_1)(\hat{\alpha}_1 - \alpha_0) + (\alpha_0 - \alpha)] f(\hat{\alpha}_1) f(\hat{\alpha}_2) d\hat{\alpha}_1 d\hat{\alpha}_2 + \int_{\hat{\alpha}_2 = 0}^{\infty} \int_{\hat{\alpha}_1 \in \bar{R}}^{\infty} (\hat{\alpha}_p - \alpha) f(\hat{\alpha}_1) f(\hat{\alpha}_2) d\hat{\alpha}_1 d\hat{\alpha}_2$$

Where $\bar{R}$ is the complement region of R in real space.

And by simple calculations, we get:

$$Bias(\tilde{\alpha}_{DS} | \hat{\alpha}_1, R) = \alpha\{(n_1 - 1)\psi(\hat{\alpha}_1) J_1(a^*, b^*) + (1 - \psi(\hat{\alpha}_1))\zeta J_0(a^*, b^*) - J_0(a^*, b^*) - \frac{1}{1+u}[(n_1 - 1) J_1(a^*, b^*) - J_0(a^*, b^*)]\} \quad \ldots(4)$$

And

$$\mathrm{MSE}(\tilde{\alpha}_{DS}|\hat{\alpha}_1,R)=E(\tilde{\alpha}_{DS}-\alpha)^2$$

$$=\int_{\hat{\alpha}_2=0}^{\infty}\int_{\hat{\alpha}_1\in R}^{\infty}[\psi(\hat{\alpha}_1)(\hat{\alpha}_1-\alpha_0)+(\alpha_0-\alpha)]^2 f(\hat{\alpha}_1)f(\hat{\alpha}_2)d\hat{\alpha}_1 d\hat{\alpha}_2+$$

$$\int_{\hat{\alpha}_2=0}^{\infty}\int_{\hat{\alpha}_1\in \bar{R}}^{\infty}(\hat{\alpha}_p-\alpha)^2 f(\hat{\alpha}_1)f(\hat{\alpha}_2)d\hat{\alpha}_1 d\hat{\alpha}_2$$

We conclude,

$$\mathrm{MSE}(\tilde{\alpha}_{DS}|\hat{\alpha}_1,R)=\alpha^2\Bigg\{(\psi(\hat{\alpha}_1))^2[(n_1-1)^2J_2(a^*,b^*)-2(n_1-1)\zeta J_1(a^*,b^*)-\zeta^2J_0(a^*,b^*)]+$$
$$2\psi(\hat{\alpha}_1)(\zeta-1)\Bigg[(n_1-1)J_1(a^*,b^*)-\zeta J_0(a^*,b^*)]+(\zeta-1)^2J_0(a^*,b^*)+\left(\frac{1}{(1+u)^2}\right)\left(\frac{1}{n_1-2}\right)+ \quad \ldots(5)$$
$$\left(\frac{u}{1+u}\right)^2\left(\frac{1}{n_1u-2}\right)\Bigg]-\Bigg[\frac{1}{(1+u)^2}[(n_1-1)^2J_2(a^*,b^*)-2(n_1-1)J_1(a^*,b^*)+J_0(a^*,b^*)]+$$
$$\left(\frac{u}{1+u}\right)^2\left(\frac{1}{n_1u-2}\right)J_0(a^*,b^*)\Bigg]\Bigg\}$$

Where $J_\ell(a^*,b^*)=\int_{a^*}^{b^*}y^{-\ell}\frac{e^{-y}y^{n-1}}{\Gamma(n)}dy;\ \ell=0,1,2$.

Also; $\zeta=\frac{\alpha_0}{\alpha}$, $a^*=\zeta^{-1}\cdot a$, $b^*=\zeta^{-1}\cdot b$, $y_i=\frac{(n_i-1)\alpha}{\hat{\alpha}_i};(i=1,2)$ and $u=n_2/n_1$, $n=n_1+n_2$.

The Expected sample size can be obtained as:

$E(n|\alpha,R)=n_1[1-\frac{u}{1+u}J_0(a^*,b^*)]$, see [14], [15].

The Efficiency of $\tilde{\alpha}_{DS}$ relative to $\hat{\alpha}_1$ is given by:

$$\mathrm{R.Eff}(\tilde{\alpha}_{DS}|\alpha,R)=\frac{\mathrm{MSE}(\hat{\alpha}_1|\alpha,R)}{\mathrm{MSE}(\hat{\alpha}_{DS}|\alpha,R)[E(n|\alpha,R)/n]}$$
$$=\frac{\alpha^2/(n_1-2)}{\mathrm{MSE}(\hat{\alpha}_{DS}|\alpha,R)[E(n|\alpha,R)/n]} \quad \ldots(6)$$

Where $[E(n|\hat{\alpha}_1,R)/n]$ is the Expected sample size proportion, see [14], [15].

