



Reducing the Costs of Transporting Multiple Products (Linear Transport Problems) Using Excel QM

Article History	
Received:	10.01.2023
Revision:	15.01.2023
Accepted:	20.01.2023
Published:	03.02.2023
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Abstract: The transportation model is a well-recognized and applied algorithm in the distribution of products of logistics operations in enterprises. Multiple forms of solution are algorithmic and technological, which are applied to determine the optimal allocation of one type of product. In this research, the general formulation of the transport model by means of linear programming, where the optimal solution is integrated for different types of related products, and through a digital, dynamic, easy illustration Develops understanding of the Computer in Excel QM program. When choosing, the implementation of the form in the organization is provided.

Keywords: Transportation Problem , Multiple Product, Cost , Excel QM.

INTRODUCTION

The transport model is a sub-class of linear programming problem, whose objective is the efficient transport (minimum cost or path) of a product. The problem involves multiple origins for storage and multiple destinations for product delivery [1]. The bases of the transport model were formulated in [2] and various approaches developed since then. \ As an example, in [3] they present the springboard method that provides an alternative way to determine the information of the simplex method. In [4] they present the Gauss Jordan pivot method, this approach is faster than the simplex method, more general than the springboard method and simpler than both. The value of the parameters as fuzzy numbers was proposed in [5], in this work it is assumed that due to the nature of some problems, it is impossible to obtain an exact value of the parameters, obtaining only approximations. More recent approaches that deal with this problem can be found in [6] [7].

A more realistic approach requires the transport of multiple products with multi-target functions [8]. A variety of approaches have been proposed in the literature to deal with this problem. For example, in [9] they propose a goal programming approach. In [10] they apply the fuzzy theory, the results indicate that the solutions obtained by this method are always the best, in this same line [11] an algorithm was developed to identify non-dominant solutions. An approach using a hyperbolic membership function was proposed in [12], the results presented show that an approach based on a fuzzy operator and a hyperbolic membership function can give better results.

The most recent study is a solution by approximation with goal programming [13] through deviations that shape the goals according to priorities and weights they receive, this model was proposed for three products. Their development was algorithmic, under a hypothetical case of assigning shipping costs per unit to each customer under handling selection of sources. The model presented is a numerical hypothetical case for four products with different supply parameters and demand, which generate a model of sixty-four (64) variables, and an optimal solution in Excel Solver, which is you can use for more products with more applications robust as the Gams. The work is organized as follows: in the Section II describes basic concepts of the problem of transportation, Section III presents the model of proposed linear programming. Section IV presents the simulation results obtained. Lastly, in the Section V presents the conclusions.

II. Problem statement :

In a production system there are costs that add value to the product and others not, these are those of distribution which have a large percentage impact on the sale price, and therefore optimizing them is a challenge for the logistics area to obtain a competitive advantage and position itself in a market dynamic. For this, the logistics area defines its distribution channels locate distribution centers or warehouses at points strategic close to potential buyers or customers, to in turn determines the means of distribution or transportation taking into account that the unit costs of distribution

They vary by product according to their weight and/or volume. The model used to optimize these costs is the transport, which is modeled for a product, if required distribute k products, a calculation will have to be made by product k and integrate the shipment quantities into a distribution consolidation. Studies carried out to solve this problem are aimed at obtaining a solution for multiple products of approach by goals, which with some complexity would reach a solution close to the optimum [14]. The linear programming model presented in this document eliminates complexity and gives an optimal solution identifying units in inventory and demand dissatisfied in real time in turn allows to generate scenarios.

III. Formulation of the model by linear programming

The formulation of the model must meet the following conditions:

- 1- The objective function and constraints must be linear. Each vector of this function represents the cost per product, there are k rows depending on the number of products and make up a matrix kij.
- 2- The sum of the capacities of the sources must be equal to the sum of the requirements of the destinations by product k, If an inequality occurs, a fictitious row or column must be added with the absolute value of the missing or surplus units and zero costs (0), as follows:

If supply is greater than demand $O_{ik} > d_{jk}$ add a fictitious column with a number of units equal to $|O_{ik} - d_{jk}|$ and costs $C_{ijk} = 0$ for all $k=a,b,...,c$, $i=1,2,3,...,n$, $j=1,2,3,...,m$. and if the demand is greater than the offer $d_{jk} > O_{ik}$, it must be add a dummy row with a number of units equal to $|d_{jk} - O_{ik}|$, and costs $C_{ijk} = 0$ for all $k=a,b,...,c$, $i=1,2,3,...,n$, $j=1,2,3,...,m$.

