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CONNECTEDNESS IN GRAPHS AND G_m-CLOSURE SPACES

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ABSTRACT

This paper is devoted to the discussion the relationships of connectedness between some types of graphs (resp., digraph) and Gm-closure spaces by using graph closure operators.

1. Introduction And Preliminaries

Graphs are some of the most important structures in discrete mathematics. Their ubiquity can be attributed to two observations. First, from a theoretical perspective, graphs are mathematically elegant. Even though a graph is a simple structure, consisting only of a set of vertices and a relation between pairs of vertices, graph theory is a rich and varied subject. This is partly due the fact that, in addition to being relational structures, graphs can also be seen as topological spaces, combinatorial objects, and many other mathematical structures. This leads to the second observation regarding the importance of graphs: many concepts can be abstractly represented by graphs, making them very useful from a practical viewpoint.

One of the most basic and important building blocks of graph theory is the notion of "connectedness". The same word also has a very important meaning in the field of general topology; indeed, arguably the latter subject grew precisely out of the efforts of several mathematicians to give the right formalization for concepts like "continuity", "convergence", "dimension" and, not least, connectedness. Although formally the two concepts are very different, one depending on finite paths and the other on open sets, the intuition behind the two versions of connectedness is

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essentially the same, and few will dispute that any link between graph theory and topology should at least reconcile them, if not be entirely dictated by this objective. In fact the usual way of modeling a graph as a topological object does achieve this, albeit in a way which, we feel, is not entirely satisfactory.

The notation we use for the graph theoretical aspects of this paper generally follow R. J. Wilson, W. D. Wallis, J. Bondy and D. S. Murty, K Diestel, and D. Nogly and M. Schladt [1, 2, 4, 5, 7]. A graph (resp., directed graph or digraph) $G = (V(G), E(G))$ consists of a vertex set $V(G)$ and an edge set $E(G)$ of unordered (resp., ordered) pairs of elements of $V(G)$. To avoid ambiguities, we assume that the vertex and edge sets are disjoint. We say that two vertices v and w of a graph (resp., digraph) G are adjacent if there is an edge of the form vw (rsep., vw or wv) joining them, and the vertices v and w are then incident with such an edge. A subgraph of a graph G is a graph, each of whose vertices belong to $V(G)$ and each of whose edges degree of a vertex v of G is the number of edges of the from vw and denoted by belong to $E(G)$. The degree of a vertex v of G is the number of edges incident with v, and written $\deg(v)$. A vertex of degree zero is an isolated vertex. In digraph, the out-$D^+(v)$, similarly, the in-degree of a vertex v of G is the number of edges of the form $\vec{wv}$, and denoted by $D^-(v)$. A graph is connected if it cannot be expressed as the union of two graphs, and disconnected otherwise. Clearly any disconnected graph G can be expressed as the union of connected graphs, each of which is a component of G. A graph whose edge-set is empty is a null graph, we denote the null graph on n vertices by N_n. A graph in which each pair of distinct vertices are adjacent is a complete graph, we dente complete graph on n vertices by K_n. A connected graph that is all vertices of degree 2 is a cycle; we denote the cycle graph on n vertices by C_n. The graph obtained from C_n by removing an edge is path graph on n vertices, and dented by P_n. The graph obtained from C_{n-1} by joining each vertex to a new vertex v is the wheel on n vertices, and denoted by W_n. If the vertex set of a graph G can be spite two disjoint sets X and Y (resp., r-disjoint sets) so that each edge of G joins a vertex of X and a vertex of Y (resp., each edge of G has ends in different r-disjoint sets) is bipartite (resp., r-partite). A complete bipartite (resp., r-partite) graph is a bipartite (resp., r-partite) in which each vertex in X is joined to each vertex in Y (resp., each vertex in any set joined to each vertex in the other set), we denote the bipartite graph with s vertices of X and t vertices of Y by $K_{s,r}$. A forest is a graph that contains no cycle, and a connected forest is a tree, we denote the tree graph with n vertices by T_n. A topological space is disconnected [3, 6] if and only if it can be written as a union of two disjoint nonempty open (resp., closed) subsets, otherwise is connected. For a set X, $|X|$ denotes the cardinal number of X.

2. Connectedness In Undirected Graphs

In this section, we introduce the graph closure operators, and we study the concepts of connectedness in some types of graphs and obtained G_m-closure spaces from these graphs by using graph closure operators.