The probability of avoiding second sample is computing by $p(\hat{\alpha}_1\in R)=J_0(a^*,b^*)$.

Finally, the percentage of overall sample saved can be obtained by:

$\frac{n_2}{n}p(\hat{\alpha}_1\in R)*(100)=\left(\frac{u}{1+u}\right)J_0(a^*,b^*)*(100)$.

See [14], [15].

4. Conclusions and Numerical Results

The computations of Relative Efficiency [R.Eff(·)] and Bias Ratio [B(·)], Expected sample size [E(n/α,R)], Expected sample size proportion

[E(n/α,R)/n], Percentage of the overall sample saved (p.o.s.s.) and probability of a voiding the second sample [$P(\hat{\alpha}_1 \in R)$] were used for the estimator $\tilde{\alpha}_{DS}$.
Also, comparison between the 1st proposed estimator which is defined in equation (2) using $\psi_1(\cdot) = \text{Exp}\left[-\left(\frac{n_1+10}{n_1}\right)\right]$ with the 2nd proposed estimator when uses the shrinkage weight factor $\psi_2(\cdot) = \text{Exp}\left[-\frac{b}{a}\right]$, where a and b are the lower and upper 100(Δ/2) percentile point of chi-square distribution with degree of freedom ($2n_1$) as defined in section three.
These computations were performed for n_1 = 4, 8,16,20, u(= n_2/n_1) = 2, 6, 10, 12, ζ = (α_0/α) = 0.25, 0.75, 1, 1.25, 1.75, 2, and Δ = 0.01,0.05. Some of these computations are given in the tables (1)-(14).
The observation mentioned in the tables lead to the following results:

1. The Relative Efficiency [R.Eff(·)] of $\tilde{\alpha}_{DS}$ are adversely proportional with small value of Δ especially when ζ = 1, i.e. Δ = 0.01 yield highest efficiency.
2. The Relative Efficiency [R.Eff(·)] of $\tilde{\alpha}_{DS}$ has maximum value when $\alpha=\alpha_0$ (ζ=1), for each n_1, Δ, and decreasing otherwise (ζ≠1).
3. Bias Ratio [B(·)] of $\tilde{\alpha}_{DS}$ are reasonably small when $\alpha=\alpha_0$ for each n_1, Δ, and increases otherwise. This property shown that the proposed estimator $\tilde{\alpha}_{DS}$ is very closely to unbiasedness property especially when $\alpha=\alpha_0$.
4. The Effective interval of $\tilde{\alpha}_{DS}$ [the value of $\tilde{\alpha}_{DS}$ which makes R.Eff(·) of $\tilde{\alpha}_{DS}$ greater than one] is at least [0.5,1.5].
5. Bias Ratio [B(·)] of $\tilde{\alpha}_{DS}$ are reasonably large with small value of u.
6. The results in (1-5) is satisfied bettery in the consider estimator which is using the shrinkage weight factor $\psi_2(\cdot)$ than the estimator with respect to $\psi_1(\cdot)$.
7. R.Eff(·) of $\tilde{\alpha}_{DS}$ is at most decreasing function with increasing of the first sample size (n_1), for each Δ and ζ with two cases of $\psi_i(\cdot)$, i = 1, 2.
8. The Expected value of sample size of $\tilde{\alpha}_{DS}$ has very good results when (ζ = 2 and Δ = 0.01) and faraway otherwise, with two cases of $\psi_i(\cdot)$, i = 1, 2.
9. R.Eff($\tilde{\alpha}_{DS}$) is an increasing function with respect to u, only with the proposed estimator $\tilde{\alpha}_{DS}$ which is using $\psi_2(\cdot)$.
   This property shown the effective of proposed estimator using small (n_1) relative to (n_2) (or large n_2) which given higher efficiency and reduce the observation cost.
10. The considered estimator $\tilde{\alpha}_{DS}$ is better than the classical estimator especially when $\alpha\approx\alpha_0$, this will given the effective of $\tilde{\alpha}_{DS}$ relative to $\hat{\alpha}_1$ and also given

an important weight of prior knowledge, and the augmentation of efficiency may be reach to tens times.

11. The considered estimator $\tilde{\alpha}_{DS}$ is more efficient than the estimators introduced by [16], in the sense of higher efficiency, with two cases of $\psi_i(\cdot)$, $i = 1, 2$.