The model is represented in a matrix see Fig. 1. The rows identify the distribution centers i and the parameters of supply O_{ik} Columns identify customers j and demand parameters d_{jk} .

In the intersection, the costs C_{ijk} per unit and the solution variables X_{ijk} are recorded, for the shipment of a quantity of products k, from a distribution center i to a customer j. For the development of this model, the matrix proposed in [14] was taken as a reference.

Indication

- i indicator that identifies the distribution center, for all $i=1,2,3,...,n$.
- j indicator that identifies the customers, for all $j=1,2,3,...,m$
- k indicator that identifies the type of product, for all $a,b,c,...,k$

parameters

- O_{ik} Quantity offered by the distribution center i of the product k.
- C_{ijk} Cost to ship a unit from distribution center i to customer j of product k.
- d_{jk} Quantity demanded by customer j of product k.

TABLE 1: Shows the cost matrix for transport model

		Destinations								supply for products
		products(k)	D1		D2		Dm		
			Required quantity	shipping costs	Required quantity	shipping costs		Required quantity	shipping costs	
distribution centers or warehouses	Dc1	a	x11a	c11a	x12a	c12a	x1ma	c1ma	O1a
		b	x11b	c11b	x12b	c12b	x1mb	c1mb	O1b
		K	x11k	c11k	x12k	c12k	x1mk	c1mk	O1k
	Dc2	a	x12a	c12a	x22a	c22a	x2ma	c2ma	O2a
		b	x12b	c12b	x22b	c22b	x2mb	c2mb	O2b
		K	x12k	c12k	x22k	c22k	x2mk	c2mk	O2k
		
	Dcn	a	xn1a	cn1a	xn2a	cn2a	xnma	cnma	Ona
		b	xn1b	cn1b	xn2b	cn2b	xnmb	cnmb	Onb
		K	xn1k	cn1k	xn2k	cn2k	xnmk	cnmk	Onk
	demand for products	d1a		d2a		dma			
		d1b		d2b		dmb			
		d1k		d2k			dmk			

Variables

X_{ijk} Number of units shipped from the distribution center i , to customer j of product k .

Objective function

$$Min Z_1 = \sum_{i=1}^n \sum_{j=1}^m \sum_{k=a}^K C_{ijk} X_{ijk} \quad \left\{ \begin{array}{l} k = a, b, c, \dots, K \\ i = 1, 2, 3, \dots, n \\ j = 1, 2, 3, \dots, m \end{array} \right\} \dots(1)$$

Subject to :

$$\sum_{i=1}^n X_{ijk} = O_{ik} \quad \left\{ \begin{array}{l} k = a, b, c, \dots, K \\ i = 1, 2, 3, \dots, n \\ j = 1, 2, 3, \dots, m \end{array} \right\} \dots (2)$$

$$\sum_{j=1}^m X_{ijk} = d_{jk} \quad \left\{ \begin{array}{l} k = a, b, c, \dots, K \\ i = 1, 2, 3, \dots, n \\ j = 1, 2, 3, \dots, m \end{array} \right\} \dots (3)$$

$$O_{ik} = d_{jk} \quad \left\{ \begin{array}{l} k = a, b, c, \dots, K \\ i = 1, 2, 3, \dots, n \\ j = 1, 2, 3, \dots, m \end{array} \right\} \dots(4)$$

$$x_{ijk} \geq 0 \quad \text{for all } i, j, k \quad \dots(5)$$

The object of equation (1) is to quantify the minimum value of the total costs of transporting products K from the different sources i , to different destinations j . The equation. (2) is the representation of the offer of each one of the distribution centers i to the different clients j of the different products k . (3) It is the representation of the demand of each one of the clients j of the different products k to the distribution centers i . (4) It is the balance equation; the sum of the offer of the product k of the distribution centers i , must be equal to the sum of the demand of the clients j . (5) It is the equation of non-negativity, it means that the values must be zero (0) or positive values.