We introduce the definition of graph closure operators in graphs as follows:

Definition 2.1

Let $G = (V(G), E(G))$ be a graph, $P(V(G))$ its power set of all subgraphs of G and $Cl_G: P(V(G)) \rightarrow$

$P(V(G))$ is a mapping associating with each subgraph $H = (V(H), E(H))$ a subgraph $Cl_G V(H)) \subseteq V(G)$ called the closure subgraph of H such that:

$$Cl_G(V(H)) = V(H) \cup \{v \in V(G)\backslash V(H);\ hv \in E(G) \text{ for all } h \in V(H)\}$$

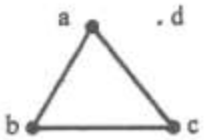
The operation Cl_G is called graph closure operator and the pair $(V(G), C_G)$ is called G-closure space, and $C_G(V(G))$ is the family of elements of Cl_G. The dual of the graph closure operator Cl_G is the graph interior operator $\text{Int}_G: P(V(G)) \rightarrow P(V(G))$ defined by $\text{Int}_G(V(H)) = V(G)\backslash Cl_G(V(G)\backslash V(H))$ for all subgraph $H \subseteq G$. A family of elements of Int_G is called interior subgraph of H and denoted by $O_G(V(G))$. Clear that $(V(G), O_G)$ is a topological space.

A subgraph H of G-closure space $(V(G), C_G)$ is called closed subgraph if $Cl_G(V(H)) = V(H)$. It is called open subgraph if its complement is closed subgraph, i.e. $Cl_G(V(G)\backslash V(H)) = V(G)\backslash V(H)$, or equivalently $\text{Int}_G(V(H)) = V(H)$.

Example 2.1

Let $G = (V(G), E(G))$ be a graph such that:

$$V(G) = \{a, b, c, d\},\ E(G) = \{ab, ac, bc\}$$



$V(H)$	$Cl_G(V(H))$	$V(H)$	$Cl_G(V(H))$
$V(G)$	$V(G)$	$\{a, d\}$	$V(G)$
φ	φ	$\{b, c\}$	$\{a, b, c\}$
$\{a\}$	$\{a, b, c\}$	$\{b, d\}$	$V(G)$
$\{b\}$	$\{a, b, c\}$	$\{c, d\}$	$V(G)$
$\{c\}$	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$
$\{d\}$	$\{d\}$	$\{a, b, d\}$	$V(G)$
$\{a, b\}$	$\{a, b, c\}$	$\{a, c, d\}$	$V(G)$
$\{a, c\}$	$\{a, b, c\}$	$\{b, c, d\}$	$V(G)$

$$C_G(V(G)) = \{V(G), \varphi, \{d\}, \{a, b, c\}\}$$

It is clear that the graph G is disconnected and G-closure spaces $(V(G), C_G)$ on a graph G is disconnected.

Proposition 2.1

If in a graph $G = (V(G), E(G)); |V(G)| > 1$ has at least one isolated vertex, then the G-closure spaces $(V(G), C_G)$ on a graph G is disconnected.

Proof:

Let $G = (V(G), E(G))$ be a graph which has an isolated vertex x. Then $x \notin Cl_G(V(H))$ for all $H(V) \subseteq G(V)\backslash\{x\}$ and $Cl_G(\{x\}) = \{x\}$ which is clopen subgraph. Therefore $\{x\}$ and $V(G)\backslash\{x\}$ are nonempty disjoint open subgrapgs and $\{x\} \cup (V(G)\setminus\{x\}) = V(G)$. Hence the G-closure space on G is disconnected.

Corollary 2.1

If in a graph $G = (V(G), E(G))$ has a vertex of degree zero ($\deg(v) = 0$), then the G-closure space $(V(G), C_G)$ on a graph G is disconnected.