12. Percentage of the overall sample saved (p.o.s.s.) $\left[\frac{n_2}{n} J_0(a^*, b^*) * 100\right]$ is increasing value with increasing value of u ($u = n_2 / n_1$) and ζ, with respect to $\psi_1(\cdot)$.

References

1. Steind, J., (1965), Random Process and the Growth of Firms, Hafner, New York.
2. Mandelbort, B., (1960), The Pareto-Levy Low and The Distribution of Income, International Economic Review, Vol.(1), pp.79-106.
3. Ord,J.K., (1975), Statistical Models for Personal Income Distributions, In Statistical Distributions in Scientific World, 2, G.P.Patil, S.Kotz and J.K.Ord.(Editors), Dordrecht, Reidel, pp.151-158.
4. Quandt, R.E., (1966), Old and New Methods of Estimation and the Pareto Distribution, Metrika, Vol.10, pp.55-82.
5. Malik, H.J., (1970), Estimation of the Parameter of the Pareto Distribution, Metrika, Vol.15, pp.126-132.
6. Ogawa, (1962),
7. Koutrouvelis, I.A., (1981), Large-Sample Quantile Estimation in Pareto Laws, Communications in Statistics, Theory and Methods, Vol.10, pp.189-201.
8. Johnson, N.L., Kotz, S. and Balakrishnan, (1994), Continuous Univariate Distributions, Vol. 1, Wiley, New York.
9. Baklizi, A., (2008), Preliminary Test Estimation in the Pareto Distribution Using Minimax Regret Significance Levels, International Mathematical Forum, Vol.3, No.10, pp.473-478.
10. Rytgaard, M., (1990), Estimation in the Pareto Distribution, Astin Bulletin, Vol.20, No.2, pp.201-216.
11. Thompson, J.R., (1968), Some Shrinkage Techniques for Estimating the Mean, J. Amer. Statist. Assoc, Vol.63, pp.113-122.
12. Casella, G., (1992-1994), Shrinkage, In: Encyclopedia of Envirnmetrics, 4 Eds.A.H.EL. Shaarawi and WW.Piegorsch), John Wiley:New York.
13. Katti, S.K., (1962), Use of Some a Prior Knowledge in the Estimation of Means fro Double Samples, Biometrics, Vol.18, pp.139-147.
14. Waikar, V.B., Schuurmann, F.J. and Raghunathar, T.E., (1984), On a Two-Stage Shrinkage Testimator of the Mean of a Normal Distribution, Commum. Statist-Theor. Meth., Vol.A13, No.15, pp.1901-1913.
15. Handa, B.K.Kambo, N.S. and Al-Hemyari, Z.A., (1988), On Double Stage Shrunken Estimator of the Mean of Exponential Distribution, IAPQR Trans, Vol.13, No.1, pp.19-33.

Table (1)

Shown Bias ratio [B(·)] and R.E.ff of $\tilde{\alpha}_{DS}$ w.r.t. $\psi_1(\cdot)$, Δ, n_1 and ζ when u = 2

			ζ					
Δ	**n_1**	**R.Eff. Bias**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
0.01	4	R.Eff(·) B(·)	0.53649 (– 0.11541)	2.399 (– 0.15975)	6.21733 (0.0183)	3.75793 (0.21725)	0.70714 (0.63004)	0.40822 (0.83555)
	8	R.Eff(·) B(·)	0.90031 (– 0.00027)	1.21347 (– 0.10393)	5.21211 (0.01001)	2.78318 (0.18422)	0.40875 (0.56323)	0.23182 (0.75064)
	16	R.Eff(·) B(·)	0.95551 (0)	0.80643 (– 0.02415)	2.4683 (0.00355)	1.77885 (0.14224)	0.23009 (0.51779)	0.12992 (0.69441)
	20	R.Eff(·) B(·)	0.96489 (0)	0.85225 (– 0.00906)	1.85225 (0.00186)	1.53236 (0.13436)	0.18957 (0.50686)	0.10702 (0.68177)
0.05	4	R.Eff(·) B(·)	0.72407 (– 0.01315)	1.5529 (– 0.10152)	3.727171 (0.3385)	3.07143 (0.2157)	0.67768 (0.6146)	0.39137 (0.80766)
	8	R.Eff(·) B(·)	0.90428 (0)	0.9557 (– 0.04664)	2.72385 (0.01352)	2.4982 (0.16394)	0.4009 (0.54134)	0.2247 (0.72167)
	16	R.Eff(·) B(·)	0.95551 (0)	0.90812 (– 0.00467)	1.43892 (0.00259)	1.72404 (0.10972)	0.22625 (0.49004)	0.126 (0.6666)
	20	R.Eff(·) B(·)	0.96489 (0)	0.94536 (– 0.00117)	1.22005 (0.00104)	1.53424 (0.08994)	0.18617 (0.47656)	0.10376 (0.65435)