Iv. Simulation result

To give applicability of the model, we have solved the example of a numerical distribution problem. A food products company that is a distributor of products (A, B, C, D), has four points Distribution (B1, B2, B3, and B4) and distributes its products to customers (C1, C2, C3, and C4).

1. Model input parameters

- Shipping costs per unit. The logistics area has determined the following rates in monetary units per transport each unit, these C_{ijk} rates are calculated with based on the weight and/or volume of the product and/or the distance between the warehouse and the customer, see Table 2.
- Warehouse inventories. The company has in the warehouses product inventory (a, b, c and d) in warehouses (B1, B2, B3 and B4), warehouse B3 does not have product a in inventory, and warehouse B4 product c, see Table 3.
- Customer demand for products. Customers demand products (A, B, C and D) from the company, customer C2 does not require product b and customer C4 product d, see Table 4.

TABLE 2. Costs Per Unit Of Transporting Product K From Each Warehouse To The Customers

		DEMAND				
		A	C1	C2	C3	C4
SUPPLY	B1	5	4	3	2	
	B2	3	8	7	6	
	B3	2	5	6	7	
	B4	9	3	2	9	

		DEMAND				
		B	C1	C2	C3	C4
SUPPLY	B1	7.5	6	4.5	3	
	B2	4.5	12	11	9	
	B3	3	8	9	11	
	B4	14	5	3	14	

		DEMAND				
		C	C1	C2	C3	C4
SUPPLY	B1	10	8	6	4	
	B2	6	16	14	12	
	B3	4	10	12	14	
	B4	18	6	4	18	

		DEMAND				
		D	C1	C2	C3	C4
SUPPLY	B1	4	3	2.4	1.6	
	B2	2.4	6	5.6	4.8	
	B3	1.6	4	4.8	5.6	
	B4	7.2	2	1.6	7.2	

TABLE 3. UNITS IN INVENTORY.

warehouses	PRODUCTS	A	B	C	D
	B1	50	600	350	220
B2	30	300	600	210	
B3	0	500	320	160	
B4	50	800	0	510	
TOTAL	130	2200	1270	1100	

TABLE 4. DEMANDED UNITS.

PRODUCTS	COSTEMERS	C1	C2	C3	C4	TOTAL
	A	40	30	60	70	200
B	500	0	500	500	1500	
C	350	400	250	450	1450	
D	240	740	100	0	1080	

B. Fulfillment of conditions to formulate the model

• Comparison of supply and demand for product k. I know verifies that the sum of the demand for products k of the different customers j, is equal to the sum of the existence of product k in warehouses i, see Table 5.

TABLE 5. COMPARISON OF THE SUPPLY AND DEMAND OF PRODUCTS k FOR BALANCING

PRODUCTS	SUPPLY TOTOAL	DMAND TOTOAL	ABSOLUTE VALUE	DIFFERINCES
A	130	200	130-200	70
B	2200	1500	2200-1500	700
C	1270	1450	1270-1450	180
D	1100	1080	1100-1080	20

Balancing supply and demand for product k. It is determined by subtracting the absolute value of supply minus demand. As a result of this operation, the following situations may occur:

- ✓ Supply equals demand. In this case it is balanced the product k.
- ✓ adding dummy columns for client C2 with 700 units of product b and 20 units to customer C4, the corresponding costs of the columns will be equal to zero C_{i2b} AND $C_{i4d}=0$
- ✓ The demand is greater than the supply. In this problem it is given for products a and c than those stored in warehouses B3 and B4. For this, the model is balanced by adding rows fictitious for warehouse B3 with 70 units of the product to and 180 units of product c in warehouse B4. The costs of the row will be equal to zero C_{3ja} and $c_{4jc} = 0$, see Table 6.