Example 2.2

Let $G = (V(G), E(G))$ be a graph such that:

$V(G) = \{a, b, c, d\}$, $E(G) = \{ab, cd\}$

$V(H)$	$Cl_G(V(H))$	$V(H)$	$Cl_G(V(H))$
$V(G)$	$V(G)$	$\{a, d\}$	$V(G)$
φ	φ	$\{b, c\}$	$V(G)$
$\{a\}$	$\{a, b\}$	$\{b, d\}$	$V(G)$
$\{b\}$	$\{a, b\}$	$\{c, d\}$	$\{c, d\}$
$\{c\}$	$\{c, d\}$	$\{a, b, c\}$	$V(G)$
$\{d\}$	$\{c, d\}$	$\{a, b, d\}$	$V(G)$
$\{a, b\}$	$\{a, b\}$	$\{a, c, d\}$	$V(G)$
$\{a, c\}$	$V(G)$	$\{b, c, d\}$	$V(G)$

$C_G(V(G)) = \{V(G), \varphi, \{a, b\}, \{c, d\}\}$

It is clear that the graph G is disconnected and G-closure spaces $(V(G), C_G)$ on a graph G is disconnected.

Proposition 2.2

In a graph $G = (V(G), E(G))$; $|V(G)| = n > 3$, if the degree of all vertices of G is one, then the G-closure spaces $(V(G), C_G)$ on a graph G is disconnected.

Proof:

Let $G = (V(G), E(G))$ be a graph and $\deg(v) = 1$ for all $v \in V(G)$. For all $v_i \in V(G)$ there exists a unique $v_j \in V(G)$ such that $Cl_G(\{v_i\}) = \{v_i, v_j\}$ (It is clear that $Cl_G(\{v_j\}) = \{v_i, v_j\}$). Clear that $|V(G)|$ is even, so $V(G)\setminus\{v_i, v_j\} = Cl_G(\{v_1, v_2, \ldots, v_{n/2-1}\})$; v_i, $i = 1, 2, \ldots, \frac{n}{2} - 1$ are no mutually adjacent.

i.e., $V(G)\setminus\{v_i, v_j\} = \bigcup_{i=1}^{\frac{n}{2}-1} Cl_G(\{v_i\})$. Therefore $\{v_i, v_j\}$ and $V(G)\setminus\{v_i, v_j\}$ are nonempty disjoint clopen subgraph and $\{v_i, v_j\} \cup (V(G)\setminus\{v_i, v_j\}) = V(G)$. Hence the G-closure spaces $(V(G), C_G)$ on G is disconnected.

Remark 2.1

The condition of $|V(G)| = n > 3$ in above proposition is necessary for example.

Example 2.3

Let $G = (V(G), E(G))$ be a graph such that $|V(G)| = 2$, then the G-closure spaces $(V(G), C_G)$ on G

is indiscrete space which is connected.

We obtain a new definition to construct topological closure spaces from G-closure spaces by redefine graph closure operator on the resultant subgraphs as a domain of the graph closure operator and stop when the operator transfers each subgraph to itself.

Definition 2.2

Let $G = (V(G), E(G))$ be a graph and $Cl_{Gm}: P(V(G)) \rightarrow P(V(G))$ an operator such that:

(a) It is called Gm-closure operator if $Cl_{Gm}(V(H)) = Cl_G(Cl_G(\ldots Cl_G(V(H))))$. m-times, for every subgraph $H \subseteq G$.

(b) it is called G_m-topological closure operator if $Cl_{Gm+1}(V(H)) = Cl_{Gm}(V(H))$ for all subgraph $H \subseteq G$.

The spaces $(V(G), C_{Gm})$ is called Gm-closure space.

Example 2.4

Let $G = (V(G), E(G))$ be a graph such that:

$V(G) = \{a, b, c\}$, $E(G) = \{ab, ac\}$

$V(H)$	$Cl_G(V(H))$	$Cl_{G2}(V(H))$
$V(G)$	$V(G)$	$V(G)$
φ	φ	φ
$\{a\}$	$V(G)$	$V(G)$
$\{b\}$	$\{a, b\}$	$V(G)$
$\{c\}$	$\{a, c\}$	$V(G)$
$\{a, b\}$	$V(G)$	$V(G)$
$\{a, c\}$	$V(G)$	$V(G)$
$\{b, c\}$	$V(G)$	$V(G)$

$C_{G2}(V(G)) = \{V(G), \varphi\}$

It is clear that a completely bipartite graph G is connected and the G_2-closure space $(V(G), C_{G2})$ on a graph G is connected.

Proposition 2.3

If a graph $G = (V(G), E(G))$ is completely bipartite $K_{s,t}$, then the Gm-closure space $(V(G), C_{Gm})$ on a graph G is connected.