Table (2)

Shown Bias ratio [B(·)] and R.E.ff of $\tilde{\alpha}_{DS}$ w.r.t. $\psi_1(\cdot)$, Δ, n_1 and ζ when u = 6

			ζ					
Δ	**n_1**	**R.Eff. Bias**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
0.01	4	R.Eff(·) B(·)	0.33509 (– 0.13411)	2.49734 (– 0.18401)	10.55162 (– 0.01209)	4.87112 (0.17311)	0.791 (0.54784)	0.45084 (0.73266)
	8	R.Eff(·) B(·)	0.94328 (– 0.00033)	0.83028 (– 0.13186)	6.76222 (– 0.01817)	3.02319 (0.15234)	0.44341 (0.51)	0.25227 (0.68402)
	16	R.Eff(·) B(·)	0.9803 (0)	0.53997 (– 0.03402)	2.35726 (0.01982)	1.58623 (0.12439)	0.23591 (0.48654)	0.13593 (0.65622)
	20	R.Eff(·) B(·)	0.98456 (0)	0.66139 (– 0.0131)	1.63615 (– 0.01692)	1.26328 (0.11134)	0.19025 (0.48079)	0.1106 (0.65035)
0.05	4	R.Eff(·) B(·)	0.6961 (– 0.01646)	1.28105 (– 0.14966)	6.22788 (– 0.02235)	4.70641 (0.14936)	0.70641 (0.52043)	0.3949 (0.69916)
	8	R.Eff(·) B(·)	0.95581 (0)	0.64374 (– 0.07028)	2.92767 (– 0.02455)	2.79569 (0.11903)	0.39143 (0.4817)	0.21814 (0.65391)
	16	R.Eff(·) B(·)	0.9803 (0)	0.79288 (– 0.00727)	1.26312 (– 0.01446)	1.54551 (0.08073)	0.20086 (0.45449)	0.11659 (0.62835)
	20	R.Eff(·) B(·)	0.98456 (0)	0.9093 (– 0.00184)	1.07625 (– 0.00945)	1.31058 (0.06593)	0.15971 (0.44654)	0.0947 (0.62298)

Table (3)

Shown Bias ratio [B(·)] and R.E.ff of $\tilde{\alpha}_{DS}$ w.r.t. $\psi_1(\cdot)$, Δ, n_1 and ζ when u = 10

			ζ					
Δ	**n_1**	**R.Eff. Bias**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
0.01	4	R.Eff(·) B(·)	0.23815 (– 0.13882)	0.08367 (– 0.19005)	9.43571 (– 0.02134)	4.64519 (0.1582)	0.80838 (0.51835)	0.46153 (0.69538)
	8	R.Eff(·) B(·)	0.95001 (– 0.00035)	0.60196 (– 0.13946)	5.85051 (– 0.02618)	2.72889 (0.14279)	0.44322 (0.4933)	0.2538 (0.66299)
	16	R.Eff(·) B(·)	0.98731 (0)	0.40249 (– 0.03673)	1.98366 (– 0.02627)	1.28566 (0.11738)	0.22847 (0.47742)	0.13444 (0.64501)
	20	R.Eff(·) B(·)	0.99008 (0)	0.53593 (– 0.0142)	1.37475 (– 0.02209)	0.98895 (0.10492)	0.18199 (0.47328)	0.1089 (0.64126)
0.05	4	R.Eff(·) B(·)	0.62246 (– 0.01735)	0.96073 (– 0.16331)	5.83112 (– 0.03946)	4.60345 (0.12782)	0.6783 (0.48747)	0.36867 (0.66042)
	8	R.Eff(·) B(·)	0.97105 (0)	0.47108 (– 0.07683)	2.50466 (– 0.03536)	2.438 (0.10585)	0.35344 (0.46325)	0.19821 (0.63265)
	16	R.Eff(·) B(·)	0.98731 (0)	0.69571 (– 0.00798)	1.08339 (0.01917)	1.26296 (0.07262)	0.17272 (0.4442)	0.10385 (0.61715)
	20	R.Eff(·) B(·)	0.99008 (0)	0.86583 (– 0.0020[illegible])	0.94675 (– 0.01234)	1.06475 (0.05927)	0.13494 (0.43797)	0.08395 (0.61391)