TABLE 6. MATRIX OF PRODUCTS ADJUSTED WITH THE PARAMETERS OF SUPPLY, DEMAND AND COSTS

		DEMAND				
		a	C1	C2	C3	C4
SUPPLY	B1	5	4	3	2	50
	B2	3	8	7	6	30
	B3	0	0	0	0	70
	B4	9	3	2	9	50
		40	30	60	70	

		DEMAND				
		b	C1	C2	C3	C4
SUPPLY	B1	7.5	0	4.5	3	600
	B2	4.5	0	10.5	9	300
	B3	3	0	9	10.5	500
	B4	13.5	0	3	13.5	800
		500	700	500	500	

		DEMAND				
		c	C1	C2	C3	C4
SUPPLY	B1	10	8	6	4	350
	B2	6	16	14	12	600
	B3	4	10	12	14	320
	B4	0	0	0	0	180
		350	400	250	450	

		DEMAND				
		d	C1	C2	C3	C4
SUPPLY	B1	4	3.2	2.4	0	220
	B2	2.4	6.4	5.6	0	210
	B3	1.6	4	4.8	0	160
	B4	7.2	2.4	1.6	0	510
		240	740	100	20	

Mathematical formulation of the proposed model

The basis of the formula is linear programming and we will solve the following linear transmission problem with some properties and it will be solved electronically using Excel QM. We will explain in a simple way the data entry process through the following steps and the windows included in the program:

Step 1: Initial data entry window to solve the transportation problem, as in Figure 1

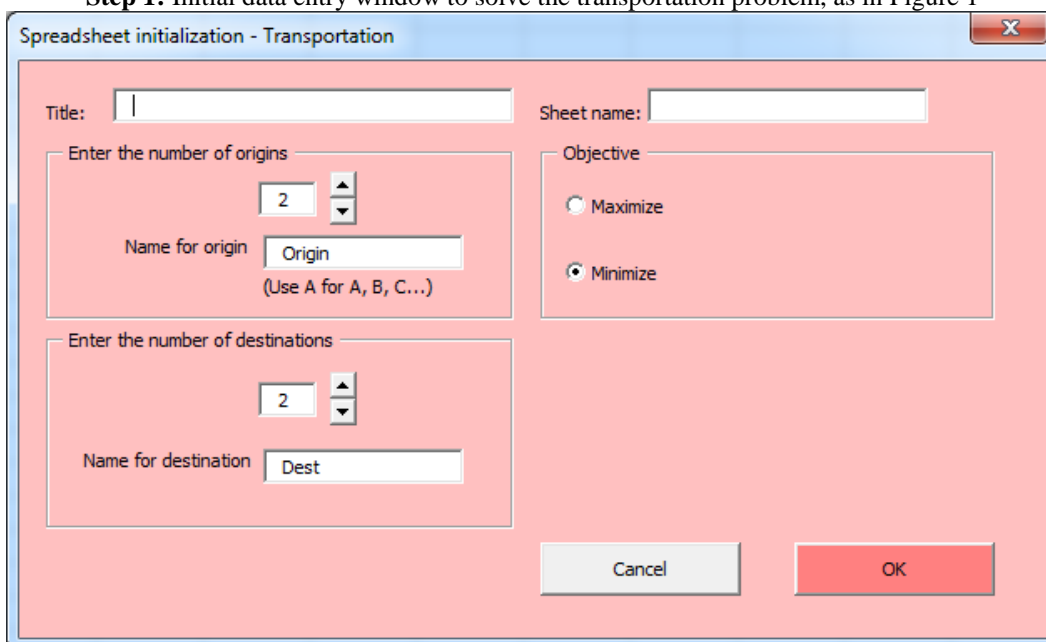


Fig .1 Entry Window

Step2: The process of entering data according to the distribution centers of the different products until they reach the customers and according to the numbers shown in the previous example, and Figure No. 2 illustrates this .