Proof:

Let $G = (V(G), E(G))$ be a completely bipartite $K_{s,t}$ graph, then there exists two subsets of a vertex set $V(G)$, say X and Y and $|X| = s$, $|Y| = t$ such that $X \cup Y = V(G)$ and $X \cap Y = \varphi$. Without lose of generality, suppose $s \leq t$. For $x \in X$, we have $Cl_G(\{x\}) = \{x\} \cup Y$ and the structure is not G-topological closure space since if $x_1 \neq x_2 \in X$, $Cl_G(\{x_1\}) = \{x_1\} \cup Y$ and $Cl_G(\{x_2\}) = \{x_2\} \cup Y$ but $Cl_G(\{x_1\}) \cap Cl_G(\{x_2\}) = Y$ which is not in this structure. Continuous in this process, we have $Cl_{G(s-1)}(\{x_1, x_2, \ldots, x_{s-1} \cup Y) = V(G)$, and the obtained space is indiscrete which is connected.

Corollary 2.2

If a graph $G = (V(G), E(G))$ is completely r-partite; $r \geq 2$, then the G_m-closure space $(V(G), C_G)$ on a graph G is connected.

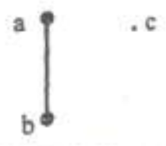
Remark 2.2

The condition of completely in above proposition is necessary for example.

Example 2.5

Let $G = (V(G), E(G))$ be a graph such that:

$V(G) = \{a, b, c\}$, $E(G) = \{ab\}$



$V(H)$	$Cl_G(V(H))$
$V(G)$	$V(G)$
φ	φ
$\{a\}$	$\{a, b\}$
$\{b\}$	$\{a, b\}$
$\{c\}$	$\{c\}$
$\{a, b\}$	$\{a, b\}$
$\{a, c\}$	$V(G)$
$\{b, c\}$	$V(G)$

$C_G(V(G)) = \{V(G), \varphi, \{c\}, \{a, b\}\}$

It is clear that the graph G is bipartite not complete but the G_m-closure spaces $(V(G), C_G)$ on a graph G is disconnected.

Proposition 2.4

If a graph $G = (V(G), E(G))$ has n components: $n \geq 2$, without isolated points, then the G_m-closure space $(V(G), C_{Gm})$ on a graph G is disconnected.

Proof:

Let $G = (V(G), E(G))$ be a graph and has n components, $V(G_1)$, $V(G_2)$, ..., $V(G_n)$. such that $V(G) = V(G_1) \cup V(G_2) \cup \ldots \cup V(G_n)$, and the number of vertices in every components $\leq k$ then $Cl_{G(k-1)}(V(G_i)) = V(G_i)$ for all $i = 1, 2, \ldots, n$. Hence $C_{G(k-1)} = \{V(G), \varphi, V(G_1), V(G_2), \ldots, V(G_n)\}$ which is disconnected.

Proposition 2.5

If a graph $G = (V(G), E(G))$ is completely K_n (cycle C_m, tree T_m, and wheel W_n), then the G_m-closure space $(V(G), C_{Gm})$ on a graph G is connected.

Proof:

Since the Gm-closure space $(V(G), C_{Gm})$ on completely K_n (cycle C_m, tree T_m, and wheel W_n) graph is indiscrete space, the proof holds.

Theorem 2.1

If $G = (V(G), E(G))$ is a graph, and $(V(G), C_{Gm})$ is G_m-closure space on G. For every subgrapg $H \subseteq G$, then

$$|Cl_{Gm}(V(H))| - 1 \leq \binom{|V(G)|}{2}; \qquad |V(G)| = n \geq 2$$

Proof:

By indication

When $n = 2$, then $|Cl_{Gm}(V(H)| - 1 = 2 - 1 = 1$

$$\text{And} \binom{|V(G)|}{2} = \binom{2}{2} = 1$$

the statement is true.

Suppose the statement is true when $n = k$. So

$$|Cl_{Gm}(V(H)| - 1 \leq \binom{|V(G)|}{2} = \binom{k}{2} = \frac{k(k-1)}{2}$$

To prove the statement is true when $n = k + 1$.

The maximum number of vertices that may adjacent with x is k, so

$$|Cl_{Gm}(V(H)| - 1 \leq k \leq \frac{k(k-1)}{2} = \binom{k+1}{2} = \binom{|V(G)|}{2}$$

Hence the statement is true.