Table (4)

Shown Bias ratio [B(·)] and R.E.ff of $\tilde{\alpha}_{DS}$ w.r.t. $\psi_1(\cdot)$, Δ, n_1 and ζ when u = 12

			ζ					
Δ	**n_1**	**R.Eff. Bias**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
0.01	4	R.Eff(·)	0.20772	1.90359	8.71696	4.47343	0.80787	0.46164
		B(·)	(– 0.14006)	(– 0.19163)	(– 0.02391)	(0.15393)	(0.50978)	(0.68452)
	8	R.Eff(·)	0.95003	0.52757	5.36318	2.57535	0.4394	0.25237
		B(·)	(– 0.00036)	(– 0.1415)	(– 0.02836)	(0.14014)	(0.78863)	(0.65709)
	16	R.Eff(·)	0.98923	0.35682	1.8219	1.16652	0.22379	0.13299
		B(·)	(0)	(– 0.03745)	(– 0.02802)	(0.11457)	(0.47491)	(0.64193)
	20	R.Eff(·)	0.99159	0.4893	1.267	0.88728	0.1774	0.1076
		B(·)	(0)	(0.0145)	(0.2349)	(0.10318)	(0.47123)	(0.63878)
0.05	4	R.Eff(·)	0.58767	0.84724	5.41071	4.39296	0.65259	0.35348
		B(·)	(– 0.01759)	(– 0.16703)	(– 0.04422)	(0.12171)	(0.47794)	(0.64917)
	8	R.Eff(·)	0.97527	0.41447	2.30163	2.25535	0.33477	0.18835
		B(·)	(0)	(– 0.0786)	(– 0.0383)	(0.10222)	(0.45811)	(0.62669)
	16	R.Eff(·)	0.98923	0.65496	1.00843	1.14888	0.16083	0.0981
		B(·)	(0)	(– 0.00817)	(– 0.02045)	(0.07042)	(0.44138)	(0.61407)
	20	R.Eff(·)	0.99159	0.84481	0.89182	0.96839	0.12487	0.07919
		B(·)	(0)	(– 0.00207)	(– 0.01312)	(0.05746)	(0.43564)	(0.61144)

Table (5)

Shown Bias ratio [B(·)] and R.E.ff of $\tilde{\alpha}_{DS}$ w.r.t. $\psi_2(\cdot)$, Δ, n_1 and ζ when u = 2

			ζ					
Δ	**n_1**	**R.Eff. Bias**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
0.01	4	R.Eff(·)	0.51316	2.3424	6.92278	3.07635	0.5083	0.29146
		B(·)	(– 0.12218)	(– 0.16987)	(0.03517)	(0.26756)	(0.75423)	(0.99736)
	8	R.Eff(·)	0.90008	1.21532	6.28806	1.74815	0.23821	0.1349
		B(·)	(– 0.00028)	(– 0.10544)	(0.03661)	(0.25662)	(0.4668)	(0.99144)
	16	R.Eff(·)	0.95551	0.8598	2.95462	0.87865	0.11951	0.06775
		B(·)	(0)	(– 0.02072)	(0.03165)	(0.22705)	(0.72434)	(0.96715)
	20	R.Eff(·)	0.96489	0.89625	2.18962	0.71856	0.09777	0.05551
		B(·)	(0)	(– 0.00734)	(0.02488)	(0.20756)	(0.71057)	(0.95143)
0.05	4	R.Eff(·)	0.72244	1.62681	3.89963	2.38171	0.47731	0.27686
		B(·)	(– 0.01336)	(– 0.09248)	(0.06499)	(0.27619)	(0.74212)	(0.96803)
	8	R.Eff(·)	0.90428	1.03522	3.04037	1.47606	0.23391	0.13336
		B(·)	(0)	(– 0.03833)	(0.04735)	(0.23795)	(0.71421)	(0.94263)
	16	R.Eff(·)	0.95551	0.93487	1.60681	0.89382	0.12523	0.07147
		B(·)	(0)	(– 0.00345)	(0.01991)	(0.16846)	(0.66097)	(0.88925)
	20	R.Eff(·)	0.96489	0.95515	1.33021	0.8011	0.10388	0.05941
		B(·)	(0)	(– 0.00085)	(0.01161)	(0.13865)	(0.63939)	(0.86827)

Table (6)

Shown Bias ratio [B(·)] and R.E.ff of $\tilde{\alpha}_{DS}$ w.r.t. $\psi_2(\cdot)$, Δ, n_1 and ζ when u = 6