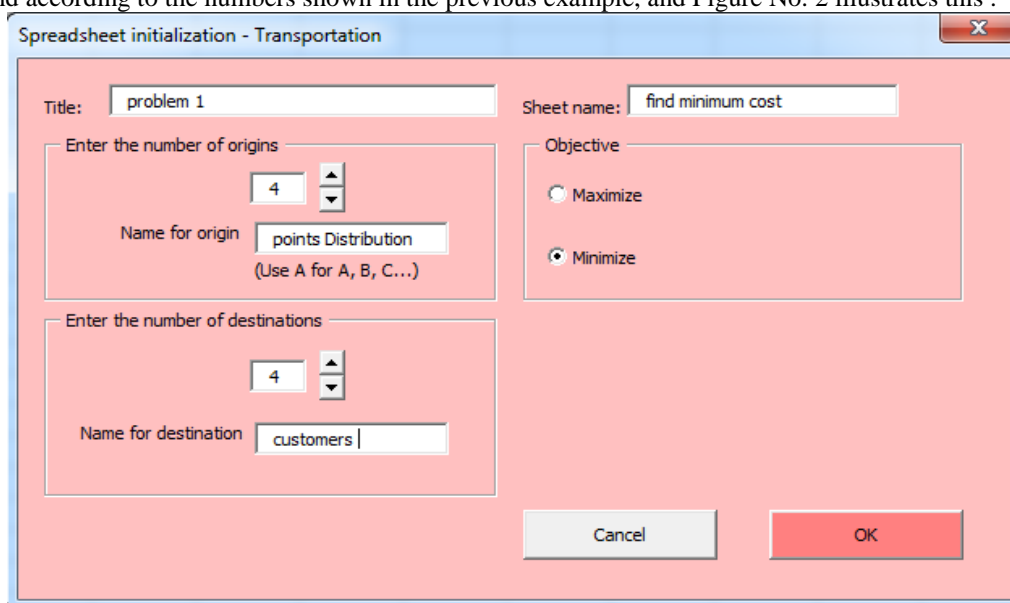


Fig.2 process of entering data

Step3: In this step, Figure 3 shows the linear programming model for transportation problems, and through it we can enter all the data related to costs, supply quantities, and demand, which will be transferred from demand centers to customers and with the lowest possible costs in case the model is solved by the well-known classic methods that are included in the program

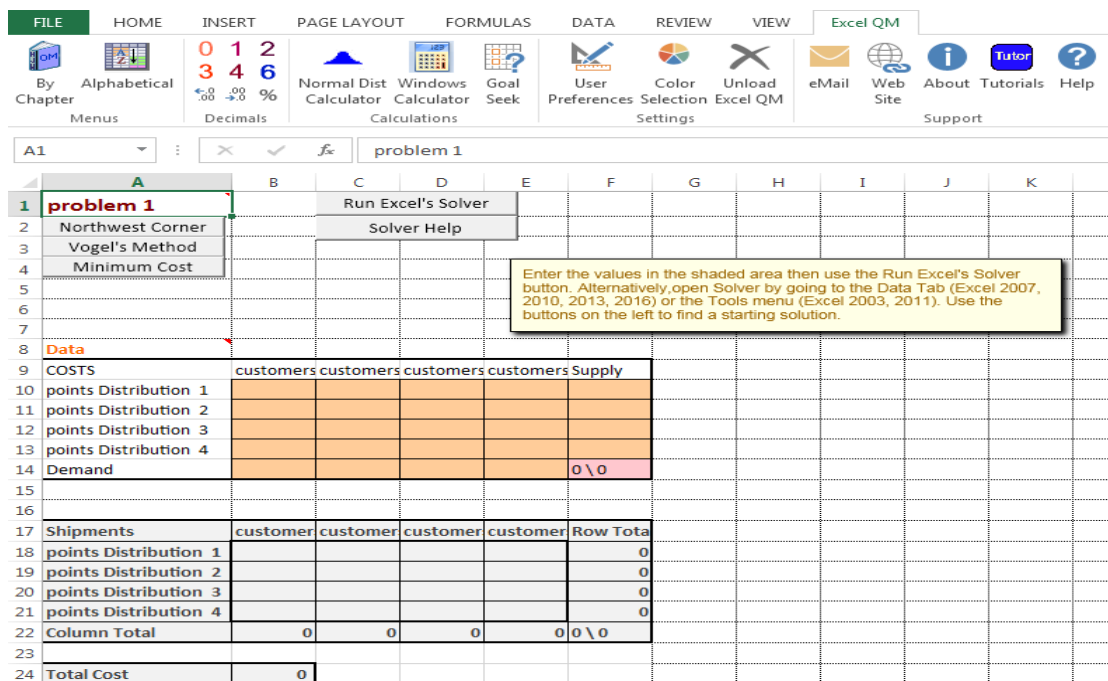


Fig.3 model for transportation problems

Step 4: Enter the costs of transporting product A, supply quantities provided by the four warehouses, as well as customer demand quantities And as in the following table No. 7