Corollary 2.3

If $G = (V(G), E(G))$ is a graph, and $(V(G), C_{Gm})$ is G_m-closure space on G. For every subgrapg $H \subseteq G$, then

$$\sum_{i=1}^{n} |Cl_{Gm}(V(H))| - n \leq n\binom{|V(G)|}{2}; \qquad |V(G)| = n \geq 2$$

3. Connectedness In Directed Graphs

In this section, we study the graph closure operators in directed graph and investigate the relationships of connectedness between some types of graphs and obtained G_m-closure spaces from these graphs by using graph closure operators.

We introduce the definition of graph closure operators in digraphs as follows:

Definition 3.1

Let $G = (V(G), E(G))$ be a digraph, $P(V(G))$ its power set of all subgraph of G and $Cl_{Gm}:P(V(G)) \to P(V(G))$ is graph closure operator such that:

$$Cl_{Gm}(V(H)) = V(H) \cup \{v \in (G) \setminus V(H); \overrightarrow{hv} \in E(G)\} \text{ for all } h \in V(H)\}$$

Where $H = (V(H), E(H))$ be a subgraph of G. and the pair $(V(G), C_{Gm})$ is called G_m-closure space.

Example 3.1

Let $G = (V(G), E(G))$ be a digraph such that: $V(G) = \{a, b, c, d\}$, $E(G) = \{(a, b), (b, c)\}$

$$C_{G2}(V(G)) = \{V(G), \varphi, \{c\}, \{d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}\}$$

$V(H)$	$Cl_G(V(H))$	$Cl_{G2}(V(H))$	$V(H)$	$Cl_G(VH))$	$Cl_{G2}(V(H))$
$V(G)$	$V(G)$	$V(G)$	$\{a, d\}$	$\{a\ b, d\}$	$V(G)$
φ	φ	φ	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$
$\{a\}$	$\{a, b\}$	$\{a\ b, c\}$	$\{b, d\}$	$\{b, c, d\}$	$\{b, c, d\}$
$\{b\}$	$\{b, c\}$	$\{b, c\}$	$\{c, d\}$	$\{c, d\}$	$\{c, d\}$
$\{c\}$	$\{c\}$	$\{c\}$	$\{a\ b, c\}$	a, b, c	a, b, c
$\{d\}$	$\{d\}$	$\{d\}$	$\{a\ b, d\}$	$V(G)$	$V(G)$
$\{a, b\}$	$\{a\ b, c\}$	$\{a\ b, c\}$	$\{a\ c, d\}$	$V(G)$	$V(G)$
$\{a, c\}$	$\{a\ b, c\}$	$\{a\ b, c\}$	$\{b, c, d\}$	$\{b, c, d\}$	$\{b, c, d\}$

It is clear that the digraph G is disconnected and the G_2-closure space $(V(G), C_{G2})$ on a graph G is disconnected.

Proposition 3.1

If in a digraph $G = (V(G), E(G))$; $|V(G)| > 1$ has at least one isolated vertex, then the G_m-closure spaces $(V(G), C_{Gm})$ on a graph G is disconnected.

Proof:

Similarly of proof proposition (2.1)

Corollary 3.1

If in a digraph $G = (V(G), E(G))$; $|V(G)| > 1$ has at least one vertex $v \in V(G)$ such that $D^+(v) = D^-(v) = 0$, then the G_m-closure spaces $(V(G), C_{Gm})$ on a graph G is disconnected.

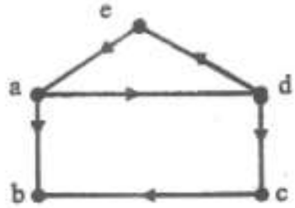
Remark 3.1

The corollary (2.1) is not satisfied in digraph. For example:

Example 3.2

Let $G = (V(G), E(G))$ be a digraph such that:

$V(G) = \{a, b, c, d, e\}$,
$E(G) = \{(a, b), (a, d), (c, b), (d, c), (d, e), (e, a)\}$