			ζ					
Δ	**n_1**	**R.Eff. Bias**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
0.01	4	R.Eff(·)	0.30149	2.44238	25.50726	3.49406	0.45391	0.25561
		B(·)	(– 0.145)	(– 0.20031)	(0.01507)	(0.2541)	(0.74778)	(0.99316)
	8	R.Eff(·)	0.94248	0.83754	14.9645	1.56693	0.2181	0.12375
		B(·)	(– 0.00035)	(– 0.13378)	0.01561)	(0.24431)	(0.74303)	(0.98989)
	16	R.Eff(·)	0.9803	0.60705	3.9132	0.62939	0.1099	0.06368
		B(·)	(0)	(– 0.03012)	(0.01215)	(0.2129)	(0.72151)	(0.96648)
	20	R.Eff(·)	0.98456	0.7284	2.52047	0.47356	0.08949	0.05242
		B(·)	(0)	(– 0.01118)	(0.00874)	(0.19293)	(0.70783)	(0.9509)
0.05	4	R.Eff(·)	0.68989	1.475	9.37476	2.5043	0.3848	0.21642
		B(·)	(– 0.0168)	(– 0.13509)	(0.02782)	(0.24682)	(0.72588)	(0.95754)
	8	R.Eff(·)	0.95581	0.76202	4.54283	1.12402	0.18827	0.10808
		B(·)	(0)	(– 0.05958)	(0.01899)	(0.21428)	(0.70418)	(0.9383)
	16	R.Eff(·)	0.9803	0.85251	1.71667	0.55756	0.0986	0.05919
		B(·)	(0)	(– 0.00584)	(0.00568)	(0.14904)	(0.65326)	(0.88727)
	20	R.Eff(·)	0.98456	0.93452	1.35591	0.48051	0.08056	0.04942
		B(·)	(0)	(– 0.00147)	(0.00259)	(0.12142)	(0.63206)	(0.86671)

Table (7)

Shown Bias ratio [B(·)] and R.E.ff of $\tilde{\alpha}_{DS}$ w.r.t. $\psi_2(\cdot)$, Δ, n_1 and ζ when u = 10

			ζ					
Δ	**n_1**	**R.Eff. Bias**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
0.01	4	R.Eff(·) B(·)	0.20838 (– 0.15122)	2.06238 (– 0.20862)	47.74391 (0.00959)	3.31252 (0.25043)	0.42824 (0.74602)	0.24013 (0.99201)
	8	R.Eff(·) B(·)	0.94864 (– 0.00036)	0.60937 (– 0.14151)	20.85194 (0.00988)	1.34796 (0.24095)	0.20642 (0.74203)	0.11772 (0.98947)
	16	R.Eff(·) B(·)	0.98731 (0)	0.46482 (– 0.03269)	4.25643 (0.00683)	0.47875 (0.20904)	0.10302 (0.72073)	0.06097 (0.96629)
	20	R.Eff(·) B(·)	0.99008 (0)	0.60983 (– 0.01222)	2.61766 (0.00433)	0.34658 (0.18894)	0.8332 (0.70709)	0.05026 (0.95076)
0.05	4	R.Eff(·) B(·)	0.61341 (– 0.01774)	1.15968 (– 0.14671)	13.33264 (0.01768)	2.12833 (0.23881)	0.32837 (0.72145)	0.18324 (0.95468)
	8	R.Eff(·) B(·)	0.97105 (0)	0.58225 (– 0.06537)	5.16228 (0.01125)	0.85503 (0.20782)	0.15964 (0.70145)	0.09253 (0.93712)
	16	R.Eff(·) B(·)	0.98731 (0)	0.77413 (– 0.0065)	1.73813 (0.0018)	0.39549 (0.14374)	0.08179 (0.65115)	0.05099 (0.88673)
	20	R.Eff(·) B(·)	0.99008 (0)	0.90393 (– 0.00164)	1.35266 (0.00013)	0.33713 (0.11673)	0.6607 (0.63006)	0.04262 (0.86628)

Table (8)

Shown Bias ratio [B(·)] and R.E.ff of $\tilde{\alpha}_{DS}$ w.r.t. $\psi_2(\cdot)$, Δ, n_1 and ζ when u = 12