Table 7 : costs of transporting product A

Data		Demand is greater than supply! Recreate this mod				
COSTS	customers	customers	customers	customers	Supply	
points Distribution 1	5	4	3	2	50	
points Distribution 2	3	8	7	6	30	
points Distribution 3	2	5	6	7	0	
points Distribution 4	9	3	2	9	50	
Demand	40	30	60	70	200 \ 130	

Not : Demand is greater than supply! Recreate this model with an extra (dummy) row that has a supply equal to the difference between total supply and total demand. Therefore, the balance condition must be available in the linear transportation problem, and then we find the optimal solution that achieves the lowest total costs for transporting multiple products from their distribution centers to customers.

Step 5 : Accordingly, we create an imaginary column or row, its transportation costs are 0, and its quantities, according to the numerical differences that occur between the supply quantity and the order quantity, And TABLE 5. shows the process of conducting the balance and the differences in the quantities that we can use according to theoretical calculations. As for solving the problem of transporting multiple products, it was through the Excel QM program - solving transportation problems and then using the best methods to reduce costs, which is the Vogel method.

Step6 : We repeat all the previously described steps for all products and get the following results.
Result :

- 1- The amount of transporting product a from its sources to its distribution centers at the lowest transportation price, as the value of transporting product a for all customers was \$290, which is the lowest transportation cost obtained, and as shown in the following table 8, where the table included the cost of transporting the product from the source to the distribution centers, according to the quantities The required and optimal quantities to be transported were allocated according to the Vogel method and using the transport form in the Excel QM program

Table 8 : Solve the transport model using linear programming for product A

Data					
COSTS	customers 1	customers 2	customers 3	customers 4	Supply
point distribution 1	5	4	3	2	50
point distribution 2	3	8	7	6	30
point distribution 3	0	0	0	0	70
point distribution 4	9	3	2	9	50
Demand	40	30	60	70	200 \ 200
The problem is balanced.					
Shipments	customers 1	customers 2	customers 3	customers 4	Row Total
point distribution 1				50	50
point distribution 2	30				30
point distribution 3	10	30	10	20	70
point distribution 4			50		50
Column Total	40	30	60	70	200 \ 200
Total Cost					
	290				

- 2- The amount of transporting product B from its sources to its distribution centers at the lowest transportation price, as the value of transporting product B for all customers was \$4,500, which is the lowest transportation cost obtained, and as shown in the following table 9, where the table included the cost of transporting the product from the source to the distribution centers, according to the quantities The required and optimal quantities to be transported were allocated according to the Vogel method and using the transport form in the Excel QM program.

Table 9 : Solve the transport model using linear programming for product B

Data					
COSTS	customers 1	customers 2	customers 3	customers 4	Supply
point distribution 1	7.5	0	4.5	3	600
point distribution 2	4.5	0	10.5	9	300
point distribution 3	3	0	9	10.5	500
point distribution 4	13.5	0	3	13.5	800
Demand	500	700	500	500	2200 \ 2200
The problem is balanced.					
Shipments	customers 1	customers 2	customers 3	customers 4	Row Total
point distribution 1		100		500	600
point distribution 2		300			300
point distribution 3	500				500
point distribution 4		300	500		800
Column Total	500	700	500	500	2200 \ 2200
Total Cost					
	4500				

- 3- The amount of transporting product c from its sources to its distribution centers at the lowest transportation price, as the value of transporting product c for all customers was \$10200, which is the lowest transportation cost obtained, and as shown in the following table 10, where the table included the cost of transporting the product from the source to the distribution centers, according to the quantities The required and optimal quantities to be transported were allocated according to the Vogel method and using the transport form in the Excel QM program.