$V(H)$	$Cl_G(V(H))$	$Cl_{G2}(V(H))$	$V(H)$	$Cl_{G2}(V(H))$	$Cl_{G2}(V(H))$
$V(G)$	$V(G)$	$V(G)$	$\{d, e\}$	$\{a, c, d, e\}$	$V(G)$
φ	φ	φ	$\{a, b, c\}$	$\{a, b, c, d\}$	$V(G)$
$\{a\}$	$\{a, b, d\}$	$V(G)$	$\{a, b, d\}$	$V(G)$	$V(G)$
$\{b\}$	$\{b\}$	$\{b\}$	$\{a, b, e\}$	$\{a, b, d, e\}$	$V(G)$
$\{c\}$	$\{b, c\}$	$\{b, c\}$	$\{a, c, d\}$	$V(G)$	$V(G)$
$\{d\}$	$\{c, d, e\}$	$V(G)$	$\{a, c, e\}$	$V(G)$	$V(G)$
$\{e\}$	$\{a, e\}$	$\{a, b, d, e\}$	$\{a, d, e\}$	$V(G)$	$V(G)$
$\{a, b\}$	$\{a, b, d\}$	$V(G)$	$\{b, c, d\}$	$\{b, c, d, e\}$	$V(G)$
$\{a, c\}$	$\{a b, c, d\}$	$V(G)$	$\{b, c, e\}$	$\{a, b, c, e\}$	$V(G)$
$\{a, d\}$	$V(G)$	$V(G)$	$\{b, d, e\}$	$V(G)$	$V(G)$
$\{a, e\}$	$\{a, b, d, e\}$	$V(G)$	$\{c, d, e\}$	$V(G)$	$V(G)$
$\{b, c\}$	$\{b, c\}$	$\{b, c\}$	$\{a, b, c, d\}$	$V(G)$	$V(G)$
$\{b, d\}$	$\{b, c, d, e\}$	$V(G)$	$\{a, b, c, e\}$	$V(G)$	$V(G)$
$\{b, e\}$	$\{a, b, e\}$	$\{a, b d, e\}$	$\{a, b, d, e\}$	$V(G)$	$V(G)$
$\{c, d\}$	$\{b, c, d, e\}$	$V(G)$	$\{a, c, d, e\}$	$V(G)$	$V(G)$
$\{c, e\}$	a, b,c, e	$V(G)$	$\{b, c, d, e\}$	$V(G)$	$V(G)$

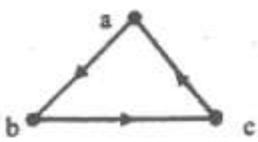
$C_{G2}(V(G)) = \{(V(G), \varphi, \{b\}, \{b, c\}, \{a, b, d, e\}\}$

It is clear that the digraph G has a vertex b of out-degree zero but the G_2-closure spaces $(V(G), C_{G2})$ on a graph G is connected.

Example 3.3

Let $G = (V(G), E(G))$ be a digraph such that:

$V(G) = \{a, b, c\}$, $E(G) = \{(a, b), (a, c), (b, c)\}$



$V(H)$	$Cl_{G}(V(H))$	$Cl_{G2}(V(H))$
$V(G)$	$V(G)$	$V(G)$
φ	φ	φ
$\{a\}$	$\{a, b\}$	$V(G)$
$\{b\}$	$\{b, c\}$	$V(G)$
$\{c\}$	$\{a, c\}$	$V(G)$
$\{a, b\}$	$V(G)$	$V(G)$
$\{a, c\}$	$V(G)$	$V(G)$
$\{b, c\}$	$V(G)$	$V(G)$

$C_{G2}(V(G)) = \{V(G), \varphi\}$

It is clear that in a digraph G all vertices has out-degree one and the G_m-closure space $(V(G), C_{G2})$ on a graph G is connected.

Proposition 3.2

In a digraph $G = (V(G), E(G))$, if the out-degree of all vertices of G is one, then the G_m-closure space $(V(G), C_{Gm})$ on a digraph G is connected.

Proof:

Let $G = (V(G), E(G))$ be a digraph and $D^{+}(v) = 1$ for all $v \in V(G)$. Then for every $v_i \in V(G)$ there exists a unique $v_j \in V(G)$ such that $Cl_G(\{v_i\}) = \{v_i, v_j\}$. So there exists a unique $v_k \in V(G)$ such that $Cl_G(\{v_i\}) \cap Cl_G(\{v_j\}) = \{v_j\} \in C_G(V(G))$. Hence $C_{G(n-1)}(V(G))$ is indiscrete space on G where n is the number of vertices of G. Which is connected.

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