			ζ					
Δ	**n_1**	**R.Eff. Bias**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
0.01	4	R.Eff(·) B(·)	0.18019 (– 0.1529)	1.89316 (– 0.21085)	59.06691 (0.00812)	3.20197 (0.24944)	0.4183 (0.74555)	0.23422 (0.9917)
	8	R.Eff(·) B(·)	0.94837 (– 0.00037)	0.53466 (– 0.14359)	23.05312 (0.00834)	1.25712 (0.24004)	0.20152 (0.74176)	0.11521 (0.98935)
	16	R.Eff(·) B(·)	0.98923 (0)	0.41579 (– 0.03338)	4.3551 (0.0054)	0.42702 (0.208)	0.10001 (0.72053)	0.05977 (0.96624)
	20	R.Eff(·) B(·)	0.99159 (0)	0.56367 (– 0.01251)	2.64239 (0.00315)	0.30527 (0.18787)	0.08062 (0.70689)	0.04929 (0.95072)
0.05	4	R.Eff(·) B(·)	0.57775 (– 0.01799)	1.03778 (– 0.14984)	14.86608 (0.01495)	1.95862 (0.23665)	0.30661 (0.72026)	0.17069 (0.95391)
	8	R.Eff(·) B(·)	0.97527 (0)	0.51949 (– 0.06693)	5.35117 (0.00917)	0.76106 (0.20608)	0.14855 (0.70071)	0.08646 (0.9368)
	16	R.Eff(·) B(·)	0.98923 (0)	0.73939 (– 0.00667)	1.74063 (0.00075)	0.34475 (0.14231)	0.07541 (0.65059)	0.04772 (0.88659)
	20	R.Eff(·) B(·)	0.99159 (0)	0.88849 (– 0.00168)	1.34856 (– 0.00053)	0.293 (0.11546)	0.06065 (0.62952)	0.03991 (0.86617)

Table (9)

Shown Expected Sample Size of $\tilde{\alpha}_{DS}$ w.r.t. Δ, u, $\psi_1(\cdot)$ and ζ when n_1 = 4

		ζ					
u	**Δ**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
2	0.01	10.27079	4.85941	4.3809	4.1918	4.07525	4.07996
	0.05	11.79328	6.65494	5.41404	4.80276	4.38445	4.40005
6	0.01	22.81238	6.57824	5.14271	4.5754	4.22575	4.23987
	0.05	27.37984	11.96483	8.24212	6.40827	5.15335	5.20014
10	0.01	35.35396	8.29706	5.90452	4.959	4.37625	4.39978
	0.05	42.9664	17.27472	11.07021	8.01379	5.92225	6.00023
12	0.01	41.62475	9.15647	6.28543	5.1508	4.4515	4.47974
	0.05	50.75968	19.92966	12.48425	8.81655	6.3067	6.40028

Table (10)

Shown Expected Sample Size of $\tilde{\alpha}_{DS}$ w.r.t. Δ, u, $\psi_2(\cdot)$ and ζ when $n_1 = 4$

		ζ					
u	**Δ**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
2	0.01	10.27079	4.85941	4.3809	4.1918	4.07525	4.07996
	0.05	11.79328	6.65494	5.41404	4.80276	4.38445	4.40005
6	0.01	22.81238	6.57824	5.14271	4.5754	4.22575	4.23987
	0.05	27.37984	11.96483	8.24212	6.40827	5.15335	5.20014
10	0.01	35.35396	8.29706	5.90452	4.959	4.37625	4.39978
	0.05	42.9664	17.27472	11.07021	8.01379	5.92225	6.00023
12	0.01	41.62475	9.15647	6.28543	5.1508	4.4515	4.47974
	0.05	50.75968	19.92966	12.48425	8.81655	6.3067	6.40028

Table (11)

Shown Expected Sample Size Proportion w.r.t. Δ, u, $\psi_1(\cdot)$ and ζ when $n_1 = 4$

		ζ					
u	**Δ**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
2	0.01	0.8559	0.40495	0.36508	0.34932	0.3396	0.34
	0.05	0.98277	0.55458	0.45117	0.40023	0.36537	0.36667
6	0.01	0.81473	0.23494	0.18367	0.16341	0.15092	0.15142
	0.05	0.97785	0.42732	0.29436	0.22887	0.18405	0.18572
10	0.01	0.8035	0.18857	0.13419	0.1127	0.9946	0.1
	0.05	0.97651	0.39261	0.2516	0.18213	0.1346	0.13637
12	0.01	0.80048	0.17609	0.12087	0.09905	0.08561	0.08615
	0.05	0.97615	0.38326	0.24008	0.16955	0.12128	0.12308

Table (12)

Shown Expected Sample Size Proportion w.r.t. Δ, u, $\psi_2(\cdot)$ and ζ, when $n_1 = 4$