Table 10 : Solve the transport model using linear programming for product c

Data					
COSTS	customers 1	customers 2	customers 3	customers 4	Supply
point distribution 1	10	8	6	4	350
point distribution 2	6	16	14	12	600
point distribution 3	4	10	12	14	320
point distribution 4	0	0	0	0	180
Demand	350	400	250	450	1450 \ 1450
The problem is balanced.					
Shipments	customers 1	customers 2	customers 3	customers 4	Row Total
point distribution 1				350	350
point distribution 2	350		150	100	600
point distribution 3		220	100		320
point distribution 4		180			180
Column Total	350	400	250	450	1450 \ 1450
Total Cost					
	10200				

- 4- The amount of transporting product d from its sources to its distribution centers at the lowest transportation price, as the value of transporting product d for all customers was \$2856, which is the lowest transportation cost obtained, and as shown in the following table 11, where the table included the cost of transporting the product from the source to the distribution centers, according to the quantities The required and optimal quantities to be transported were allocated according to the Vogel method and using the transport form in the Excel QM program.

Table 11 : Solve the transport model using linear programming for product D

Data					
COSTS	customers 1	customers 2	customers 3	customers 4	Supply
point distribution 1	4	3.2	2.4	0	220
point distribution 2	2.4	6.4	5.6	0	210
point distribution 3	1.6	4	4.8	0	160
point distribution 4	7.2	2.4	1.6	0	510
Demand	240	740	100	20	1100 \ 1100
The problem is balanced.					
Shipments	customers 1	customers 2	customers 3	customers 4	Row Total
point distribution 1		100	100	20	220
point distribution 2	210				210
point distribution 3	30	130			160
point distribution 4		510			510
Column Total	240	740	100	20	1100 \ 1100
Total Cost					
	2856				

- 5- The results obtained by using the model solution in Excel QM. The model gives the optimal solution for the shipping quantities for product k, and the optimal and total economic value for transporting the four products amounted to 17846 \$.
- 6- The optimal quantities of products A, B, C, and D were obtained, which will be transported from their sources to the destinations requesting them, and at the lowest costs obtained, according to the transportation cost matrix and the following table 12 that represents the quantity allocated to each product with its transportation cost and the total economic cost that was reached by means of Solving the mathematical model using Excel QM

TABLE 12 : represents the quantity allocated to each product with its transportation cost and the total economic cost

	optimal quantities	transportation cost per unit	total economic cost
K=a	X14 = 50	C14 = 2	290\$
	X21= 30	C21= 3	
	X31 = 10	C31 = 0	
	X32 = 30	C32 = 0	
	X33 = 10	C33 = 0	
	X34 = 20	C34 = 0	
	X43 = 50	C43 = 2	
K=b	optimal quantities	transportation cost per unit	total economic cost
	X12= 100	C12=0	4500\$
	X14 = 500	C14 = 3	
	X22 = 300	C22 = 0	
	X31 = 500	C31= 3	
	X42 = 300	C42 = 0	
	X43 = 500	C43 = 3	
K= c	optimal quantities	transportation cost per unit	
	X14 = 350	C14= 4	10200\$
	X21 = 350	C21 = 6	
	X23 = 150	C23 = 14	
	X24 = 100	C24 = 12	
	X32 = 220	C32 = 10	
	X33 = 100	C33 = 12	
X42 = 180	C42 = 0		
K= d	optimal quantities	transportation cost per unit	total economic cost
	X12 = 100	C12 = 3.2	2856\$
	X13 = 100	C13 = 2.4	
	X14 = 20	C14 = 0	
	X21 = 210	C21 = 2.4	
	X31 = 30	C31 = 1.6	
	X32 = 130	C32 = 4	
X42 = 510	C42 = 2.4		
		Total	17846\$

CONCLUSIONS:

In this work, the optimal solution was obtained by product each k and the results were compared with the model proposed, obtaining the same solution. Therefore, it can ensure that the model meets the objective by integrate all the products in a single model and give a optimal solution. Additionally, in the proposed model identify the quantities x_{ijk} of shipment to each of the customers, identifying units in inventory and unsatisfied demand. As well as the costs per product and totals. The proposed model is easy to apply, exceeds the complexity of similar solution proposals, with this application can generate results in real time with high added values and provide solutions to scenarios multiple, for this one(s) becomes a parameter variable(s) and/or offer parameters are modified and demand. The proposed formulation can be applied to problems with a large number of variables, using higher capacity software such as GAMS.

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