		ζ					
u	**Δ**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
2	0.01	0.8559	0.40495	0.36508	0.34932	0.3396	0.34
	0.05	0.98277	0.55458	0.45117	0.40023	0.36537	0.36667
6	0.01	0.81473	0.23494	0.18367	0.16341	0.15092	0.15142
	0.05	0.97785	0.42732	0.29436	0.22887	0.18405	0.18572
10	0.01	0.8035	0.18857	0.13419	0.1127	0.9946	0.1
	0.05	0.97651	0.39261	0.2516	0.18213	0.1346	0.13637
12	0.01	0.80048	0.17609	0.12087	0.09905	0.08561	0.08615
	0.05	0.97615	0.38326	0.24008	0.16955	0.12128	0.12308

Table (13)

Shown Probability of a Voiding Second Sample w.r.t. Δ, u, n_1 and ζ for $\psi_1(\cdot)$, $\psi_2(\cdot)$

			ζ					
u	**n_1**	**Δ**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
2	4	0.01	0.21615	0.89257	0.95239	0.97602	0.99059	0.99001
		0.05	0.02584	0.66813	0.82324	0.89966	0.95194	0.94999
6	8	0.01	530182×10^{-4}	0.62016	0.85138	0.94181	0.98834	0.99
		0.05	32179×10^{-6}	0.29981	0.61242	0.80612	0.94447	0.94999
10	16	0.01	277939×10^{-12}	0.14749	0.55437	0.83655	0.98344	0.99
		0.05	0	0.02914	0.26435	0.60564	0.92901	0.95
12	20	0.01	0	0.05522	0.40878	0.77071	0.98082	0.99
		0.05	0	722154×10^{-3}	0.15876	0.51094	0.92114	0.95

Table (14)
Shown Percentage of Overall Sample Saved w.r.t. Δ, u, n_1, $\psi_1(\cdot)$ and ζ

			ζ					
u	**n_1**	**Δ**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
2	4	0.01	14.41007	59.5049	63.49246	65.06833	66.03958	66.00036
		0.05	1.72266	44.54213	54.88299	59.97702	63.46291	63.33295
6	8	0.01	0.04544	53.15653	72.97562	80.72648	84.71523	84.85722
		0.05	275819×10^{-4}	25.69784	52.49297	69.0962	80.95432	81.42783
10	16	0.01	252672×10^{-10}	13.40818	50.3969	76.0516	89.40397	90
		0.05	134787×10^{-14}	2.64914	24.03157	55.05818	84.45573	86.36319
12	20	0.01	291725×10^{-15}	5.09685	37.73399	71.14217	90.53717	91.38451
		0.05	0	0.6666	14.6547	47.16403	85.02801	87.69248

مُقدر التقلص ذو المرحلتين في توزيع باريتو

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المستخلص:

يقترح هذا البحث استخدام مقدر الاختبار الاولي المقلص ذو المرحلتين (PTDSSE) لتقدير معلمة الشكل (α) لتوزيع باريتو، عندما تكون لمعلمة القياس أقل قيمة في المنطقة (R) حول المعلومات المسبقة (α_0) المتوفرة حول المعلمة الحقيقية (α) بشكل تقدير ابتدائي، لتقليل متوسط مربعات الخطأ وتقليل كلفة المعاينة في التجارب.

عندما يكون استهلاك الوقت أو كلفة المعاينة في التجارب عالٍ جداً فان طريقة التقلص ذات المرحلتين تكون مناسبة للحصول على مقدر يقلل من حجم العينة المتوقع وبالتالي التقليل من هذه الكلف.

ومن خواص هذا المقدر ايضاً انه ذو متوسط مربعات خطأ (MSE) صغير خصوصاً عند اختيار عامل تقلص موزون $\psi(\cdot)$ ومنطقة قبول R بشكل مناسب.

اشتقت معادلات التحيز، متوسط مربعات الخطأ (MSE)،الكفاءة النسبية $[R.Eff(\cdot)]$، حجم العينة المتوقع $[E(n/\alpha,R)]$، حجم العينة المتوقع النسبي $[E(n/\alpha,R)/n]$، احتمالية تجنب العينة الثانية $[p(\hat{\alpha}_1 \in R)]$ ونسبة الادخار الكلي المئوية للعينة $[\frac{n_2}{n_1} p(\hat{\alpha}_1 \in R) * 100]$ للمقدر المقترح (DSSE).

أعطيت النتائج العددية والاستنتاجات الخاصة بالمقدر المقترح (PTDSSE) عندما يكون المقدر المقترح هو مقدر الاختبار الأولي بمستوى معنوية Δ.

اجريت المقارنات مع المقدر الكلاسيكي وبعض المقدرات المقترحة في الدراسات الاخيرة لبيان فائدة المقدر المقترح.